Trig Functions

Definition: If x is an angle in radians, then the following are trigonometric functions:

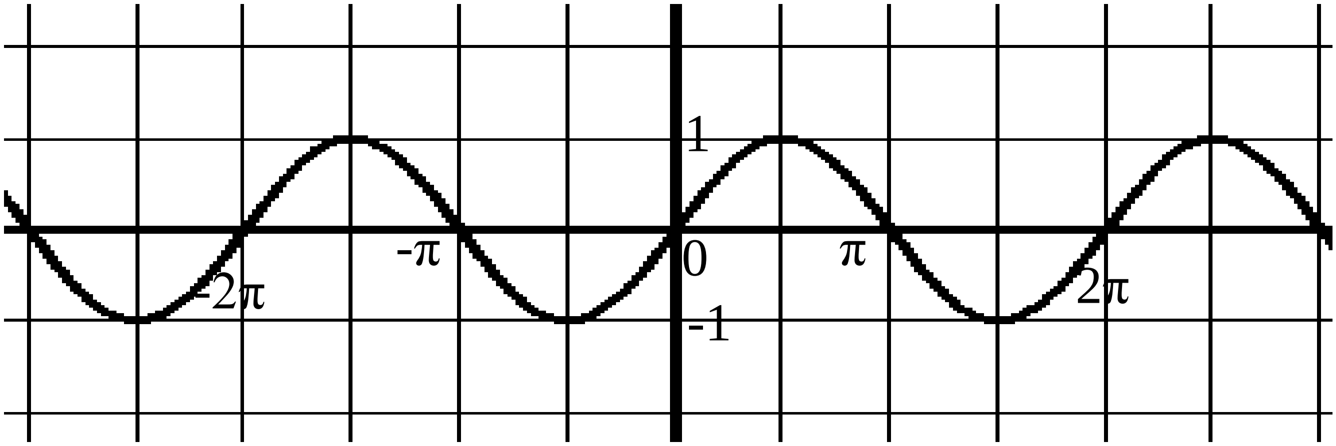
$$f(x) = \sin x$$
$$f(x) = \cos x$$
$$f(x) = \tan x$$

Graph $f(x) = \sin x$. This is the same as $y = \sin x$ and $y = \sin(x)$

x	y
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{5\pi}{6}$	$\frac{1}{2}$
π	0

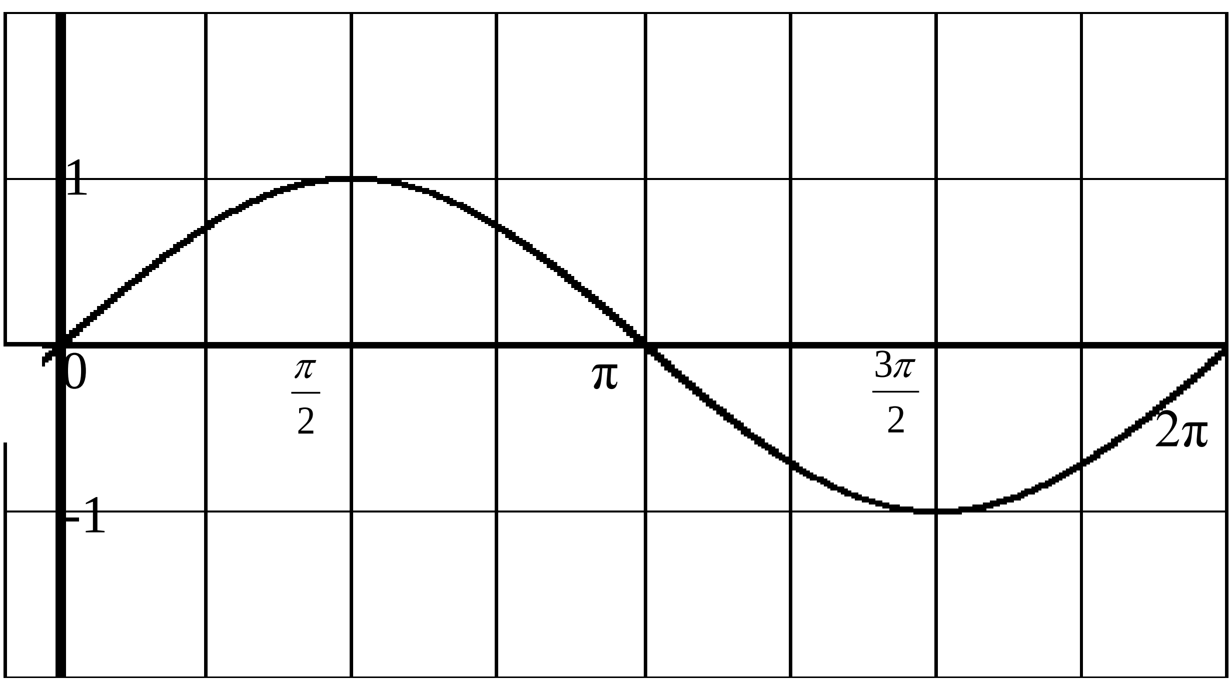
x	y
π	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{5\pi}{4}$	$\frac{-\sqrt{2}}{2} \approx -0.7$
$\frac{4\pi}{3}$	$\frac{-\sqrt{3}}{2} \approx -0.87$
$\frac{3\pi}{2}$	-1
$\frac{5\pi}{3}$	$\frac{-\sqrt{3}}{2} \approx -0.87$
$\frac{7\pi}{4}$	$\frac{-\sqrt{2}}{2} \approx -0.7$
$\frac{11\pi}{6}$	$-\frac{1}{2}$
2π	0

x	y
0	0
$\frac{13\pi}{6}$	$\frac{1}{2}$
$\frac{9\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{7\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{5\pi}{2}$	1
$\frac{8\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{11\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{17\pi}{6}$	$\frac{1}{2}$
3π	0



The Domain of $y = \sin x$ is the set of all real numbers.
The Range of $y = \sin x$ is $-1 \leq y \leq 1$.

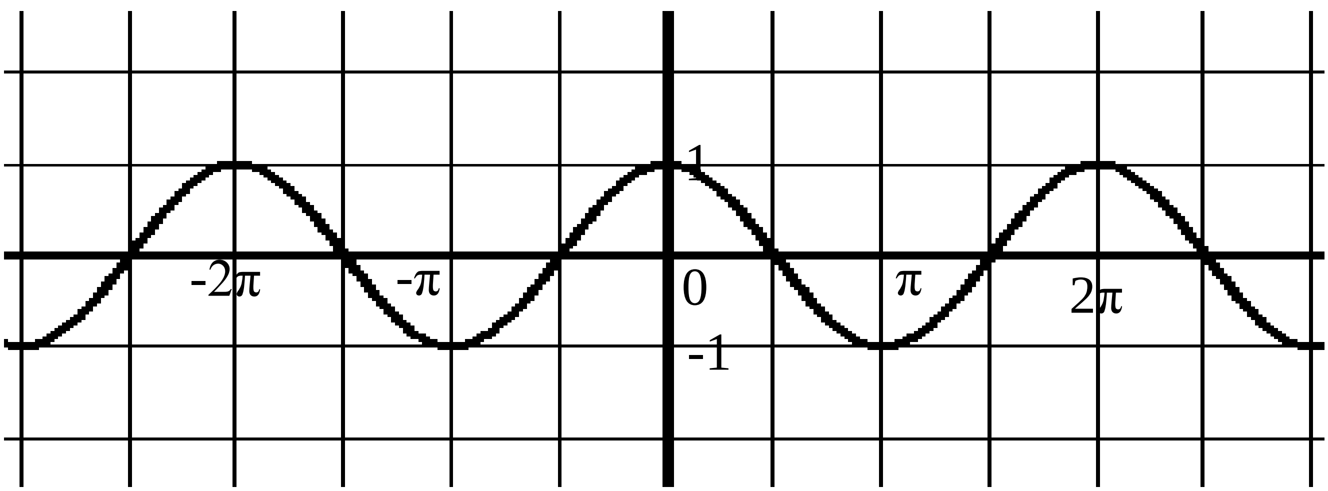
The portion of the graph of $y = \sin x$ that includes one period is called one cycle of the sine curve.



Every period of the sine curve has **5 key points**: the intercepts and a minimum and maximum point.

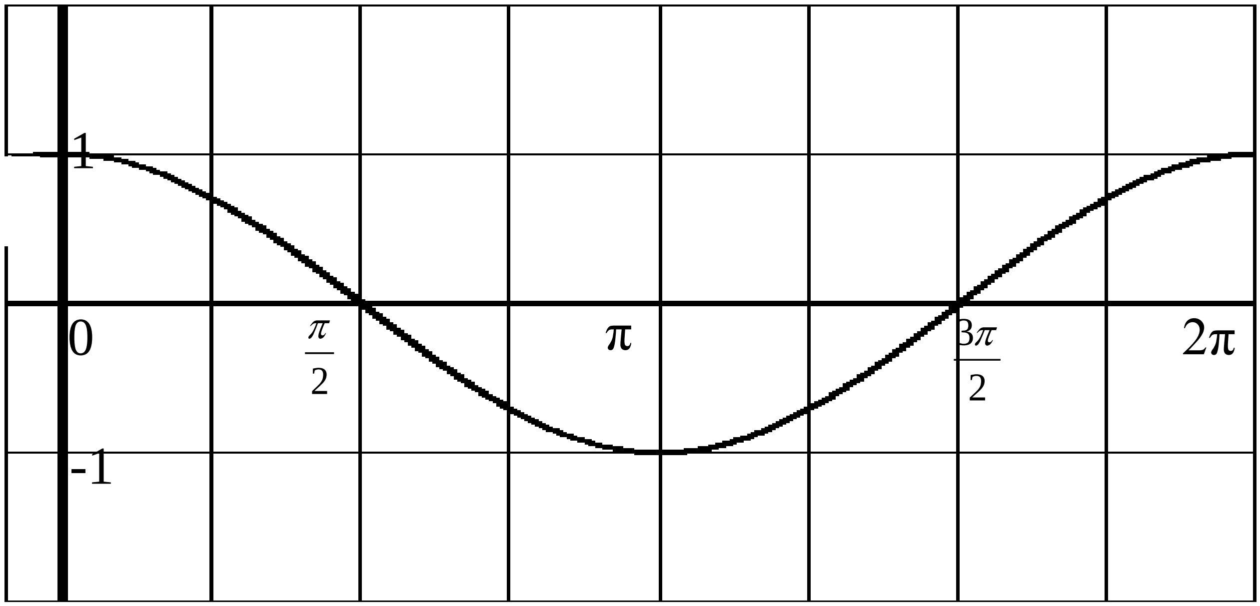
For one period of the sine curve, the x-intercepts occur at (0, 0), (π, 0), and (2π, 0). The maximum point is (π/2, 1) and the minimum point is (3π/2, -1).

Graph $y = \cos x$.



The Domain of $y = \cos x$ is the set of all real numbers.
The Range of $y = \cos x$ is $-1 \leq y \leq 1$.

The portion of the graph of $y = \cos x$ that includes one period is called one cycle of the cosine curve.



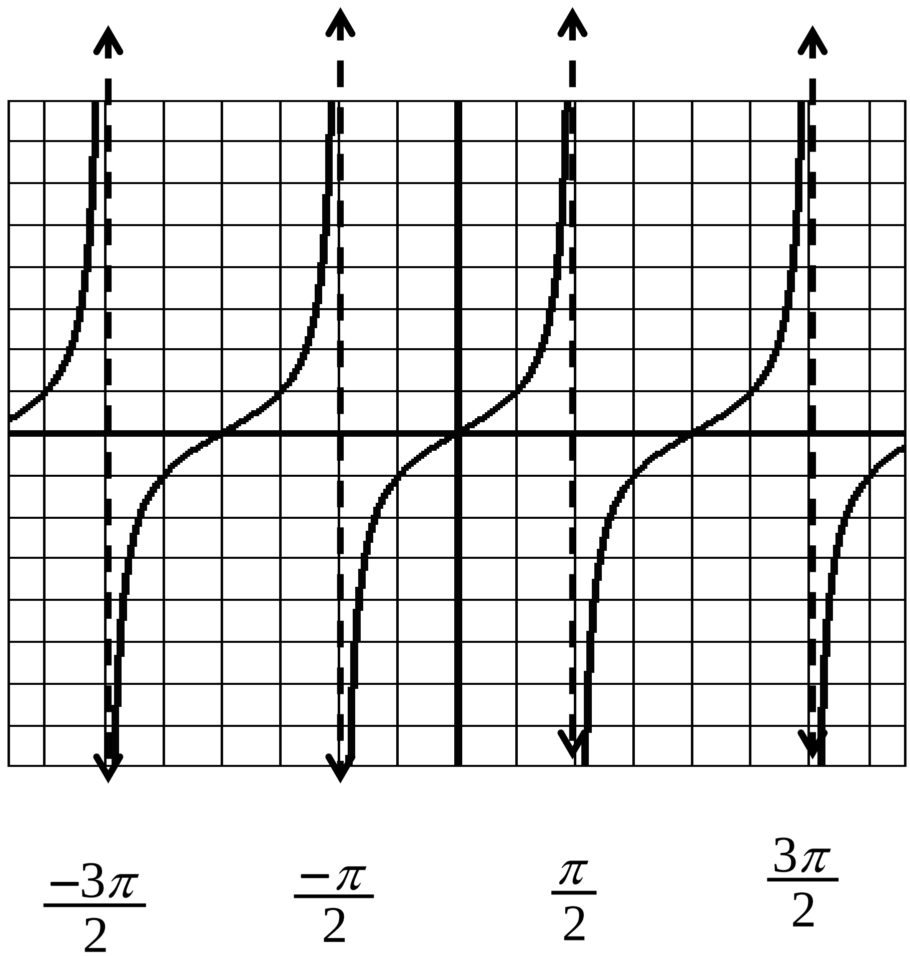
Every period of the cosine curve has **5 key points**: the intercepts and a minimum and maximum point.

For one period of the sine curve, the x-intercepts occur at (π/2, 0), and (3π/2, 0). The maximum point is (0, 1) and (2π, 0) and the minimum point is (π, -1).

**Both sine and cosine curves have a period of 2π. We consider the interval from 0 to 2π as the basic cycle.

Graph $y = \tan x$.

x	y
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx .58$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.7$
$\frac{\pi}{2}$	undefined
$\frac{2\pi}{3}$	$-\sqrt{3} \approx -1.7$
$\frac{3\pi}{4}$	-1
$\frac{5\pi}{6}$	$\frac{-\sqrt{3}}{3} \approx -.58$
π	0

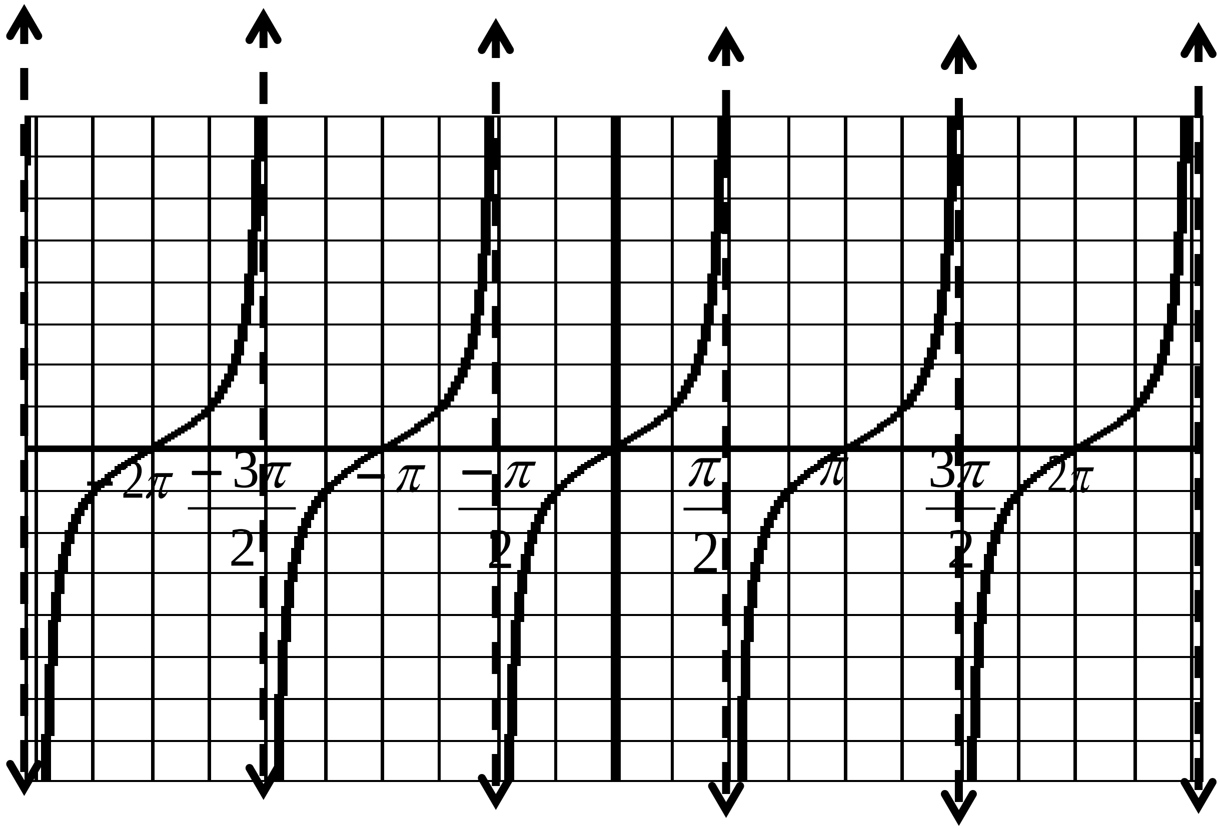


The <u>Domain</u> is all real numbers except multiples of $\frac{\pi}{2}$.

(We say the domain is all $x \neq \frac{\pi}{2} + n\pi$)

The <u>Range</u> is the set of all real numbers.

$y = \tan x$



- The period for tangent is π.
- One cycle is $\frac{-\pi}{2} < x < \frac{\pi}{2}$. (Note that it's not $\leq$)
- One cycle goes from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.
- There is a vertical asymptote at $x = \frac{\pi}{2} \pm n\pi$ (at every x-value for which the tangent is undefined.)
- The Domain is all $x \neq \frac{\pi}{2} + n\pi$
- The Range is all real numbers.

- All three trig functions are <u>periodic functions</u> because there is a repeating pattern.
  - For sine and cosine, the basic period is 2π.
  - For tangent, the basic period is π.
- The graphs of sine and cosine are <u>continuous</u> because there are no breaks.
- The graph of tangent is <u>discontinuous</u> because there are jumps/breaks (where the asymptotes are).

<u>Amplitude</u>

On a graphing calculator, graph $y = \sin x$

$y = 2\sin x$

$y = 5\sin x$

$y = \frac{1}{2} \sin x$

What can you conclude?

As the number being multiplied out front increases, the graph of $y = \sin x$ *stretches vertically.*

Definition: The amplitude of $y = a \sin x$ and $y = a \cos x$ represents half the distance between the maximum and minimum values of the function and is given by Amplitude = $|a|$.

***Note**: If a is a negative number, the graph of the function will be reflected over the x-axis.

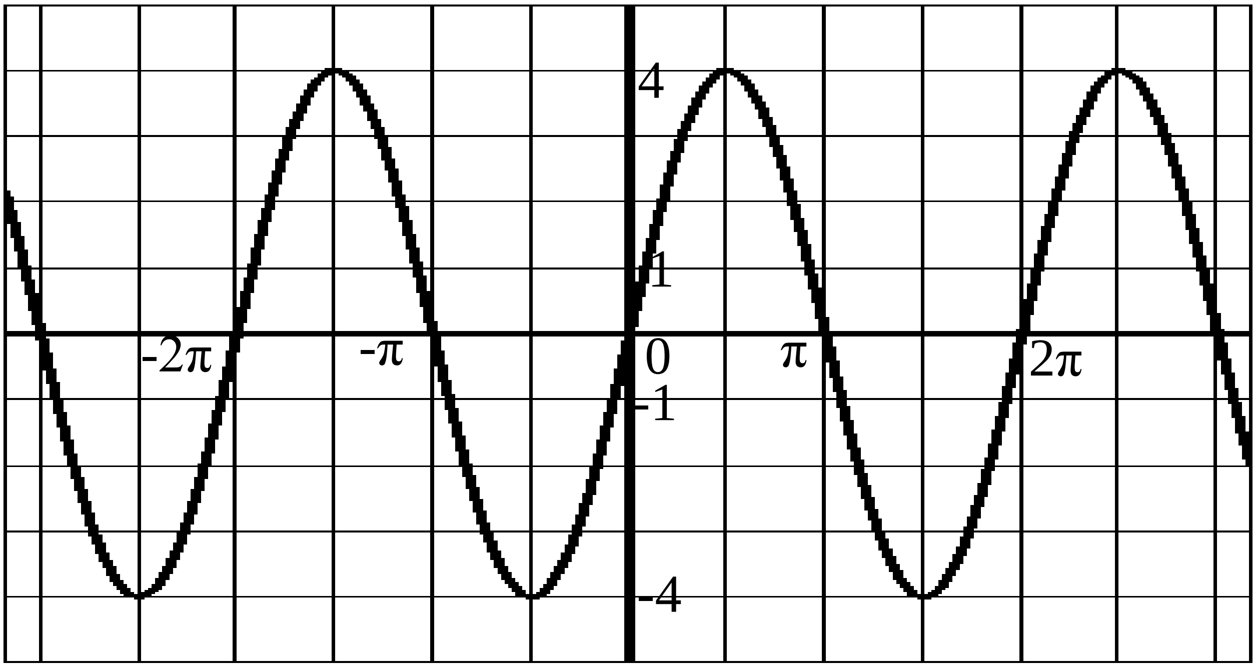
Example: Graph y = -sin x.
Graph y = -2sin$x$.

See that these are the same as y = sin x and y = 2sin x, but they are "up-side-down."

Example of amplitudes:

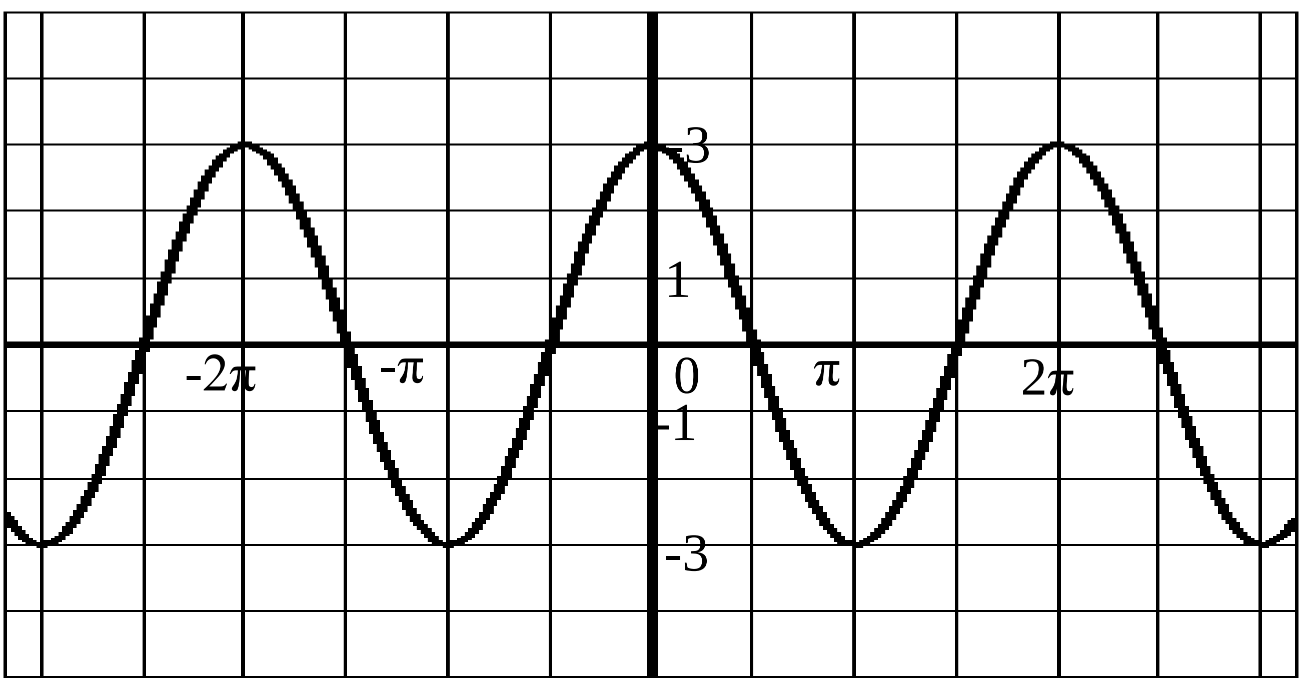
The amplitude of $y = \sin x$ is 1.
The amplitude of $y = 2 \sin x$ is 2.
The amplitude of $y = 5 \sin x$ is 5.
The amplitude of $y = \frac{1}{2} \sin x$ is ½ .
The amplitude of $y = -13 \sin x$ is 13. (*not* -13)

Graph $y = 4 \sin x$

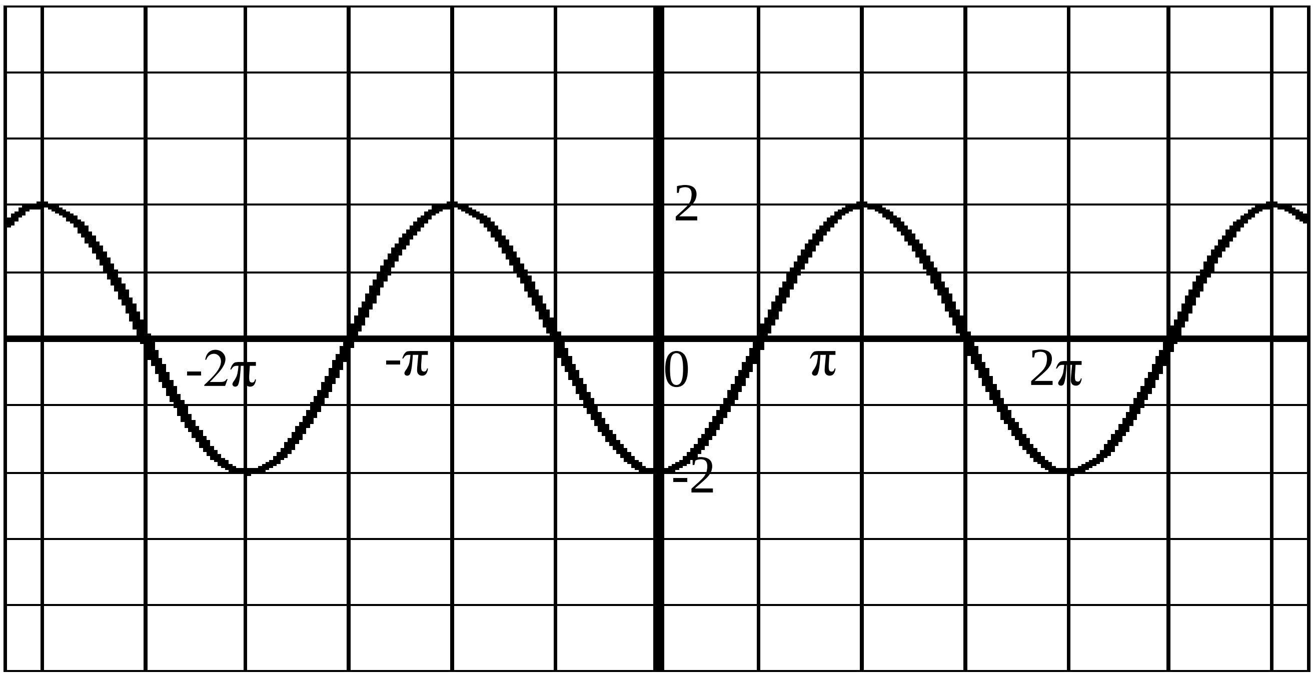


The period remains the same, but the amplitude changes.

Graph $y = 3 \cos x$



Graph $y = -2\cos x$



Changing the Period of Sine and Cosine

On a graphing calculator graph: $y = \sin x$
$y = \sin 2x$

What do you notice?

The length of one cycle is half as long for $y = \sin 2x$.

Definition: Let b be a positive real number. The period of $y = a\sin bx$ and $y = a\cos bx$ is $2\pi/b$.

Example: Find the period of $y = \cos 6x$.

The period = $\frac{2\pi}{b} = \frac{2\pi}{6} = \frac{\pi}{3}$

Example: Find the period of $y = sin\ \frac{x}{5}$.

The period = $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{5}} = 2\pi \cdot \frac{5}{1} = 10\pi$

<u>Note</u>: Once you know the basic shape of the sine and cosine curves, it is basically a matter of making adjustments to the axes labels.

Example: Graph $y = 3sin\ 4x$.