CHAT Pre-Calculus – First Semester Extra Credit

Review of Trig Functions

Instructions: Complete the following problems on separate paper. Show your work. Be neat and circle your answers. Omit the problems that are crossed out.

Note: This is an open book activity. You can use your notes and your book.

Extra Credit Amount:

5 points for scores of 50% - 60%
8 points for scores of 61% - 84%
10 points for scores of 85% - 100%

CHAT Pre-Calculus
Extra Credit

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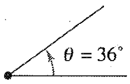
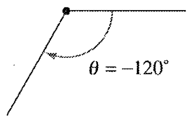
In Exercises 1 and 2, determine two coterminal angles in degree measure (one positive and one negative) for each angle.

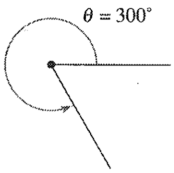
1. (a)



(b)



2. (a)



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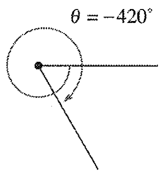
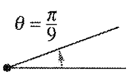


In Exercises 3 and 4, determine two coterminal angles in radian measure (one positive and one negative) for each angle.

3. (a)



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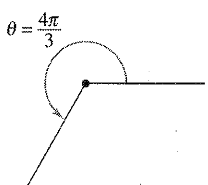
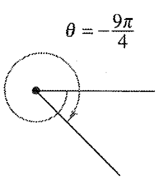


4. (a)



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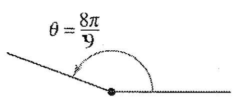


In Exercises 5 and 6, rewrite each angle in radian measure as a multiple of π and as a decimal accurate to three decimal places.

5. (a) 30° (b) 150° (c) 315° (d) 120°

6. (a) −20° (b) −240° (c) −270° (d) 144°

In Exercises 7 and 8, rewrite each angle in degree measure.

7. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$ (c) $-\frac{7\pi}{12}$ (d) -2.367

8. (a) $\frac{7\pi}{3}$ (b) $-\frac{11\pi}{30}$ (c) $\frac{11\pi}{6}$ (d) 0.438

9. Let r represent the radius of a circle, θ the central angle (measured in radians), and s the length of the arc subtended by the angle. Use the relationship $s = r\theta$ to complete the table.

r	8 ft	15 in.	85 cm		
s	12 ft			96 in.	8642 mi
θ		1.6	$\frac{3\pi}{4}$	4	$\frac{2\pi}{3}$

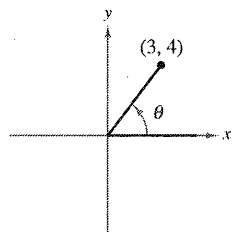
~~**10.** *Angular Speed* A car is moving at the rate of 50 miles per hour, and the diameter of its wheels is 2.5 feet.~~

~~(a) Find the number of revolutions per minute that the wheels are rotating.~~

~~(b) Find the angular speed of the wheels in radians per minute.~~

In Exercises 11 and 12, determine all six trigonometric functions for the angle θ.

11. (a)



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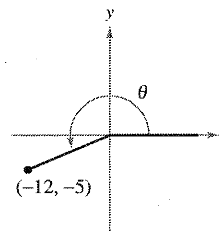
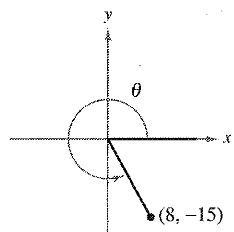


12. (a)



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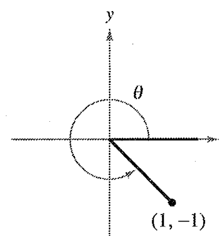


In Exercises 13 and 14, determine the quadrant in which θ lies.

13. (a) $\sin\theta < 0$ and $\cos\theta < 0$

(b) $\sec\theta > 0$ and $\cot\theta < 0$

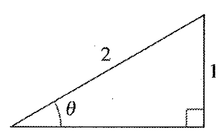
14. (a) $\sin\theta > 0$ and $\cos\theta < 0$

(b) $\csc\theta < 0$ and $\tan\theta > 0$

In Exercises 15–18, evaluate the trigonometric function.

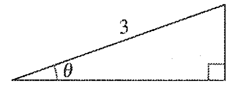
15. $\sin\theta = \frac{1}{2}$

$\cos\theta =$ ▒



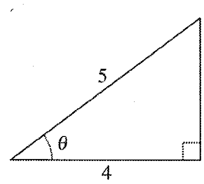
16. $\sin\theta = \frac{1}{3}$

$\tan\theta =$ ▒



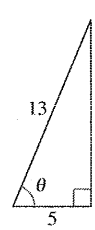
17. $\cos\theta = \frac{4}{5}$

$\cot\theta =$ ▒



18. $\sec\theta = \frac{13}{5}$

$\csc\theta =$ ▒



In Exercises 19–22, evaluate the sine, cosine, and tangent of each angle *without* using a calculator.

19. (a) $60°$
(b) $120°$
(c) $\frac{\pi}{4}$
(d) $\frac{5\pi}{4}$

20. (a) $-30°$
(b) $150°$
(c) $-\frac{\pi}{6}$
(d) $\frac{\pi}{2}$

21. (a) $225°$
(b) $-225°$
(c) $\frac{5\pi}{3}$
(d) $\frac{11\pi}{6}$

22. (a) $750°$
(b) $510°$
(c) $\frac{10\pi}{3}$
(d) $\frac{17\pi}{3}$

In Exercises 23–26, use a calculator to evaluate each trigonometric function. Round your answers to four decimal places.

23. (a) $\sin 10°$
(b) $\csc 10°$

24. (a) $\sec 225°$
(b) $\sec 135°$

25. (a) $\tan \frac{\pi}{9}$
(b) $\tan \frac{10\pi}{9}$

26. (a) $\cot(1.35)$
(b) $\tan(1.35)$

In Exercises 27–30, find two solutions of each equation. Give your answers in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

27. (a) $\cos \theta = \frac{\sqrt{2}}{2}$
(b) $\cos \theta = -\frac{\sqrt{2}}{2}$

28. (a) $\sec \theta = 2$
(b) $\sec \theta = -2$

29. (a) $\tan \theta = 1$
(b) $\cot \theta = -\sqrt{3}$

30. (a) $\sin \theta = \frac{\sqrt{3}}{2}$
(b) $\sin \theta = -\frac{\sqrt{3}}{2}$

In Exercises 31–38, solve the equation for θ ($0 \le \theta < 2\pi$).

31. $2\sin^2 \theta = 1$

32. $\tan^2 \theta = 3$

33. $\tan^2 \theta - \tan \theta = 0$

34. $2\cos^2 \theta - \cos \theta = 1$

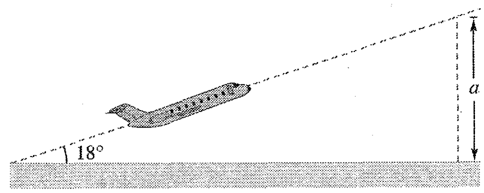
35. $\sec \theta \csc \theta = 2 \csc \theta$

36. $\sin \theta = \cos \theta$

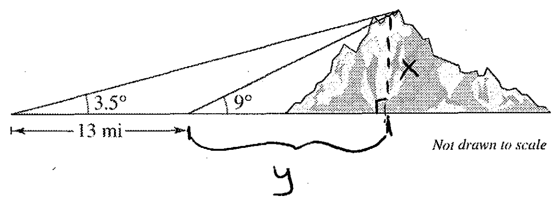
37. $\cos^2 \theta + \sin \theta = 1$

38. $\cos \frac{\theta}{2} - \cos \theta = 1$

39. ***Airplane Ascent*** An airplane leaves the runway climbing at an angle of $18°$ with a speed of 275 feet per second (see figure). Find the altitude a of the plane after 1 minute.

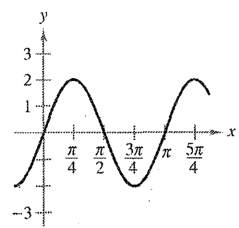


40. ***Height of a Mountain*** In traveling across flat land, you notice a mountain directly in front of you. Its angle of elevation (to the peak) is $3.5°$. After you drive 13 miles closer to the mountain, the angle of elevation is $9°$. Approximate the height of the mountain.

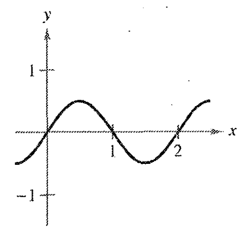


In Exercises 41–44, determine the period and amplitude of each function.

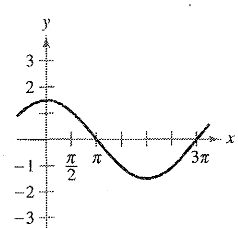
41. (a) $y = 2\sin 2x$



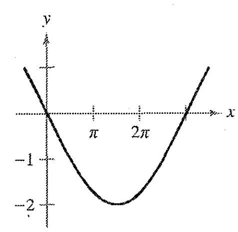
(b) $y = \frac{1}{2}\sin \pi x$



42. (a) $y = \frac{3}{2}\cos \frac{x}{2}$



(b) $y = -2\sin \frac{x}{3}$



43. $y = 3\sin 4\pi x$

44. $y = \frac{2}{3}\cos \frac{\pi x}{10}$

In Exercises 45–48, find the period of the function.

45. $y = 5 \tan 2x$

46. $y = 7 \tan 2\pi x$

47. $y = \sec 5x$

48. $y = \csc 4x$

Writing **In Exercises 49 and 50, use a graphing utility to graph each function f in the same viewing window for $c = -2$, $c = -1$, $c = 1$, and $c = 2$. Give a written description of the change in the graph caused by changing c.**

49. (a) $f(x) = c \sin x$
(b) $f(x) = \cos(cx)$
(c) $f(x) = \cos(\pi x - c)$

50. (a) $f(x) = \sin x + c$
(b) $f(x) = -\sin(2\pi x - c)$
(c) $f(x) = c \cos x$

In Exercises 51–62, sketch the graph of the function.

51. $y = \sin \dfrac{x}{2}$

52. $y = 2 \cos 2x$

53. $y = -\sin \dfrac{2\pi x}{3}$

54. $y = 2 \tan x$

55. $y = \csc \dfrac{x}{2}$

56. $y = \tan 2x$

57. $y = 2 \sec 2x$

58. $y = \csc 2\pi x$

59. $y = \sin(x + \pi)$

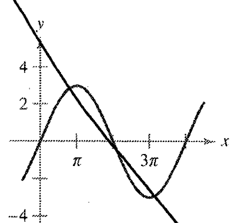
60. $y = \cos\left(x - \dfrac{\pi}{3}\right)$

61. $y = 1 + \cos\left(x - \dfrac{\pi}{2}\right)$

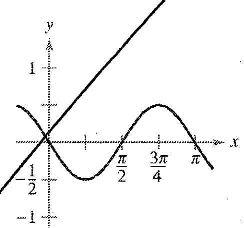
62. $y = 1 + \sin\left(x + \dfrac{\pi}{2}\right)$

Graphical Reasoning **In Exercises 63 and 64, find a, b, and c such that the graph of the function matches the graph in the figure.**

63. $y = a \cos(bx - c)$



64. $y = a \sin(bx - c)$



65. *Think About It* Sketch the graphs of $f(x) = \sin x$, $g(x) = |\sin x|$, and $h(x) = \sin(|x|)$. In general, how are the graphs of $|f(x)|$ and $f(|x|)$ related to the graph of f?

66. *Think About It* The model for the height h of a Ferris wheel car is

$h = 51 + 50 \sin 8\pi t$

where t is measured in minutes. (The Ferris wheel has a radius of 50 feet.) This model yields a height of 51 feet when $t = 0$. Alter the model so that the height of the car is 1 foot when $t = 0$.

67. *Sales* The monthly sales S (in thousands of units) of a seasonal product are modeled by

$$S = 58.3 + 32.5 \cos \frac{\pi t}{6}$$

where t is the time (in months) with $t = 1$ corresponding to January. Use a graphing utility to graph the model for S and determine the months when sales exceed 75,000 units.

68. *Investigation* Two trigonometric functions f and g have a period of 2, and their graphs intersect at $x = 5.35$.

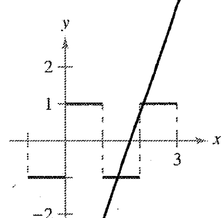
(a) Give one smaller and one larger positive value of x where the functions have the same value.

(b) Determine one negative value of x where the graphs intersect.

(c) Is it true that $f(13.35) = g(-4.65)$? Give a reason for your answer.

Pattern Recognition **In Exercises 69 and 70, use a graphing utility to compare the graph of f with the given graph. Try to improve the approximation by adding a term to $f(x)$. Use a graphing utility to verify that your new approximation is better than the original. Can you find other terms to add to make the approximation even better? What is the pattern? (In Exercise 69, sine terms can be used to improve the approximation and in Exercise 70, cosine terms can be used.)**

69. $f(x) = \dfrac{4}{\pi}\left(\sin \pi x + \dfrac{1}{3} \sin 3\pi x\right)$



70. $f(x) = \dfrac{1}{2} - \dfrac{4}{\pi^2}\left(\cos \pi x + \dfrac{1}{9} \cos 3\pi x\right)$