section 7.7

Construction 3: Bisect a Segment

Given: Line n containing point X.

1. Place the point of the compass on X and make intersecting arcs on line n on each side of X. Label these points A and B.

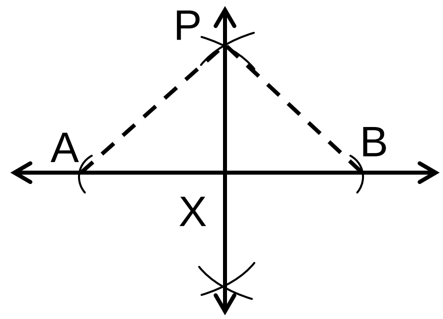
1. Place the point of the compass at A and then at B, making intersecting arcs above and below the line segment.

2. Connect the two intersecting points to form the line perpendicular to line n at the point B. (The line will also be the bisector of $\overline{AB}$.)

**To prove the construction, we must show that the angle formed through our construction is a right angle.

Justification:

Show that $\angle PXA$ is a right angle, and thus the lines are perpendicular ($\overleftrightarrow{PX} \perp \overleftrightarrow{AB}$)

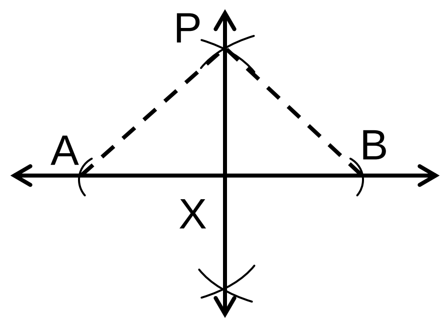


Statement	Reason
1. $\overleftrightarrow{AB}$ contains point X	1. Given
2. $\overline{PA} \cong \overline{PB}$, $\overline{AX} \cong \overline{BX}$	2.
3. $\overline{PX} \cong \overline{PX}$	3.
4. $\Delta PAX \cong \Delta PBX$	4.
5. $\angle PXA \cong \angle PXB$	5.
6. $\angle PXA$ & $\angle PXB$ form a linear pair	6.
7. $\angle PXA$ & $\angle PXB$ are supplementary	7.
8. $\angle PXA$ is right angle	8.
9. $\overleftrightarrow{PX} \perp \overleftrightarrow{AB}$	9.

Solution:

***Justification*:**

Show that ∠PXA is a right angle, and thus the lines are perpendicular ($\overleftrightarrow{PX} \perp \overleftrightarrow{AB}$)



Statement	Reason
1. $\overleftrightarrow{AB}$ contains point X	1. Given
2. $\overline{PA} \cong \overline{PB}$, $\overline{AX} \cong \overline{BX}$	2. Radii of ≅ circles ≅
3. $\overline{PX} \cong \overline{PX}$	3. Reflexive
4. ΔPAX ≅ ΔPBX	4. SSS
5. ∠PAX ≅ ∠PBX	5. Def. of ≅ Δ's
6. ∠PXA & ∠PXB form a linear pair	6. Def. of linear pair
7. ∠PXA & ∠PXB are supplementary	7. ∠'s that form a linear pair are supp. (4.3)
8. ∠PXA is right angle	8. Congruent supplementary ∠'s are right angles (4.6)
9. $\overleftrightarrow{PX} \perp \overleftrightarrow{AB}$	9. Def. of perpendicular

<u>Construction 9</u>: Copy a triangle.

***Given*:** ΔABC
***Construct*:** A triangle congruent to ΔABC

1. Draw a line. Choose a point on the line and call it A'.

2. Using a compass, measure length AB. Place the point of the compass at A' and mark off a segment congruent to AB on the line Call the point B'.

3. Using measure AC and using A' as center, construct an arc above A'B'.

4. Repeat step 3, using measure BC with B' as center.

5. The arcs intersect at a point. Call it C'. Connect C' with A' and B' to form a triangle congruent to the original triangle.

Construction 10: Copy a polygon.

Given: A polygon
Construct: A polygon congruent to the given polygon

1. Subdivide the given polygon into triangles.
2. Copy one of the triangles (construction 9).
3. Copy adjacent triangles until the polygon is complete.

Construction 11: Construct a line parallel to a given line through a point not on the line.

Given: Line k and point A not on line k.
Construct: A line containing the point A that is parallel to line k.

1. Draw a line that goes through point A and intersect the given line at point B.
2. Construct an angle, $\angle$BAE, congruent to $\angle$ABC, so that E and C are in opposite half-planes (construction 4).
3. The two angles will be congruent alternate interior angles, thus AE is parallel to K by the Parallel Postulate.

Chapter 7 Vocabulary:

altitude
centroid
circumcenter
concurrent lines
exterior angle of a triangle
HACongruence Theorem
Hinge Theorem
HL Congruence Theorem
incenter
LA Congruence Theorem
LL Congruence Theorem
longer side inequality
median of a triangle
orthocenter
remote interior angle
triangle inequality

section 8.1

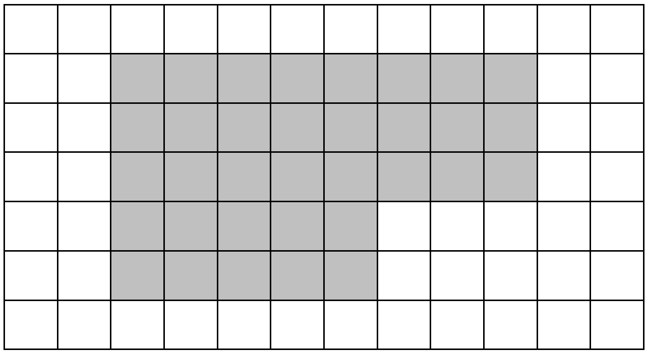
Definition:

The area of a region is the number of square units needed to cover it completely.

***Note**: We find the area of regions, not the polygon itself. A polygonal region is the union of the polygon and its interior.

Linear measure is one-dimensional
Area measure is two-dimensional

Example:



Area = 34 square units

Area Postulate (8.1): Every region has an area given by a unique positive real number.

(An area can't have 2 different areas)

Congruent Regions Postulate (8.2): Congruent regions have the same area.

Area of Square Postulate (8.3): The area of a square is the square of the length of one side: $A = s^2$

Area Addition Postulate (8.4): If the interior of 2 regions do not intersect, then the area of the union is the sum of their areas.

Theorem 8.1: The area of a rectangle is the product of its base and height: $A = bh$

Sample Problems:

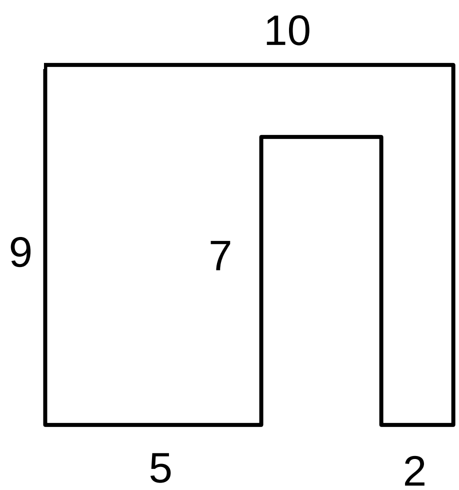
1. Find the area of a rectangle that measures 6' x 12'

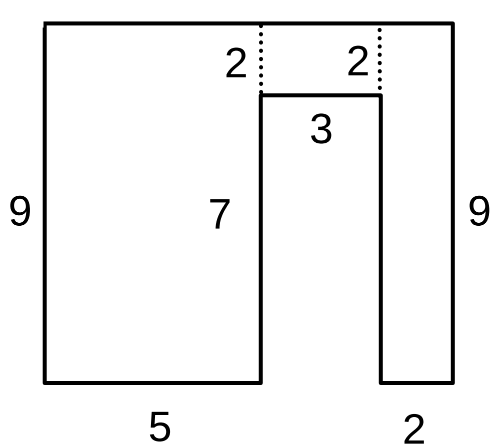
   $A = bh = 6(12) = 72$ sq. ft.

2. Find the area of a square with side 5 mm.

   $A = s^2 = 5^2 = 25\ mm^2$

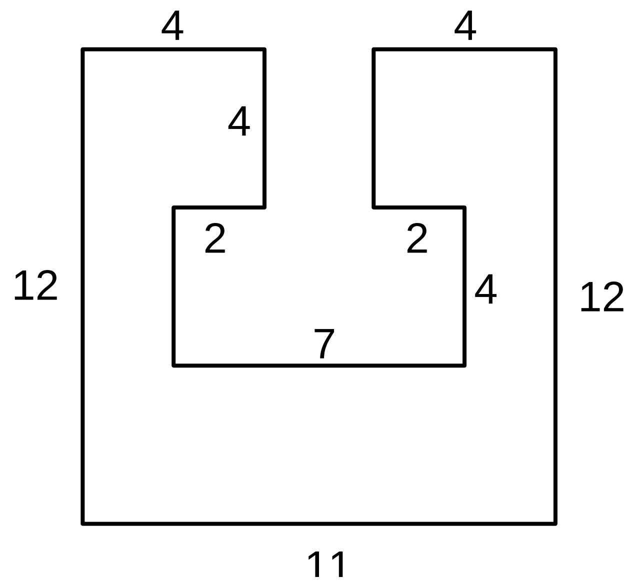
3. Find the area of the following region.



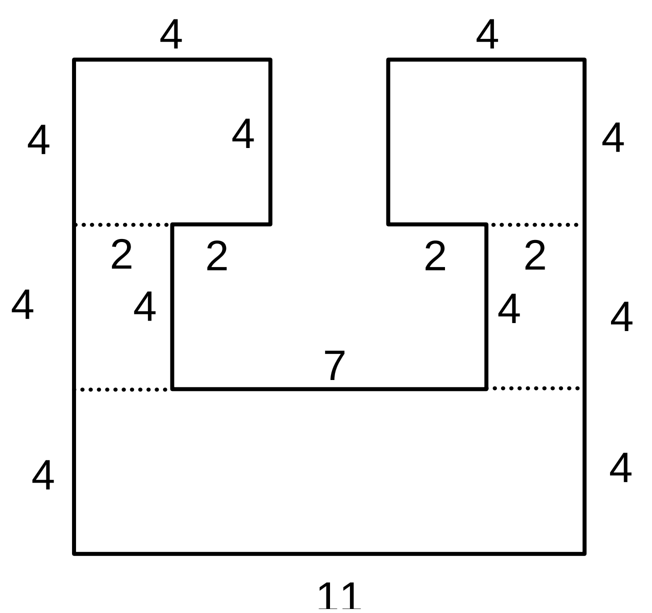


Area = (9x5)+(3x2)+(9x2)
Area = 45 + 6 + 18
Area = 69 sq. units

4. Find the area of the following region:



Answer:



Area = 2(4x4)
+ 2(2x4)
+ 1(4x11)

Area = 32+16+44

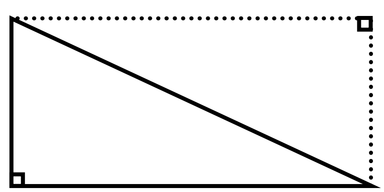
Area = 92 sq. units

5. A rectangle has a base of 16 feet and an area of 128 square feet. What is the height of the rectangle?

$$A = bh$$
$$\frac{128}{16} = \frac{16h}{16}$$
$$h = 8 \text{ ft.}$$

section 8.2

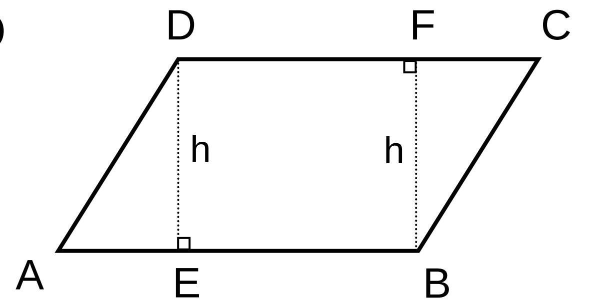
<u>Theorem 8.2</u>: The <u>area of a right triangle</u> is one-half of the product of the lengths of the legs: A = ½bh



<u>Theorem 8.3</u>: The <u>area of a parallelogram</u> is the product of the base and the altitude: A = bh

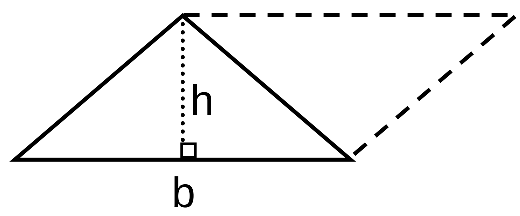
Note: The <u>altitude</u> is a perpendicular line from the base to the opposite side.

Given: Parallelogram ABCD
Draw: Altitudes DE and FB
Prove: A = bh

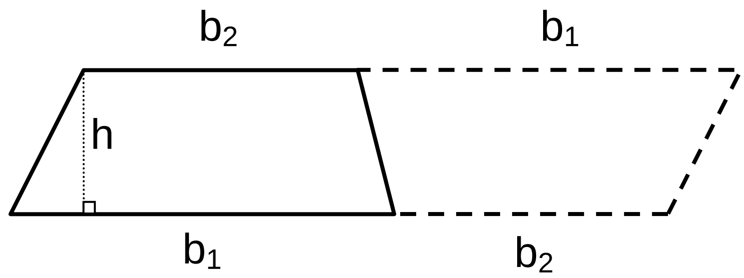


Statement		Reason	
1.	Parallelogram ABCD	1.	Given
2.	$\overline{BC} \cong \overline{AD}$	2.	Opposite sides ≅
3.	∠A ≅ ∠ C	3.	Opposite angles ≅
4.	$\overleftrightarrow{DE} \perp \overleftrightarrow{AB}$; $\overleftrightarrow{BF} \perp \overleftrightarrow{CD}$	4.	Def. of altitudes
5.	∠AED and ∠CFB are right angles	5.	Def. of perpendicular
6.	ΔADE ≅ ΔCBF	6.	HA (or SSS)
7.	Area ΔADE = ½(AE)h	7.	Area rt. Δ Thm.
8.	Area ΔCBF = ½(AE)h	8.	Congr. Regions Post.
9.	Area BEDF = (BE)h	9.	Area of Rect. Thm.
10.	Area ABCD = Area ΔADE + Area ΔCBF + Area BEDF	10.	Area Add. Post.
11.	Area ABCD = ½(AE)h + ½(AE)h + (BE)h	11.	Substitution (steps 7,8,9 into 10)
12.	Area ABCD=(AE+BE)h	12.	Distributive Prop.
13.	AE + BE = AB = b	13.	Def. of betweenness
14.	Area ABCD = bh	14.	Subst. (step 13 into 12)

<u>Theorem 8.4</u>: The <u>area of a triangle</u> is one-half of the base times the height: $A = \frac{1}{2}bh$



<u>Theorem 8.5</u>: The <u>area of a trapezoid</u> is one-half the product of the altitude and the sum of the lengths of the bases: $A = \frac{1}{2}(b_1 + b_2)$



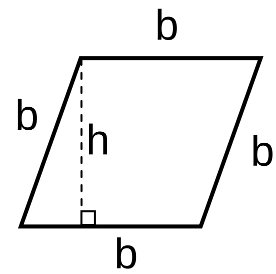
Area of the parallelogram = $(b_1 + b_2)h$

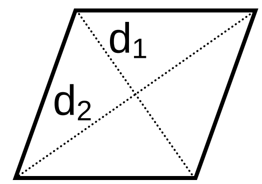
Area of Trapezoid is ½ of the area of the parallelogram

Area of the trapezoid = $1/2(b_1 + b_2)h$

****Note:** Since rectangles, parallelograms, rhombi, and squares are all trapezoids, the formula for the area of a trapezoid should work with all of these regions, too.

<u>Theorem 8.6:</u> The <u>area of a rhombus</u> is one-half of the product of the lengths of the diagonals: $A = \frac{1}{2} d_1 d_2$





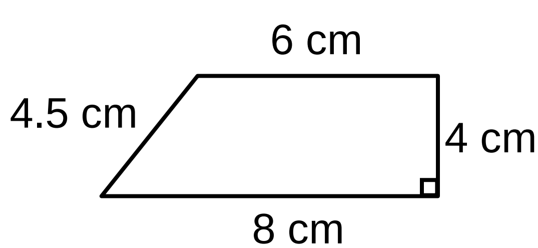
A = bh **$A = \frac{1}{2} d_1 d_2$**

Summary of Area

Figure	Formula
rectangle	$A = bh$
square	$A = s^2$
triangle	$A = \frac{1}{2} bh$
parallelogram	$A = bh$
trapezoid	$A = \frac{1}{2}(b_1+b_2)h$
rhombus	$A = bh$ or $A = \frac{1}{2} d_1 d_2$

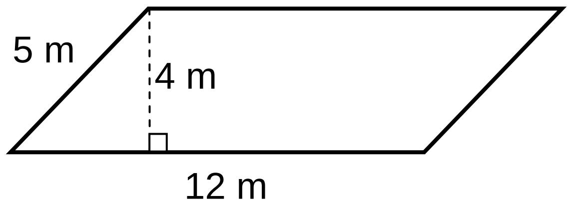
<u>Sample Problems</u>: Find the areas.

1.



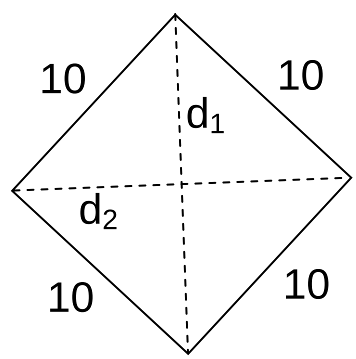
$A = \frac{1}{2} (6+8)(4) = \frac{1}{2}(14)(4) = 28\ cm_2$

2.



A = bh = 12(4) = 48 m_2

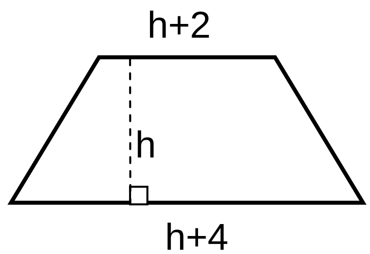
3.



$d_1 = 16$
$d_2 = 12$

A = ½ $d_1 d_2$ = ½(16)(12) = 8(12) = 96 sq. units

4. The bases of a trapezoid are 2 and 4 feet longer than the height respectively. If the area is 54 sq. feet, find the height.



Area = ½([h+2] + [h+4])h
Area = ½(2h + 6)h
Area = ½ h(2h + 6)
Area = $h^2 + 3h$

- Since Area = 54, we have

$54 = h^2 + 3h$
$h^2 + 3h – 54 = 0$
(h+9)(h-6) = 0
h+9 = 0 or h-6 = 0
h = -9 or h = 6

- Since distance can’t be negative,

height = 6 feet