ch. 5 review

Chapter 5 Vocabulary:

biconditional
conclusion
conditional
conjunction
connective
contrapositive
converse
deductive reasoning
disjunction
existential quantifier
fallacy
hypotheses
implication
inductive reasoning
inverse
Law of Deduction
logically equivalent
modus ponens
modus tollens
negation
premise
proof
reasoning
sound argument
statement
transitivity
truth table
truth value
universal quantifier

Symbols:

$\sim$ not
$\rightarrow$ conditional
$\leftrightarrow$ biconditional
$\exists$ There exists
$\forall$ For all
$\wedge$ conjunction
$\vee$ disjunction
$\therefore$ therefore

For the test you must:

- Know the symbols
- Match terms with definitions
- Write negations and conditional statements
- Analyze statements and arguments
- Make truth tables
- Identify fallacies
- Identify the type of reasoning in a deductive argument

section 6.1

Hints to keep in mind when proving a theorem:

1. Identify the premise (given information).
2. Identify the conclusion you are trying to obtain.
3. Draw a picture to make sure you understand the theorem.
4. Write down any definitions, postulates, or previously proved theorems that relate to the theorem you are trying to prove.
5. Work backwards if necessary, starting with the conclusion.

Example of using the Law of Deduction

Prove: A line and a point not on that line are contained in one and only one plane.

Answer:

1. Consider definitions and incidence postulates that you have learned.

Review Incidence Postulates

Expansion Postulate: A line contains at least two points. A plane contains at least three noncollinear points. Space contains at least 4 noncoplanar points.

Line Postulate: Any two points in space lie in exactly one line.

Plane Postulate: Three distinct noncollinear points lie in exactly one plane.

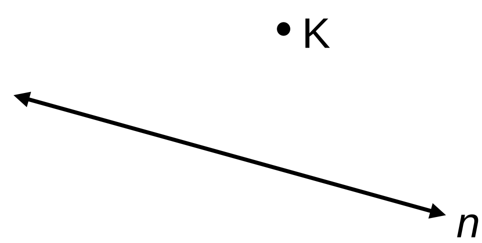
Flat Plane Postulate: If two points lie ina plane, then the line containing these two points lies in the same plane.

Plane Intersection Postulate: If two planes intersect, then their intersection is exactly one line.

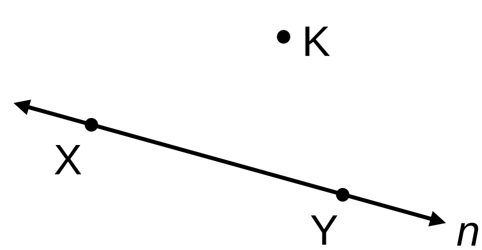
2. Determine the premise and the conclusion.

Premise: There is a line and a point not on that line.
Conclusion: The line and point lie in exactly one plane.

3. Draw a picture to help you understand the premise and the conclusion.



4. *n* is the given line, and K is the point that is not on the line. You want to show that only one plane passes through both the line and the point. According to the Expansion Postulate, a line contains at least two points, so *n* contains at least 2 points. Call them X and Y.



5. Now you see 3 distinct points, X, Y, and K. According to the Plane Postulate, 3 non-collinear points determine exactly one plane.

6. By the Flat Plane Postulate, since X and Y lie in a plane, the entire line *n* that contains them lies in the plane. Thus, line *n* and point K lie in one plane.

The same proof in a 2-column form:

Prove: A line and a point not on that line are contained in one and only one plane.

Answer: *Given*: a line n, a point K not on line n
Prove: a plane p containing n and K

Statement	Reason
1. There is a line, *n*, and a point, K, not on the line.	1. Premise (given)
2. Line *n* contains two points, X and Y	2. Expansion Postulate
3. X, Y, and K are noncollinear	3. Definition of noncollinear
4. X, Y, and K determine exactly one plane *p*.	4. Plane Postulate
5. Line *n* and point K are in plane *p*.	5. Flat Plane Postulate

Note: The only time that you have to actually show the Law of Deduction step to complete the proof is when you are proving an "if-then" statement.

****Properties of Real numbers are often helpful in geometric proofs:

Properties of Real Numbers

Property	Addition	Multiplication
Commutative	a+b = b+a	ab = ba
Associative	(a+b)+c=a+(b+c)	(ab)c = a(bc)
Distributive		a(b+c) = ab+ac
Identity	a+0 = 0+a = a	a·1 = 1·a = a
Inverse	a+(-a) = 0	a·(1/a) = 1

Equality Properties	
Property	**Meaning**
Addition	If a=b, then a+c = b+c
Multiplication	If a=b, then ac=bc

More Equality Properties	
Reflexive	a=a
Symmetric	If a=b, then b=a
Transitive	If a=b and b=c, then a=c

Note: Combining like terms is really distributive. You can write "combining like terms" in your proofs.

Abbreviations acceptable for proofs:

definition → def. or defn.
theorem → th. or thm.
property → prop.
addition → add. or +
multiplication → mult.
equality → =
congruent → ≅

segment → seg.
angle → ∠
triangle → Δ
perpendicular → ⊥
parallel → // or ||

<u>Problem</u>:

Prove: <u>Congruent Segment Bisector Theorem</u>: If two congruent segments are bisected, then the 4 resulting segments are congruent.

First, draw a picture.

Next, look for p in your conditional, p→r. Remember that p is your given and that r is the conclusion to be proved from p.

Given: Two congruent segments: $\overline{XY} \cong \overline{KL}$

Prove: If A and B are the midpoints of the congruent segments $\overline{XY}$ and $\overline{KL}$, then $\overline{AY} \cong \overline{XA} \cong \overline{KB} \cong \overline{BL}$

Statement	Reason
1. $\overline{XY} \cong \overline{KL}$; A and B are midpoints	1.
2. XY = KL	2.
3. ½XY = ½KL	3.
4. XA = ½XY; KB = ½KL	4.
5. XA = KB	5.
6. XA=AY, KB=BL	6.
7. $\overline{XA} \cong \overline{KB}$, $\overline{XA} \cong \overline{AY}$, $\overline{KB} \cong \overline{BL}$	7.
8. $\overline{AY} \cong \overline{XA} \cong \overline{KB} \cong \overline{BL}$	8.
9. If A and B are the midpoints of the congruent segments XY and KL, then AY ≅ XA ≅ KB ≅ BL	9.

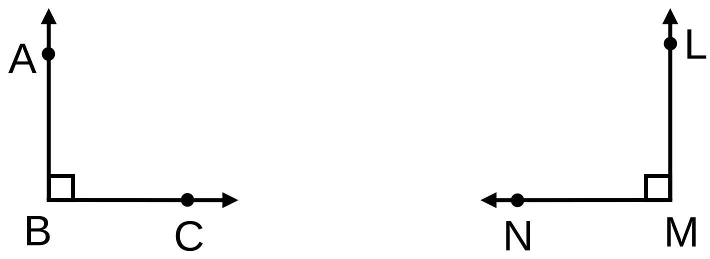
ANSWER:

Given: Two congruent segments: $\overline{XY} \cong \overline{KL}$

Prove: If A and B are the midpoints of the congruent segments $\overline{XY}$ and $\overline{KL}$, then $\overline{AY} \cong \overline{XA} \cong \overline{KB} \cong \overline{BL}$

Statement	Reason
1. $\overline{XY} \cong \overline{KL}$; A and B are midpoints	1. Given
2. XY = KL	2. Def. of congruent segments
3. ½XY = ½KL	3. Mult. Property of Equality
4. XA = ½XY; KB = ½KL	4. Midpoint Theorem
5. XA = KB	5. Substitution (step 4 into 3)
6. XA=AY, KB=BL	6. Def. of midpoint
7. $\overline{XA} \cong \overline{KB}$, $\overline{XA}\cong\overline{AY}$, $\overline{KB}\cong\overline{BL}$	7. Def. of congruent segments
8. $\overline{AY} \cong \overline{XA} \cong \overline{KB} \cong \overline{BL}$	8. Transitive property of congruent segments
9. If A and B are the midpoints of the congruent segments $\overline{XY}$ and $\overline{KL}$, then $\overline{AY} \cong \overline{XA} \cong \overline{KB} \cong \overline{BL}$	9. Law of Deduction

Prove: All right angles are congruent.

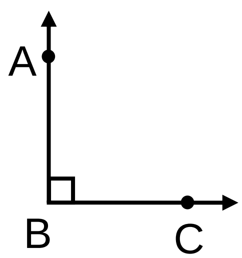


After the picture is drawn of 2 arbitrary right angles, prove that the right angles are congruent.

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.

ANSWER:

Prove: All right angles are congruent.

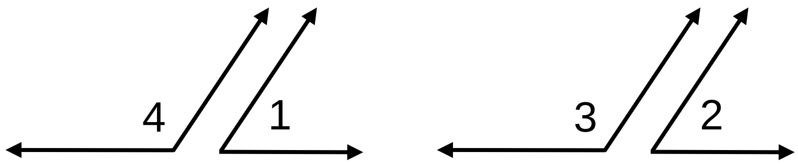


After the picture is drawn of 2 arbitrary right angles, prove that the right angles are congruent.

Statement	Reason
1. $\angle M$ and $\angle B$ are right angles.	1. Given
2. $m\angle B = 90°$; $m\angle M = 90°$;	2. Def. of right triangles
3. $m\angle B = m\angle M$	3. Transitivity prop. of equality
4. $\angle B \cong \angle M$	4. Def. of congruent angles

Prove: Supplements of congruent angles are congruent.

Draw 2 congruent angles and their supplements.



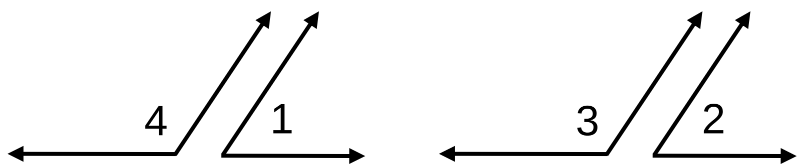
Assume that $\angle 1 \cong \angle 2$, $\angle 3$ is a supplement of $\angle 2$, and $\angle 4$ is a supplement of $\angle 1$. Prove that $\angle 3 \cong \angle 4$.

Statement	Reason
1. $\angle 1 \cong \angle 2$; $\angle 1$ and $\angle 4$ are supplements	1.
2. $m\angle 1 = m\angle 2$	2.
3. $m\angle 1 + m\angle 4 = 180°$ $m\angle 2 + m\angle 3 = 180°$	3.
4. $m\angle 1+m\angle 4=m\angle 2+m\angle 3$	4.
5. $m\angle 4 = m\angle 3$	5.
6. $\angle 3 \cong \angle 4$	6.

ANSWER:

Prove: Supplements of congruent angles are congruent.

Draw 2 congruent angles and their supplements.



Assume that $\angle 1 \cong \angle 2$, $\angle 3$ is a supplement of $\angle 2$, and $\angle 4$ is a supplement of $\angle 1$. Prove that $\angle 3 \cong \angle 4$.

Statement	**Reason**
1. $\angle 1 \cong \angle 2$; $\angle 1$ and $\angle 4$ are supplements	1. Given
2. $m\angle 1 = m\angle 2$	2. Def. of congruent angles
3. $m\angle 1 + m\angle 4 = 180°$ $m\angle 2 + m\angle 3 = 180°$	3. Definition of supplementary angles
4. $m\angle 1+m\angle 4=m\angle 2+m\angle 3$	4. Transitive property of equality
5. $m\angle 4 = m\angle 3$	5. Addition property of equality
6. $\angle 3 \cong \angle 4$	6. Definition of congruent angles

<u>Review Chapter 4 Theorems</u>:

Theorem 4.1: All right angles are congruent.

Theorem 4.2: If two angles are adjacent and supplementary, then they form a linear pair.

Theorem 4.3: Angles that form a linear pair are supplementary.

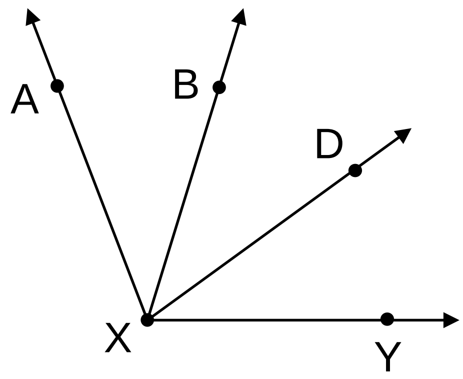
Theorem 4.4: If one angle of a linear pair is a right angle, then the other angle is also a right angle.

Theorem 4.5: <u>*Vertical Angle Theorem*</u>. Vertical angles are congruent.

Theorem 4.6: Congruent supplementary angles are right angles.

Theorem 4.7: <u>*Angle Bisector Theorem*</u>. If $\overrightarrow{AB}$ bisects $\angle CAD$, then $m\angle CAB = \frac{1}{2} m\angle CAD$

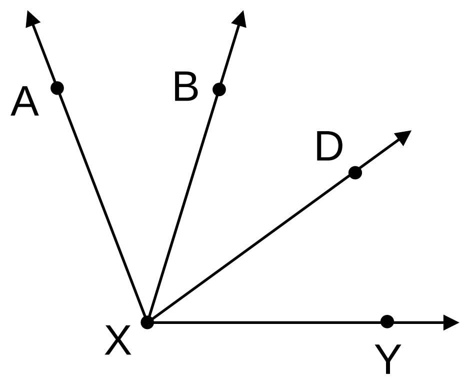
Prove: If $m\angle AXB = m\angle DXY$, then $m\angle AXD = m\angle BXY$



Statement	Reason
1. $m\angle AXB = m\angle DXY$	1.
2. $m\angle AXB + m\angle BXD =$ $m\angle BXD + m\angle DXY$	2.
3. $m\angle AXB+m\angle BXD=m\angle AXD$ $m\angle BXD+m\angle DXY=m\angle BXY$	3.
4. $m\angle AXD = m\angle BXY$	4.
5. If $m\angle AXB = m\angle DXY$, then $m\angle AXD = m\angle BXY$	5.

ANSWER:

Prove: If $m\angle AXB = m\angle DXY$, then $m\angle AXD = m\angle BXY$



Statement	Reason
1. $m\angle AXB = m\angle DXY$	1. Given
2. $m\angle AXB + m\angle BXD = m\angle BXD + m\angle DXY$	2. Add. Property of Equality
3. $m\angle AXB+m\angle BXD=m\angle AXD$ $m\angle BXD+m\angle DXY=m\angle BXY$	3. Angle Add. Postulate
4. $m\angle AXD = m\angle BXY$	4. Substitution (step 3 into 2)
5. If $m\angle AXB = m\angle DXY$, then $m\angle AXD = m\angle BXY$	5. Law of Deduction

Theorem 6.4: Complements of congruent angles are congruent.

Theorem 6.5: Angle congruence is an equivalence relation

Theorem 6.6: *Adjacent Angle Sum Theorem*. If two adjacent angles are congruent to another pair of adjacent angles, then the larger angles formed are congruent.

Theorem 6.7: *Adjacent Angle Portion Theorem*. If two angles, one in each of two pairs of adjacent angles, are congruent, and the larger angles formed are also congruent, then the other two angles are congruent.

Theorem 6.8: *Congruent Angle Bisector Theorem*. If two congruent angles are bisected, the four resulting angles are congruent.

***In general, congruent figures have the same size and shape.

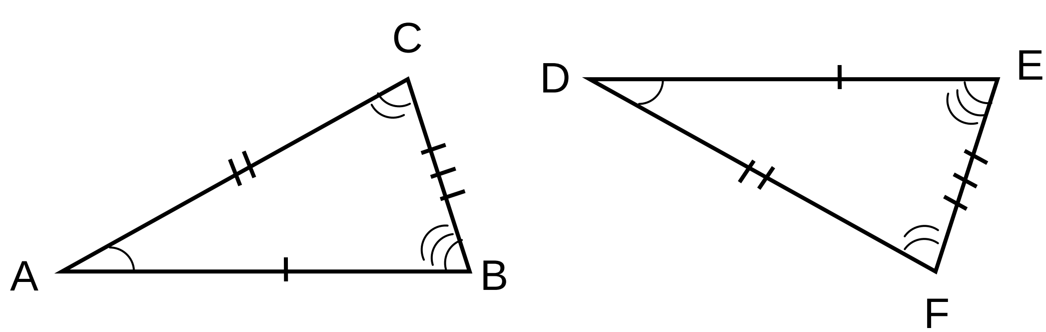
Definitions:

Congruent circles are circles with congruent radii.

Congruent polygons are polygons that have 3 properties:

1. Same number of sides
2. Corresponding sides are congruent
3. Corresponding angles are congruent

Example:



$\Delta ABC \cong \Delta DEF$

*Corresponding vertices must be listed carefully!

Definition:

Congruent triangles are triangles in which corresponding angles and corresponding sides are congruent.

Congruence Properties

Property	Meaning
Reflexive	$X \cong X$
Symmetric	If $X \cong Y$, then $Y \cong X$
Transitive	If $X \cong Y$ and $Y \cong Z$, then $X \cong Z$

***Remember that if a relation is reflexive, symmetric, and transitive, it is called an equivalence relation.

Theorem 6.9: Triangle congruence is an equivalence relation.

Theorem 6.10: Circle congruence is an equivalence relation.

Note: Since any polygon can be divided into triangles, we can show polygons congruent by showing that the triangles that make them up are congruent.

Sample Problem: If quadrilateral ABCD and PQRS are congruent, list all possible conclusions.

Answer:

$\overline{AB} \cong \overline{PQ}$ $\angle A \cong \angle P$
$\overline{BC} \cong \overline{QR}$ $\angle B \cong \angle Q$
$\overline{CD} \cong \overline{RS}$ $\angle C \cong \angle R$
$\overline{AD} \cong \overline{PS}$ $\angle D \cong \angle S$