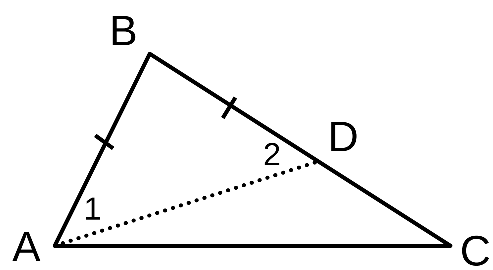
section 7.4

Longer Side Inequality Theorem: One side of a triangle is longer than another side of a triangle if and only if the measure of the angle opposite the longer side is greater than the angle opposite the shorter side.

Proof of Part 1:

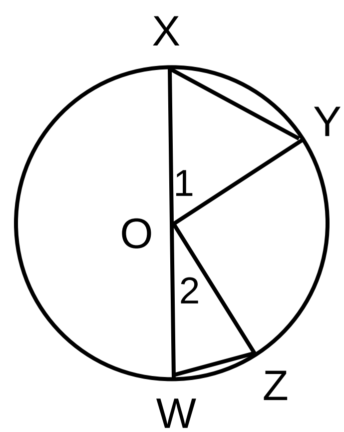


Given: ΔABC, BC>AB
Construct: AD such that B-D-C and $\overline{AB} \cong \overline{AD}$
Prove: M∠A > m∠C

Statement	Reason
1. ΔABC; BC>AB; $\overline{AB} \cong \overline{AD}$	1. Given
2. ΔABD is an isosceles Δ	2. Def. of Isosceles Δ
3. ∠1 ≅ ∠2	3. Isosceles Δ Thm.
4. m∠1 = m∠2	4. Def. of congr. angle
5. m∠CAD+m∠1=m∠CAB	5. Angle Add. Post.
6. m∠CAB > m∠1	6. Def. of greater than
7. m∠CAB > m∠2	7. Substitution
8. m∠2 > m∠C	8. Exterior ∠ Ineq.
9. m∠CAB > m∠C	9. Trans. prop of ineq.
10. If one side of a Δ is longer than another side of a Δ, then the measure of the ∠ opposite the longer side is greater than the ∠opposite the shorter side.	10. Law of Deduction

Hinge Theorem: Two triangles have 2 pairs of congruent sides. If the measure of the included angle of the first triangle is larger than the measure of the other included angle, then the opposite (3^{rd}) side of the first triangle is longer than the opposite (3^{rd}) side of the 2^{nd} triangle.

Sample Problem:

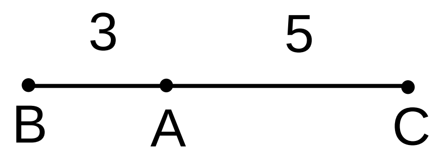


Given: Circle O
with $m\angle 1 > m\angle 2$

Prove: XY > WZ

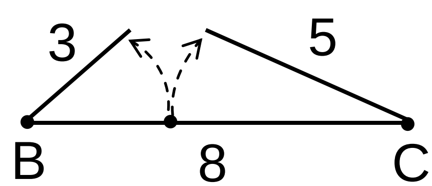
Statement	Reason
1. Circle O with $m\angle 1 > m\angle 2$	1. Given
2. $\overline{OX} \cong \overline{OW}$ $\overline{OZ} \cong \overline{OY}$	2. All radii of a circle are congruent
3. XY > WZ	3. Hinge Theorem

Look at:

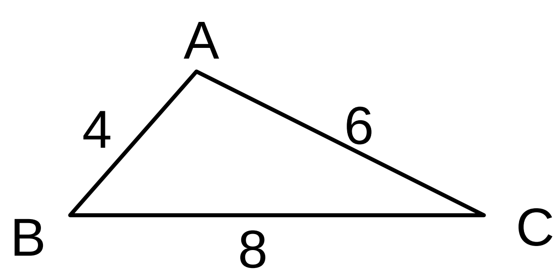


If BC = 8, then we must have B-A-C, and B, A, and C must be collinear.

If we try to make a triangle from these given lengths, we cannot.

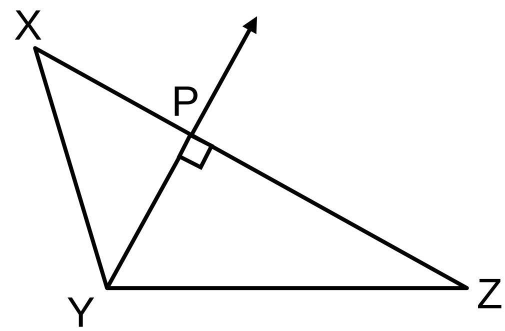


What if BA and AC are larger, so that BA + AC > BC?



We will get a triangle as long as the sum of the lengths of 2 sides is greater than the length of the third side.

Triangle Inequality Theorem (7.14): The sum of the lengths of any 2 sides of a triangle is greater than the length of the third side.



Proof:

Given: ΔXYZ with no side longer than $\overline{XZ}$

Auxilliary line: $\overleftrightarrow{PY} \perp \overleftrightarrow{XZ}$ at P

Prove: $XZ+YZ > XY$
$XZ+XY > YZ$
$XY+YZ > XZ$

Trichotomy guarantees that you can list the 3 sides of a triangle in order of length from shortest to longest. Label side XZ so that there is no longer side. Since $XZ \geq XY$, it follows that $XZ+XY > YZ$ (distance $YZ>0$). Similarly, $XZ \geq YZ$, ao $XZ+XY > YZ$. These 2 inequalities were easy to prove, but the third is harder and requires the altitude from Y to $\overline{XZ}$ as an auxiliary line.

Statement	Reason
1. ΔXYZ with $\overline{XZ}$ the longest side, altitude $\overline{YP}$	1. Given
2. $\overline{YP} \perp \overline{XZ}$	2. Def.of altitude
3. $\angle XPY$ & $\angle ZPY$ are rt $\angle$'s	3. Def. of $\perp$
4. ΔXPY & ΔZPY are rt Δ's	4. Def. of right Δ's
5. $XY > XP$, $YZ > PZ$	5. Hypot. is longest
6. $XY+YZ > XP + PZ$	6. Add. of Inequalities
7. $XP + PZ = XZ$	7. Def. of between
8. $XY+YZ > XZ$	8. Sub. (step 7 into 6)

Sample Problem:

1. Can a triangle be constructed with lengths 5, 8,13?

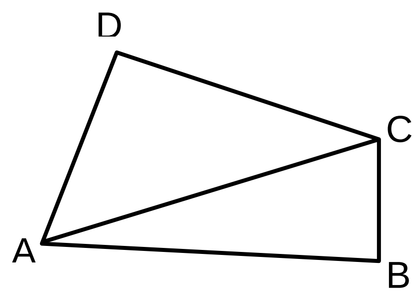
   no, 5+8 is not greater than 13

2. Given sides 4 and 7, what is the range of possible values for the 3^{rd} side?

   (4+7) must be greater than the 3rd side
   (3rd side + 4) and be greater than 7
   So, the 3rd side must be between 3 and 11

3. *Given*: Quadrilateral ABCD

***Prove*:** AD+DC+BC > AB

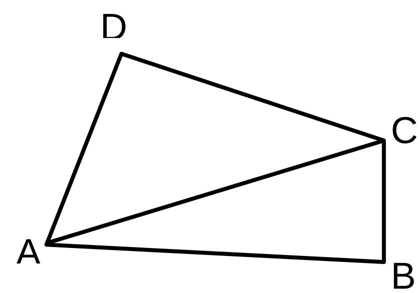


Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.

3. Solution:

Given: Quadrilateral ABCD

Prove: AD+DC+BC > AB



Statement	Reason
1. Quadrilateral ABCD	1. Given
2. AD+DC > AC AC+BC > AB	2. Triangle inequality theorem
3. AD+DC+ AC+BC>AC+AB	3. Addition of ineq.
4. AD+DC+BC>AB	4. Add. prop. of ineq.

section 7.6

Definition:

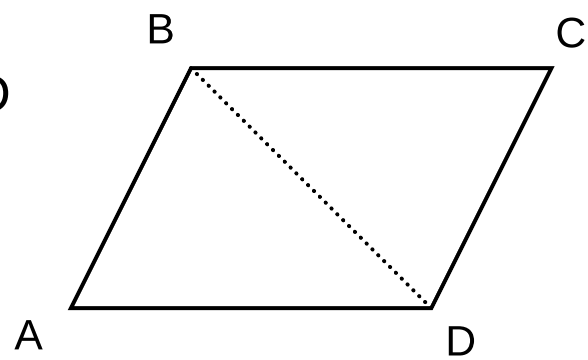
A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.

Theorem 7.15: The opposite sides of a parallelogram are congruent.

Proof**:**

Given: Parallelogram ABCD

Prove: $\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{AD}$



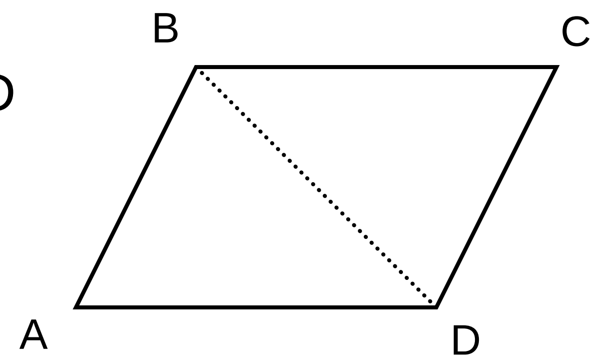
Statement	**Reason**
1. ABCD is a parallelogram	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

Theorem 7.15: The opposite sides of a parallelogram are congruent.

Proof**:**

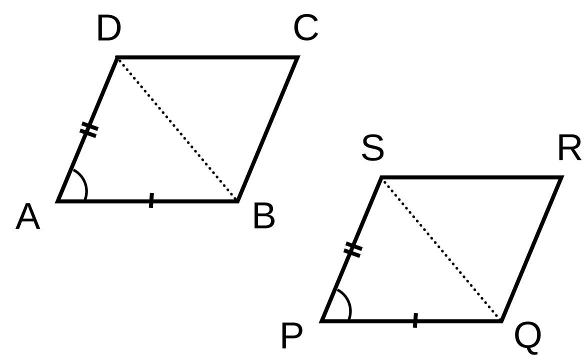
Given: Parallelogram ABCD

Prove: $\overline{AB} \cong \overline{CD}$
$\overline{BC} \cong \overline{AD}$



Statement	Reason
1. ABCD is a parallelogram	1. Given
2. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$	2. Def. of parallelogram
3. Draw $\overleftrightarrow{BD}$	3. Line Postulate
4. $\angle ABD \cong \angle CDB$ $\angle CBD \cong \angle ADB$	4. Parallel Postulate
5. $\overline{BD} \cong \overline{BD}$	5. Reflexive
6. $\Delta ABD \cong \Delta CBD$	6. ASA
7. $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$	7. Def. of congr. Δ's

Thm. 7.16: SAS Congruence for Parallelograms:
If two sides and the included angle of a parallelogram are congruent to the corresponding two sides and included angle of another parallelogram, then the parallelograms are congruent.

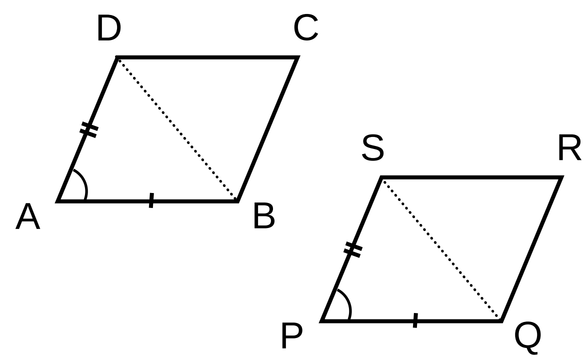


Given: Parallelograms ABCD and PQRS
$\overline{AB} \cong \overline{PQ}$, $\overline{AD} \cong \overline{PS}$,
$\angle A \cong \angle P$

Prove: ABCD $\cong$ PQRS

Statement	Reason
1. Parallelograms ABCD and PQRS, $\overline{AB} \cong \overline{PQ}$, $\overline{AD} \cong \overline{PS}$, $\angle A \cong \angle P$	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.

Thm. 7.16: SAS Congruence for Parallelograms:
If two sides and the included angle of a parallelogram are congruent to the corresponding two sides and included angle of another parallelogram, then the parallelograms are congruent.



Given: Parallelograms ABCD and PQRS
$\overline{AB} \cong \overline{PQ}$, $\overline{AD} \cong \overline{PS}$,
$\angle A \cong \angle P$

Prove: ABCD ≅ PQRS

Statement	Reason
1. Parallelograms ABCD and PQRS, $\overline{AB} \cong \overline{PQ}$, $\overline{AD} \cong \overline{PS}$, $\angle A \cong \angle P$	1. Given
2. Draw $\overleftrightarrow{BD}$ and $\overleftrightarrow{QS}$	2. Auxiliary lines
3. $\Delta ABD \cong \Delta PQS$	3. SAS
4. $\overline{BD} \cong \overline{QS}$	4. Def. of congr. Δ
5. $\overline{AB} \cong \overline{CD}$, $\overline{PQ} \cong \overline{RS}$, $\overline{AD} \cong \overline{BC}$, $\overline{PS} \cong \overline{QR}$	5. Opp. sides of a parallelogram congr
6. $\overline{BC} \cong \overline{QR}$, $\overline{CD} \cong \overline{RS}$	6. Transitive of congr.
7. $\Delta BCD \cong \Delta QRS$	7. SSS
8. ABCD ≅ PQRS	8. Subdivision into corres. congr. Δ's

Theorem 7.17: A quadrilateral is a parallelogram if and only if the diagonals bisect one another.

Theorem 7.18: Diagonals of a rectangle are congruent.

Theorem 7.19: The sum of the measures of the 4 angles of every convex quadrilateral is 360°

Theorem 7.20: Opposite angles of a parallelogram are congruent.

Theorem 7.21: Consecutive angles of a parallelogram are supplementary.

Theorem 7.22: If the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram

Theorem 7.23: A quadrilateral with one pair of parallel sides that are congruent is a parallelogram.