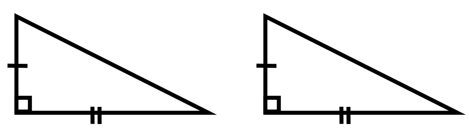
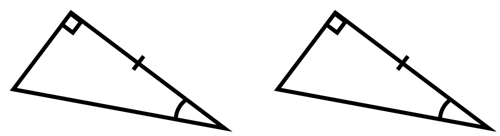
section 7.1

Triangle congruence can be proved by:
- SAS
- ASA
- SSS
- SAA

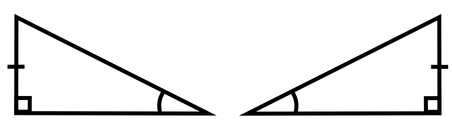
Identify the congruence theorem or postulate:



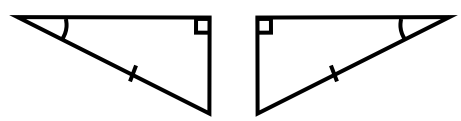
SAS



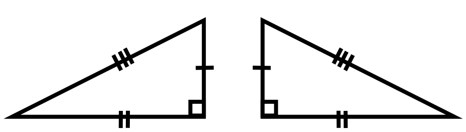
ASA



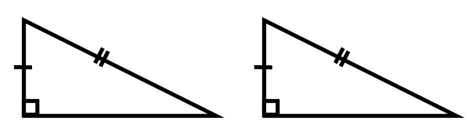
SAA



SAA

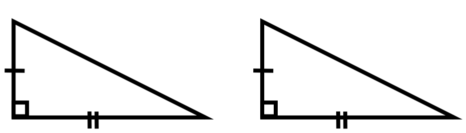


SSS or SAS

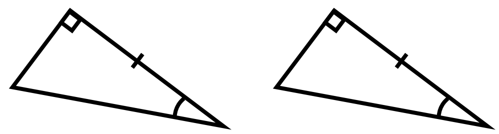


SSA*
(*There is no SSA theorem.)

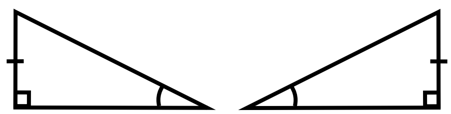
Now replace each S with an L if it's a leg and with an H if it's the hypotenuse. Leave out any A that stands for a right angle.



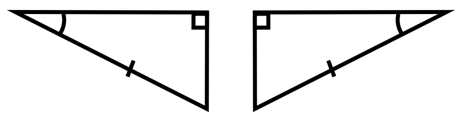
SAS → LL



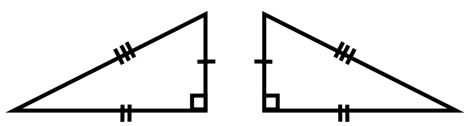
ASA → LA



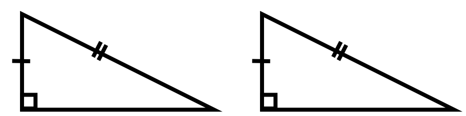
SAA → LA



SAA → HA



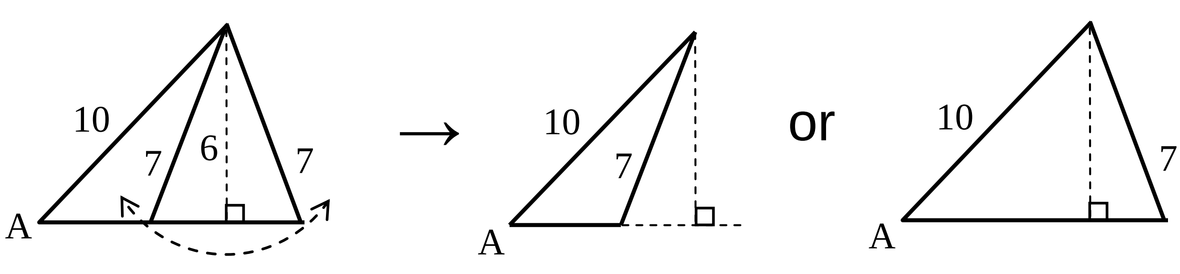
SSS or SAS → LL
HL



SSA → HL

*SSA only works for right triangles.

<u>SSA with an acute triangle may produce 2 triangles.</u>

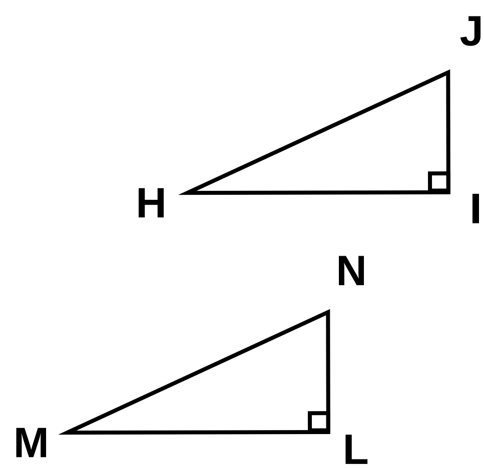


If A is acute, and $h<a<b$, then there are <u>two</u> possible triangles.

HL Congruence Theorem: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the two triangles are congruent.

LL Congruence Theorem: If the two legs of one right triangle are congruent to the two legs of another right triangle, then the two triangles are congruent.

Proof:



Given: ΔHIJ and ΔMLN are right triangles; $\overline{HI} \cong \overline{ML}$; $\overline{JI} \cong \overline{NL}$

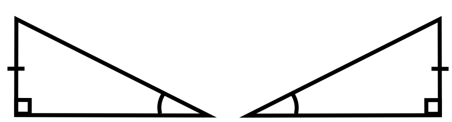
Prove: $\Delta HIJ \cong \Delta MLN$

Statement	Reason
1. ΔHIJ & ΔMLN are rt Δ's $\overline{HI} \cong \overline{ML}$; $\overline{JI} \cong \overline{NL}$	1. Given
2. $\angle I$ & $\angle L$ are rt. angles	2. Def. of right triangle
3. $\angle I \cong \angle L$	3. Rt. angles are congr.
4. $\Delta HIJ \cong \Delta MLN$	4. SAS
5. If the two legs of one right triangle are congruent to the two legs of another right triangle, then the two triangles are congruent.	5. Law of Deduction

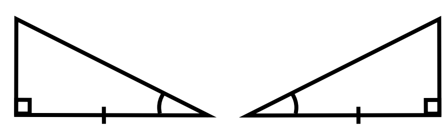
HA Congruence Theorem: If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the two triangles are congruent.

LA Congruence Theorem: If a leg and one of the acute angles of a right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the two triangles are congruent.

***Note**: LA has 2 cases, depending on whether the leg is opposite or adjacent to the angle.



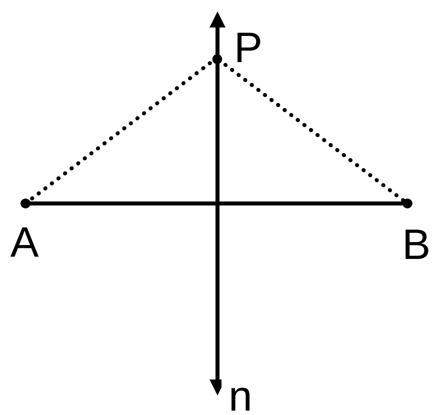
LA with leg opposite the angle



LA with leg adjacent to the angle

*****The LL, LA, and HA Congruence Theorems follow directly from SAS, ASA, and SAA.

Theorem 7.5: Any point lies on the perpendicular bisector of a segment if and only if it is equidistant from the two endpoints.

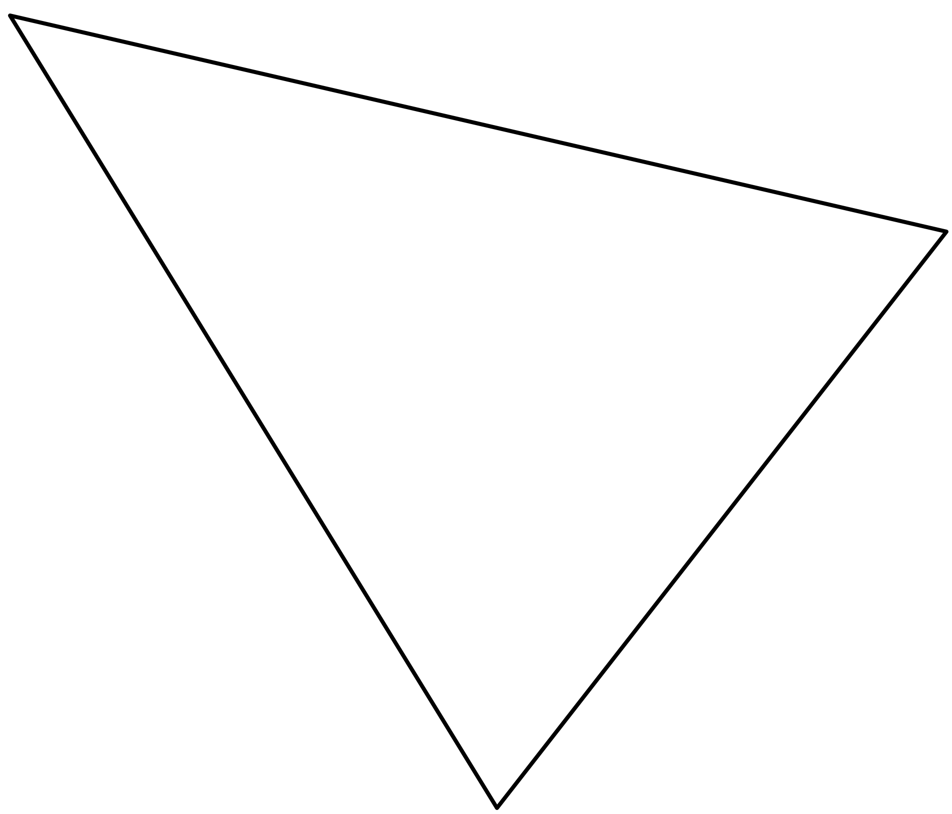


If n is the perpendicular bisector of $\overline{AB}$, then AP must equal BP.

If AP = BP, then line n must be the perpendicular bisector of $\overline{AB}$.

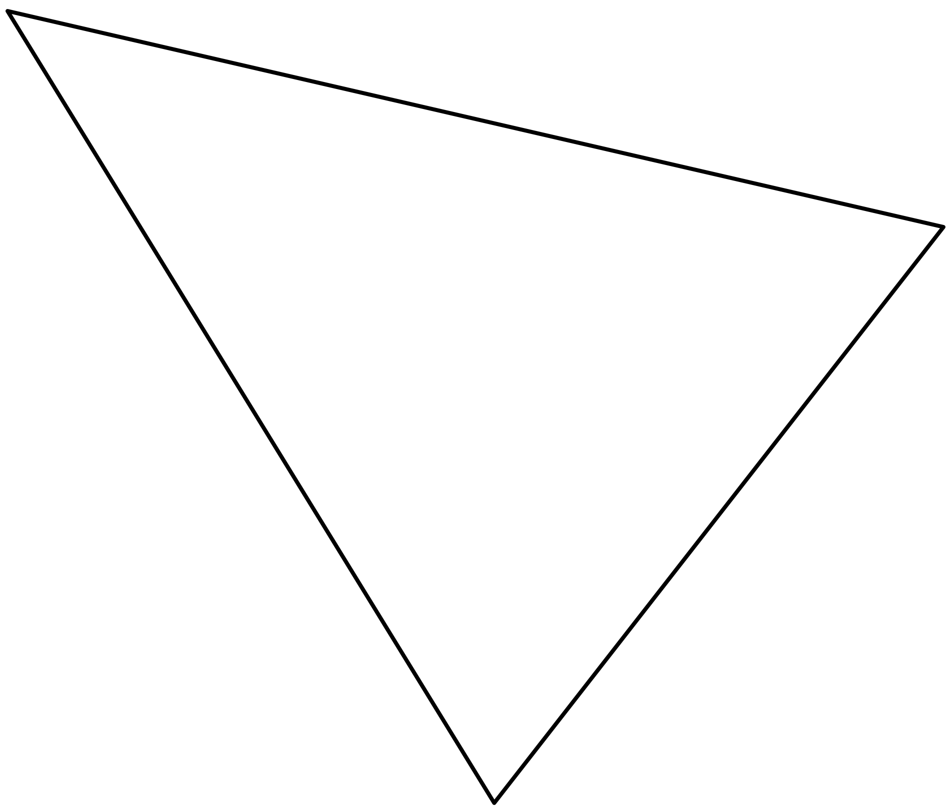
CHAT Geometry Section 7.2

Construct the perpendicular bisectors of each side of the triangle:



CHAT Geometry Section 7.2

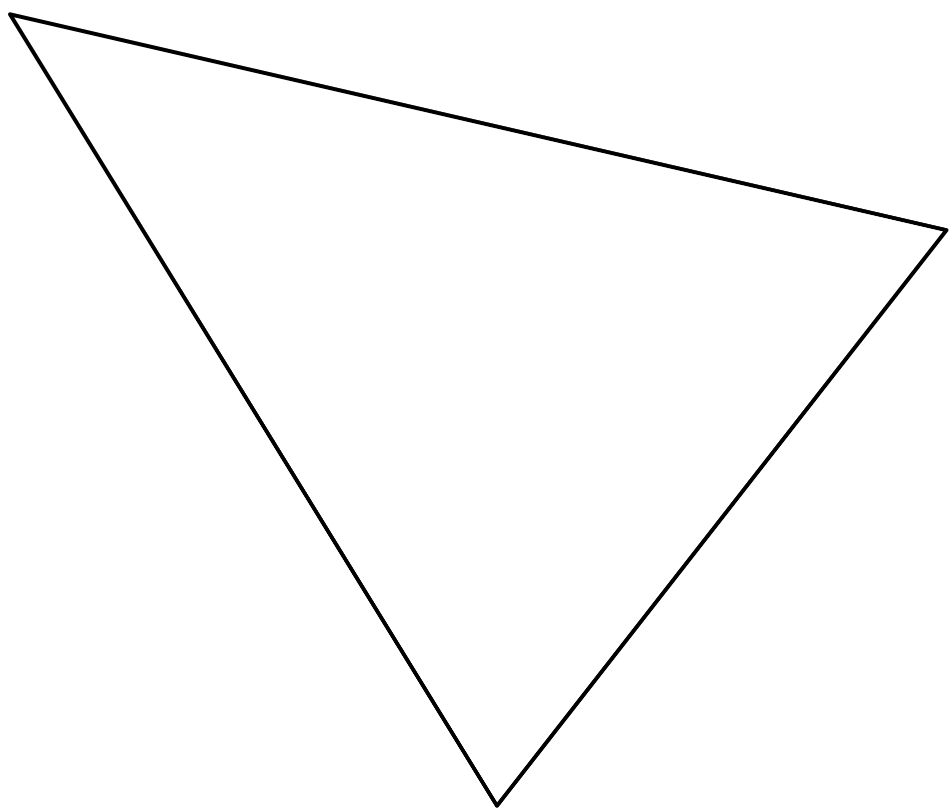
Construct the angle bisectors of each angle of the triangle:



CHAT Geometry Section 7.2

Definition: An altitude of a triangle is a segment that extends from a vertex and is perpendicular to the opposite side.

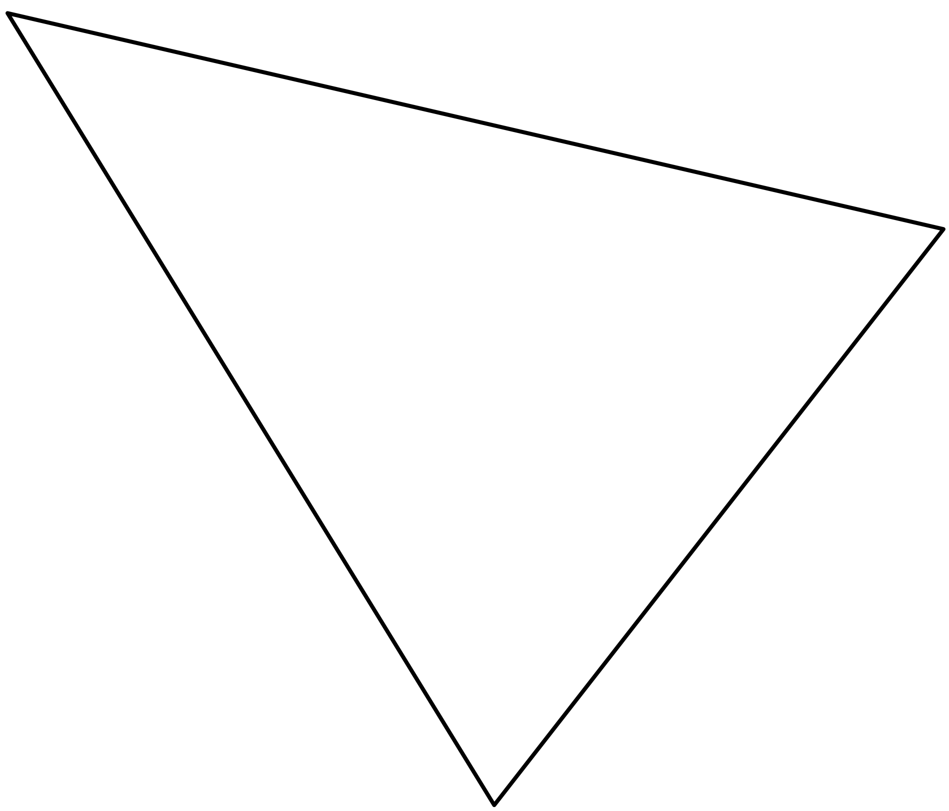
Construct the 3 altitudes of this triangle:



CHAT Geometry Section 7.2

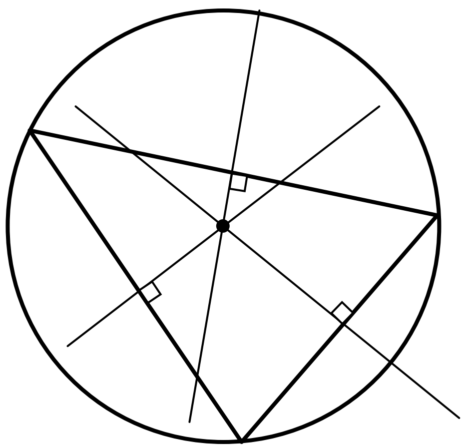
Definition: A median of a triangle is a segment that extends from a vertex to the midpoint of the opposite side.

Construct the 3 medians of the triangle:



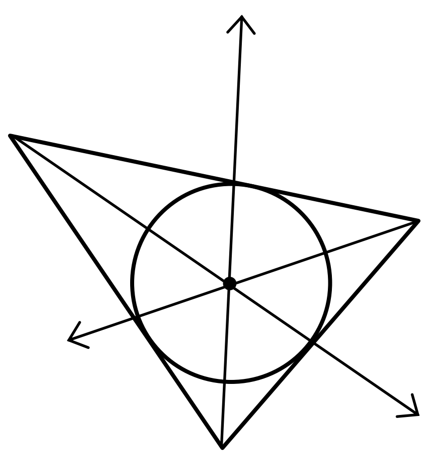
Hint: You will first have to construct the perpendicular bisector of each side to find the midpoint. (Draw them very lightly, as they are not part of your answer.)

Circumcenter Theorem: The perpendicular bisectors of the sides of any triangle are concurrent at the circumcenter, which is equidistant from each vertex of the triangle.



The circumcenter is the center of the circle that is circumscribed around the triangle.

Incenter Theorem: The angle bisectors of the angles of a triangle are concurrent at the incenter, which is equidistant from the sides of the triangle.

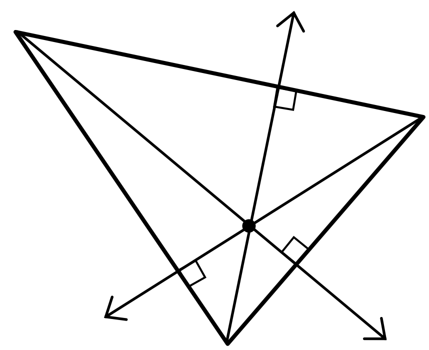


The incenter is the center of the circle that is inscribed in the triangle.

Definition:

The <u>altitude of a triangle</u> is a segment that extends from a vertex and is perpendicular to the opposite side.

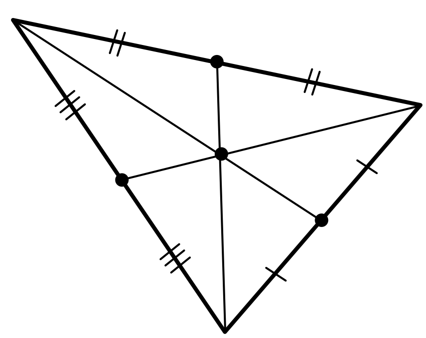
Orthocenter Theorem: The lines that contain the three altitudes of a triangle are concurrent at the orthocenter.



Definition:

A <u>median of a triangle</u> is a segment extending from a vertex to the midpoint of the opposite side.

Centroid Theorem: The three medians of a triangle are concurrent at the centroid.



Summary:

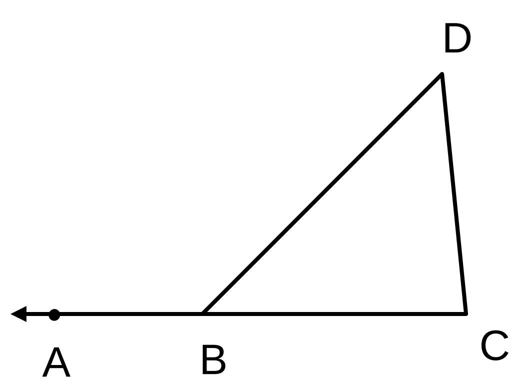
perpendicular bisectors → circumcenter
angle bisectors → incenter
altitudes → orthocenter
medians → centroid

section 7.3

Definitions:

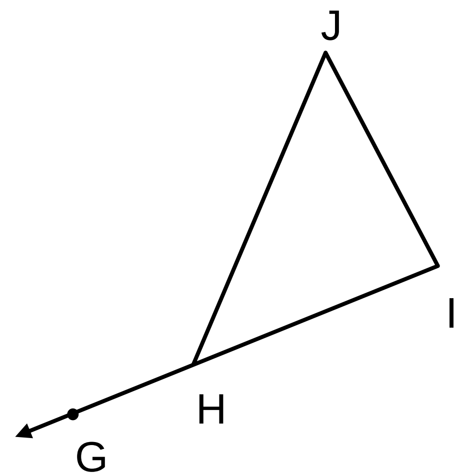
An exterior angle of a triangle is an angle that forms a linear pair with one of the angles of the triangle.

The remote interior angles of an exterior angle are the 2 angles of the triangle that do not form a linear pair with a given exterior angle.



∠ABD is an exterior angle
∠C and ∠D are remote interior angles of ∠ABD

Exterior Angle Theorem: The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.



Given: ∆HIJ with exterior ∠JHG

Prove: m∠JHG = m∠I + m∠J

Statement	Reason
1. ∆HIJ with exterior ∠JHG	1. Given
2. ∠IHJ and ∠JHG form a linear pair	2. Definition of exterior angle
3. ∠IHJ and ∠JHG are supplementary angles	3. Linear pairs are supplementary
4. m∠IHJ + m∠JHG = 180	4. Def. of supp. angles
5. m∠IHJ+ m∠I+ m∠J=180	5. The measures of the ∠'s of ∆ total 180°
6. m∠IHJ + m∠JHG = m∠IHJ+ m∠I+ m∠J	6. Transitive
7. m∠JHG = m∠I+ m∠J	7. Add. Prop. of Equality

Inequality Properties

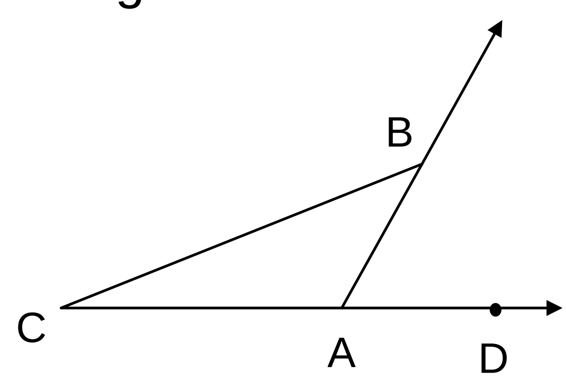
Property	Meaning
Addition	If a>b, then a+c > b+c
Multiplication	If a>b and c>0, then ac>bc If a>b and c<0, then ac<bc
Transitive	If a>b and b>c, then a>c
Definition of greater than	If a=b+c, and c>0, then a>b

Exterior Angle Inequality Theorem: The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

Proof:

Given: ΔABC and exterior $\angle DAB$

Prove: $m\angle DAB > m\angle B$ and $m\angle DAB > m\angle C$

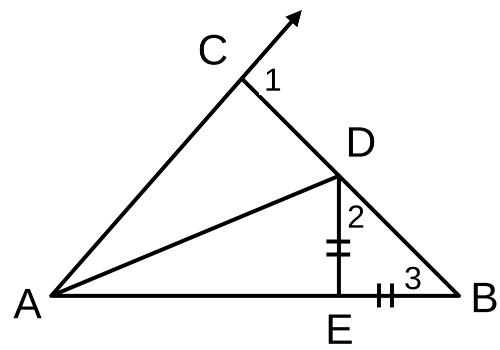


Statement	Reason
1. ΔABC and exterior $\angle DAB$	1. Given
2. $m\angle DAB = m\angle B + m\angle C$	2. Exterior Angle Thm.
3. $m\angle DAB > m\angle B$	3. Def. of greater than
4. $m\angle DAB > m\angle C$	4. Def. of greater than

Sample Problem:

Given: $\overline{DE} \cong \overline{BE}$

Prove: $m\angle 1 > m\angle 2$



Statement	**Reason**
1. $\overline{DE} \cong \overline{BE}$	1. Given
2. $\angle 2 \cong \angle 3$	2. Isosceles Δ Thm.
3. $m\angle 2 = m\angle 3$	3. Def. of congruent ∠
4. $m\angle 1 > m\angle 3$	4. Exterior Angle Inequality Thm.
5. $m\angle 1 > m\angle 2$	5. Subst. (step 3 into 4)