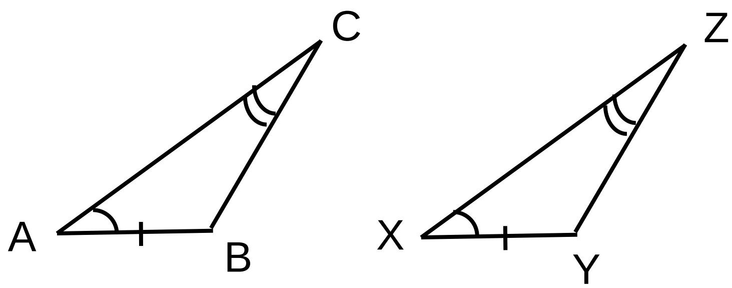
section 6.7

Theorem 6.19: SAA Congruence Theorem: If two angles of a triangle and a side opposite one of the two angles are congruent to the corresponding angles and side of another triangle, then the two triangles are congruent.

Proof :



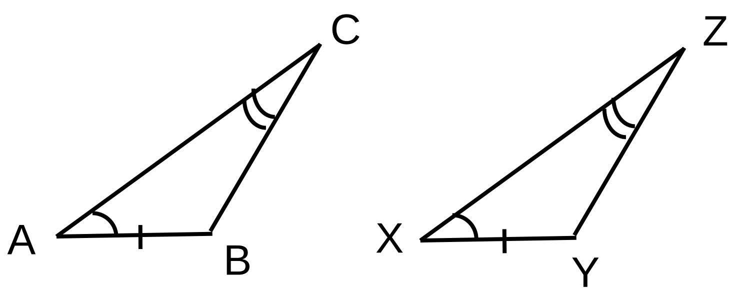
Given: $\angle A \cong \angle X$; $\angle C \cong \angle Z$; $\overline{AB} \cong \overline{XY}$

Prove: $\Delta ABC \cong \Delta XYZ$

Statement	**Reason**
1.	1.
2.	2.
3.	3.
4.	4.

Solution:

Proof of Theorem 6.19: SAA Congruence Theorem: If two angles of a triangle and a side opposite one of the two angles are congruent to the corresponding angles and side of another triangle, then the two triangles are congruent.



Given: $\angle A \cong \angle X$; $\angle C \cong \angle Z$; $\overline{AB} \cong \overline{XY}$

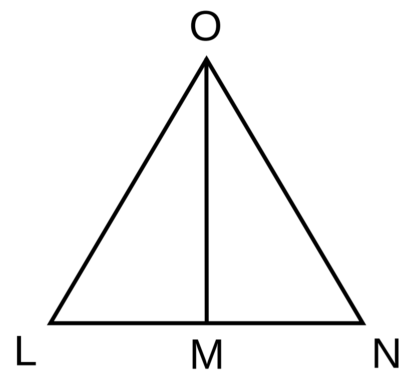
Prove: $\Delta ABC \cong \Delta XYZ$

Statement	**Reason**
1. $\angle A \cong \angle X$; $\angle C \cong \angle Z$; $\overline{AB} \cong \overline{XY}$	1. Given
2. $\angle B \cong \angle Y$	2. Third angles are congruent (6.17)
3. $\Delta ABC \cong \Delta XYZ$	3. ASA
4. If $\angle A \cong \angle X$; $\angle C \cong \angle Z$; $\overline{AB} \cong \overline{XY}$, then $\Delta ABC \cong \Delta XYZ$	4. Law of Deduction

Example 1:

Given: M is the midpoint of $\overline{LN}$

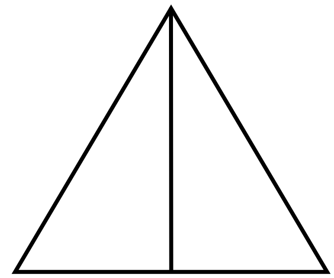
$\overleftrightarrow{OM} \perp \overleftrightarrow{LN}$



Prove: $\Delta LMO \cong \Delta NMO$

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

Solution for Example 1:



Given: M is the midpoint of $\overline{LN}$;
$\overleftrightarrow{OM} \perp \overleftrightarrow{LN}$

Prove: $\Delta LMO \cong \Delta NMO$

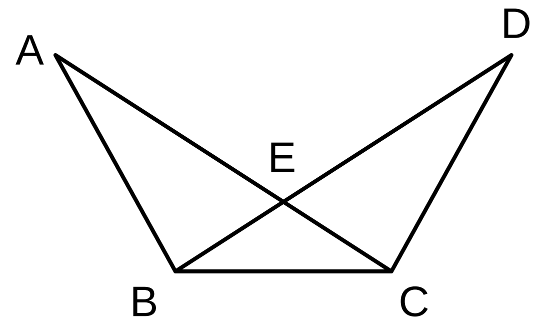
Statement	Reason
1. M is the midpoint of $\overline{LN}$; $\overleftrightarrow{OM} \perp \overleftrightarrow{LN}$	1. Given
2. LM = NM	2. Definition of midpoint
3. $\overline{LM} \cong \overline{NM}$	3. Def. of congruent segments
4. $\overline{OM} \cong \overline{OM}$	4. Reflexive
5. $\angle LMO$ and $\angle NMO$ are right angles	5. Def. of perpendicular
6. $\angle LMO \cong \angle NMO$	6. All right angles are congruent
7. $\Delta LMO \cong \Delta NMO$	7. SAS

Example 2:

Given: $\angle A \cong \angle D$

$\angle ABC \cong \angle DCB$

Prove: $\overline{AC} \cong \overline{DB}$

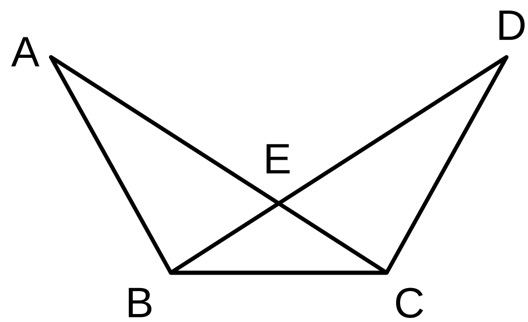


Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Solution for Example 2:

Given: $\angle A \cong \angle D$
$\angle ABC \cong \angle DCB$

Prove: $\overline{AC} \cong \overline{DB}$



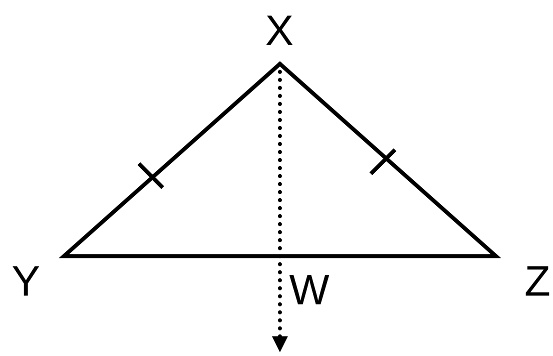
Statement	Reason
1. $\angle A \cong \angle D$; $\angle ABC \cong \angle DCB$	1. Given
2. $\overline{BC} \cong \overline{BC}$	2. Reflexive
3. $\Delta ABC \cong \Delta DCB$	3. SAA
4. $\overline{AC} \cong \overline{DB}$	4. Def. of congruent triangles

Theorem 6.20: Isosceles Triangle Theorem: In an isosceles triangle the two base angles are congruent.

Proof:

Given: Isosceles ΔXYZ
$\overline{XY} \cong \overline{XZ}$

Draw auxilliary $\overleftrightarrow{XW}$
to bisect $\angle YXZ$



Prove: $\angle Y \cong \angle Z$

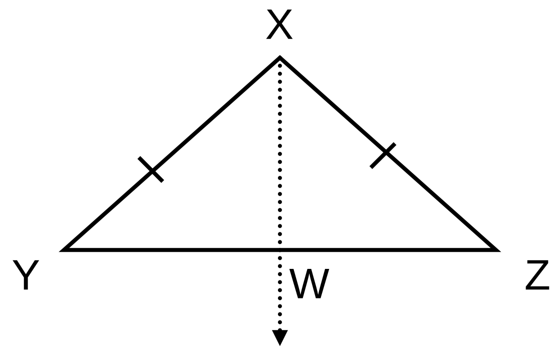
Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Solution:

Theorem 6.20: Isosceles Triangle Theorem: In an isosceles triangle the two base angles are congruent.

Given: Isosceles ΔXYZ
$\overline{XY} \cong \overline{XZ}$

Draw auxilliary $\overleftrightarrow{XW}$
to bisect $\angle YXZ$

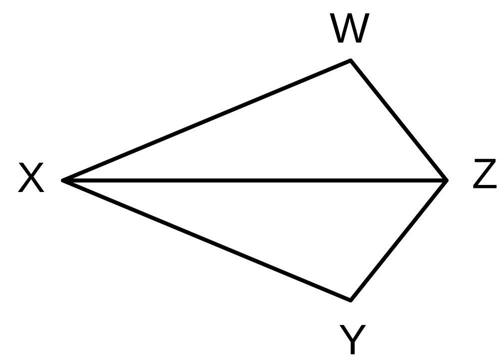


Prove: $\angle Y \cong \angle Z$

Statement	Reason
1. $\overline{XY} \cong \overline{XZ}$	1. Given
2. $\overleftrightarrow{XW}$ bisects $\angle X$	2. Auxiliary ray
3. $\angle YXW \cong \angle ZXW$	3. Def. of angle bisector
4. $\overline{XW} \cong \overline{XW}$	4. Reflexive Property
5. $\Delta XYW \cong \Delta XZW$	5. SAS
6. $\angle Y \cong \angle Z$	6. Corresponding parts of congruent triangles are congruent.

Example 3:

Given: $\overleftrightarrow{ZX}$ bisects ∠WZY
and ∠WXY

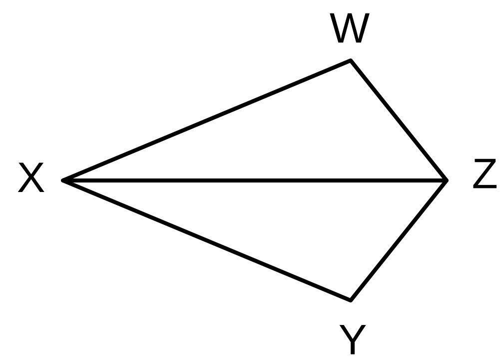


Prove: $\overline{XY} \cong \overline{XW}$

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Solution for Example 3:

Given: $\overleftrightarrow{ZX}$ bisects $\angle WZY$ and $\angle WXY$



Prove: $\overline{XY} \cong \overline{XW}$

Statement	Reason
1. $\overleftrightarrow{ZX}$ bisects $\angle WZY$ and $\angle WXY$	1. Given
2. $\angle WXZ \cong \angle YXZ$ $\angle WZX \cong \angle YZX$	2. Def. of angle bisector
3. $\overline{XZ} \cong \overline{XZ}$	3. Reflexive
4. $\Delta XZW \cong \Delta XZY$	4. ASA
5. $\overline{XY} \cong \overline{XW}$	5. Def. of congruent triangles (CPCTC)

Theorem 6.20: Isosceles Triangle Theorem: In an isosceles triangle the two base angles are congruent.

Theorem 6.21: If two angles of a triangle are congruent, then the sides opposite those angles are congruent, and the triangle is an isosceles triangle.

Theorem 6.22: A triangle is equilateral if and only if it is equiangular.

section 6.5

SSS Congruence Theorem: If each side of one triangle is congruent to the corresponding side of a second triangle, then the two triangles are congruent.

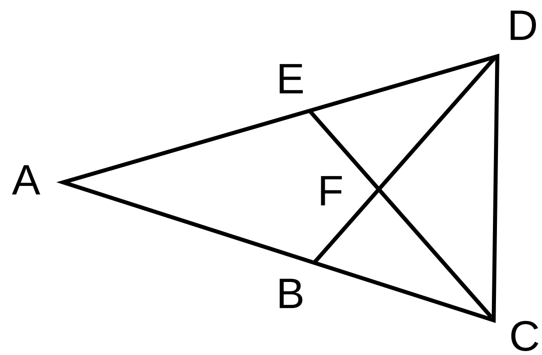
Summary:

Ways you CAN use to prove congruence	Ways you CAN'T use to prove congruence
SAS	SSA
ASA	AAA
SSS	
SAA	

Example 2, page 253

Given: $\overline{ED} \cong \overline{BC}$, $\overline{EC} \cong \overline{BD}$

Prove: $\overline{AE} \cong \overline{AB}$

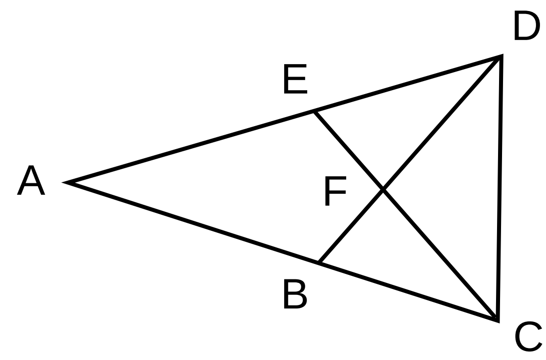


Statement	Reason
1. $\overline{ED} \cong \overline{BC}$, $\overline{EC} \cong \overline{BD}$	1. Given
2. $\overline{DC} \cong \overline{DC}$	2. Reflexive
3. $\Delta BCD \cong \Delta EDC$	3. SSS
4. $\angle EDC \cong \angle BCD$, $\angle BDC \cong \angle ECD$	4. CPCTC
5. $\angle EDF \cong \angle BCF$	5. Adjacent Angle Portion Theorem
6. $\angle A \cong \angle A$	6. Reflexive
7. $\Delta ABD \cong \Delta AEC$	7. SAA
8. $\overline{AE} \cong \overline{AB}$	8. CPCTC

Alternate Proof for Example 2, page 253

Given: $\overline{ED} \cong \overline{BC}$, $\overline{EC} \cong \overline{BD}$

Prove: $\overline{AE} \cong \overline{AB}$



Statement	Reason
1. $\overline{ED} \cong \overline{BC}$, $\overline{EC} \cong \overline{BD}$	1. Given
2. $\overline{DC} \cong \overline{DC}$	2. Reflexive
3. $\Delta BCD \cong \Delta EDC$	3. SSS
4. $\angle EDC \cong \angle BCD$	4. CPCTC
5. $\overline{AD} \cong \overline{AC}$	5. Thm. 6.21 (conv. of Isos Thm)
6. AD = AC	6. Def. of congruent
7. AD = AE + ED AC = AB + BC	7. Def. of betweenness
8. AE + ED = AB + BC	8. Subst.(step 6 into 7)
9. ED = BC	9. Def. of congruent
10. AE = AB	10. Add. Prop. (Subtr. Eq.)
11. $\overline{AE} \cong \overline{AB}$	11. Def. of Congr. seg.

Vocabulary:

Adjacent Angle Portion Theorem
Adjacent Angle Sum Theorem
alternate exterior angles
alternate interior angles
ASA Congruence Postulate
congruent circles,
congruent polygons
congruent triangles
corresponding angles
Equilateral Triangle Theorem
included angle
included side
Isosceles Triangle Theorem
SAA Congruence Theorem
SAS Congruence Theorem
SSS Congruence Theorem
two-column proof
transversal