Special Functions
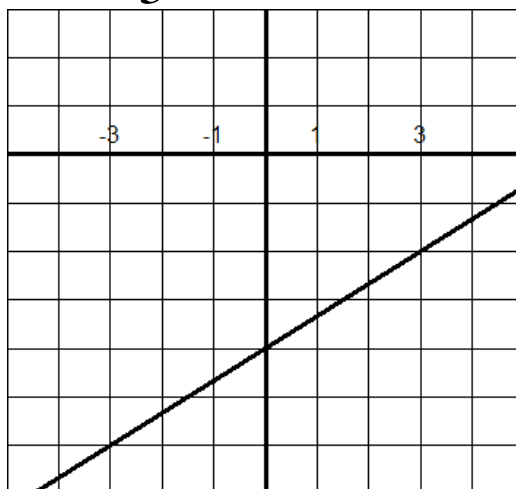
Linear Functions

The graph of every linear function $f(x) = mx + b$ is a line with slope $= m$ and y-intercept $(0, b)$. The graph of a linear function has the following features:

- Domain: All real numbers
- Range: All real numbers
- One y-intercept at $(0, b)$
- The graph is increasing if $m>0$, decreasing if $m<0$, and constant if $m = 0$.

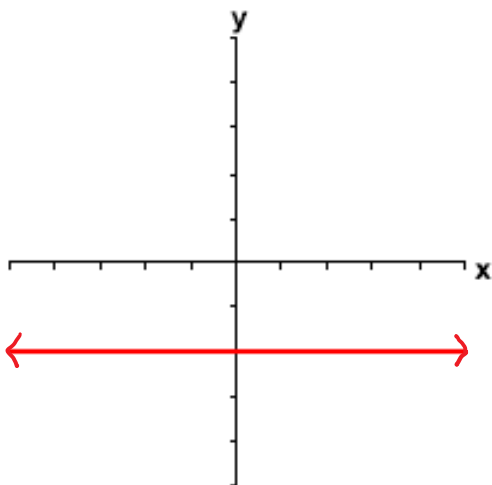**Example**: Graph the linear function $y = \dfrac{2}{3}x - 4$.

Solution: The slope is $\dfrac{2}{3}$ and y-intercept $(0,-4)$.
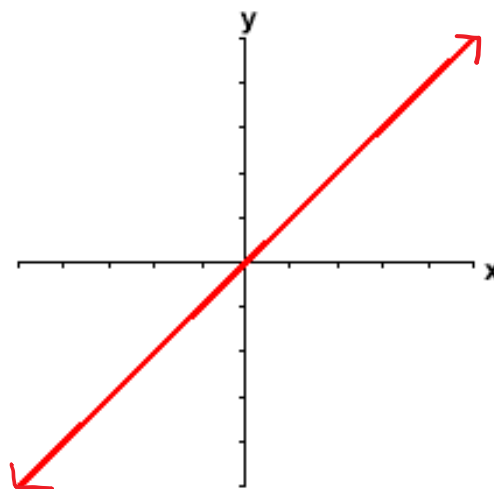
There are 2 special linear functions:

| Constant Function | Identity Function |

A constant function has the form $f(x) = c$.

The graph is a horizontal line.

The identity function has the form $f(x) = x$.

The graph is a line with slope 1 and passing through the origin.

Greatest Integer Function

The greatest integer function, denoted by $y = [x]$ where

$$[x] = \text{the greatest integer} \leq x$$

(If the number is *not* an integer, we want the integer to the left of that number on the number line.)

This is commonly referred to as a step function. It is the type of function telephone companies use to bill us for long distance calls or cell phone minutes.
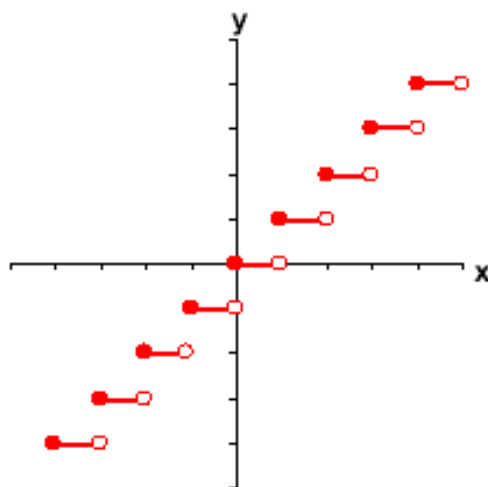
Example: Find the following:

a) [3]

   solution: 3

b) [5.9]

   solution: 5

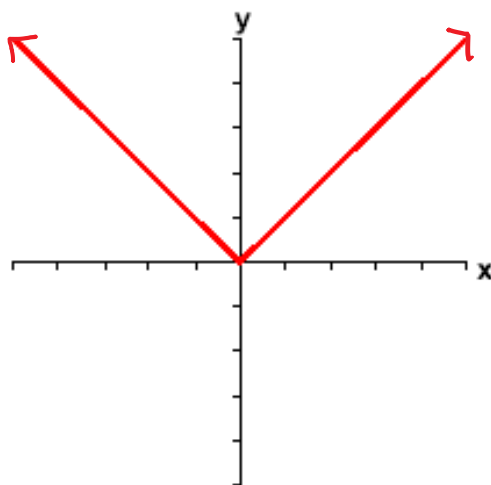c) [-4.2]

   solution: -5

Greatest Integer Function

$$y = [x]$$

The Absolute Value Function

The absolute value function $f(x) = |x|$ is always V-shaped. Make a table to find the vertex.

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3

$$f(x) = |x|$$

The Exponential Function

An exponential function is of the form $f(x) = a^x$ where $a > 0$, $a \neq 1$, and x is any real number.

*The reason $a \neq 1$ is because 1 raised to any power is 1, so we would have f(x) = 1, which is a horizontal line.

$$f(x) = 2^x$$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8