section 8.6

Review:

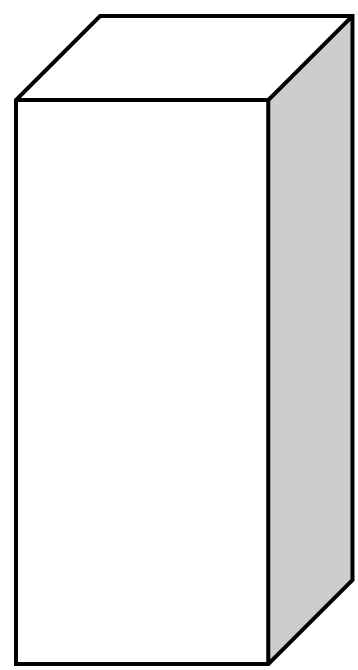
area – the number of squares needed to cover a region

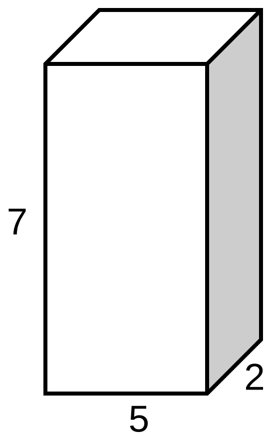
surface area – the number of squares needed to cover the outer shell of a sold in space

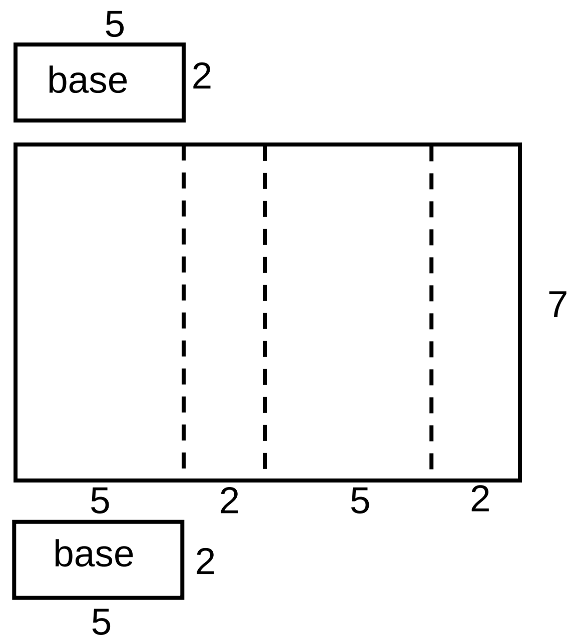
surface – the boundary of a 3-dimensional figure

Prism Terminology:

bases
lateral faces
height of a prism
right prism
lateral surface area







Lateral area = (14)(7) = 98
Area of base = (5)(2) = 10
Surface area = 98 + 2(10) = 118

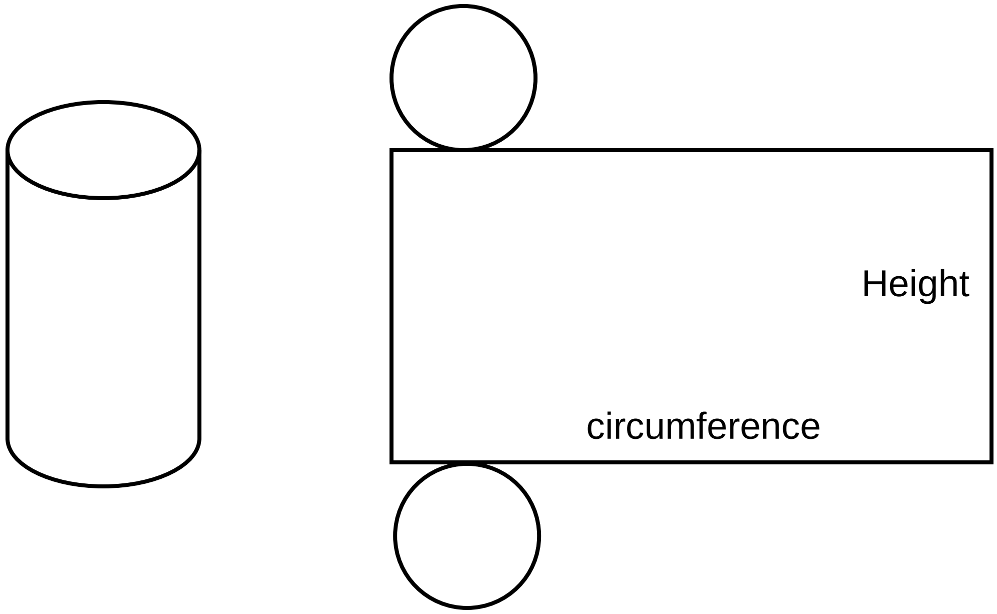
Theorem 8.14: The surface area of a prism is the sum of the lateral surface area and the area of the bases:

$$S = L + 2B$$

The lateral surface area of a right prism is the product of its height and the perimeter of its base:

$$L = pH$$

Special Cases: cubes ($S = 6s^2$)
regular prism – right prism w/regular polygon as its base

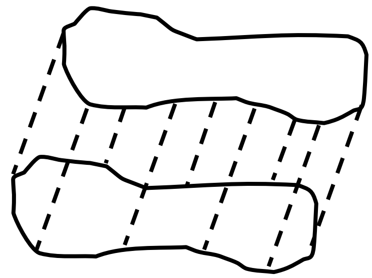


L = pH
L = cH (the perimeter of the base is the circumference)

<u>Theorem 8.15</u>: The <u>surface area of a cylinder</u> is the sum of the lateral surface area and the area of the bases: S = L + 2B

The <u>lateral surface area of a right cylinder</u> is the product of its circumference and height: L = cH

Sample Problem: The base of the following diagram has perimeter of 23 m and area of 20 sq. m. If the height is 11m, find the surface area.



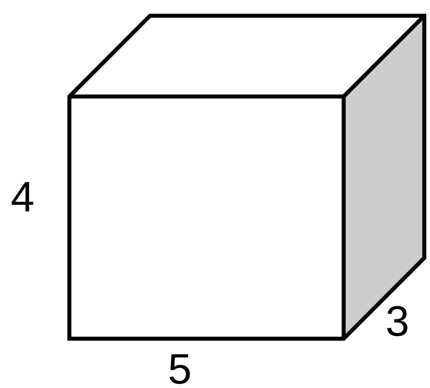
Answer: S = L + 2B

S = pH + 2B

S = 293 sq. meters

Sample Problems: Find the lateral and total surface area of the following solid figures.

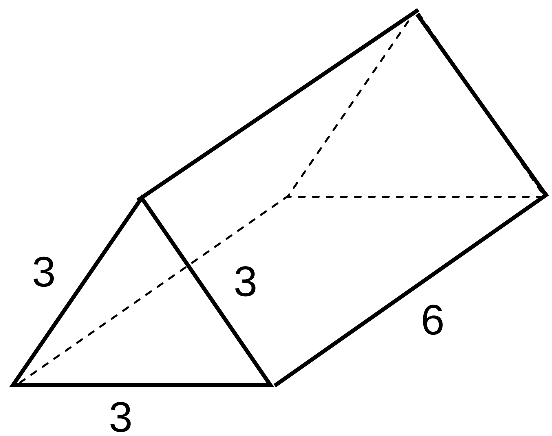
1.



Answer: L = pH = (16)(4) = 64

S = L + 2B = 64 + 30 = 94

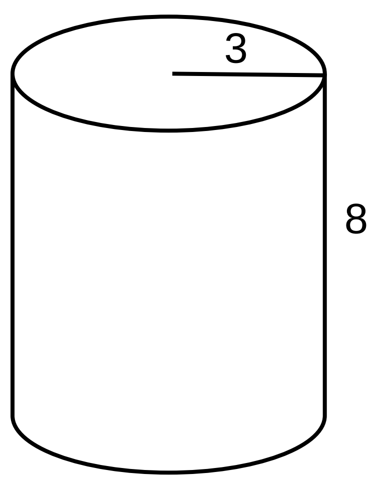
2.



Answer: L = pH = 54

S = L + 2B = 54 + $\frac{9\sqrt{3}}{2}$

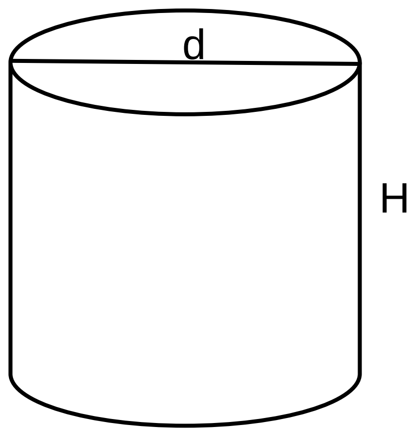
3.



Answer: L = cH = 48π

S = L + 2B = 66π

<u>Sample Problem:</u> A right cylinder has the same height as diameter. If the total surface area is 96π sq. inches, what is the size of the cylinder?



<u>Answer</u>: Let x = height and diameter

$$S = L + 2B$$

$$96\pi = cH + 2B$$

$$96\pi = \pi dH + 2(\pi\, r^2)$$

$$96\pi = \pi xx + 2\pi\left(\frac{x}{2}\right)^2$$

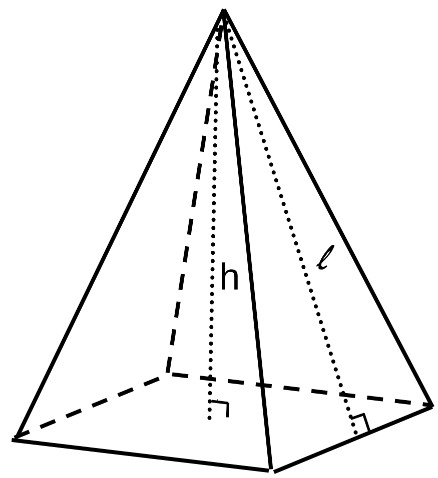
$$96\pi = \pi x^2 + \frac{\pi x^2}{2}$$

$$192\pi = 2\pi x^2 + \pi x^2$$

$$192\pi = 3\pi x^2$$

$$x = 8$$

Terminology for Pyramids



base
lateral faces
vertex
altitude
lateral edges

Surface Area = Lateral surface area + base area

$$S = L + B$$

Definition: A regular pyramid is a right pyramid that has a regular polygon as its base.

***For a regular pyramid, all of the lateral faces are congruent, isosceles triangles.

**The lateral surface area of a regular pyramid is ½ of the perimeter times the slant height.

$$L = \frac{1}{2} p\ell$$

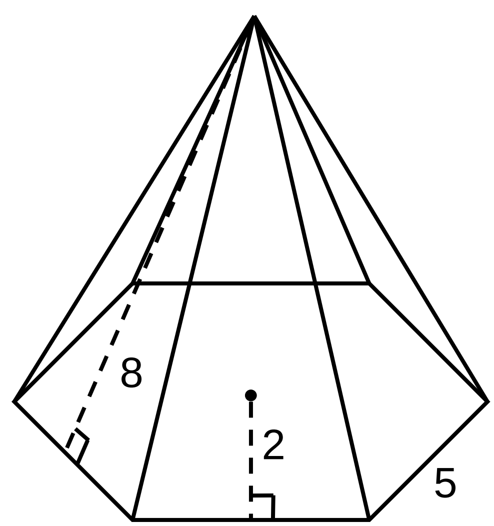
<u>Theorem 8.16</u>: The <u>surface area of a pyramid</u> is the sum of the lateral surface area and the area of the base: $S = L + B$

We know: $L = \frac{1}{2} p\ell$
$B = \frac{1}{2} pa$

So, $S = L + B$
$S = \frac{1}{2} p\ell + \frac{1}{2} pa$
<u>or</u>: $S = \frac{1}{2} p(\ell + a)$

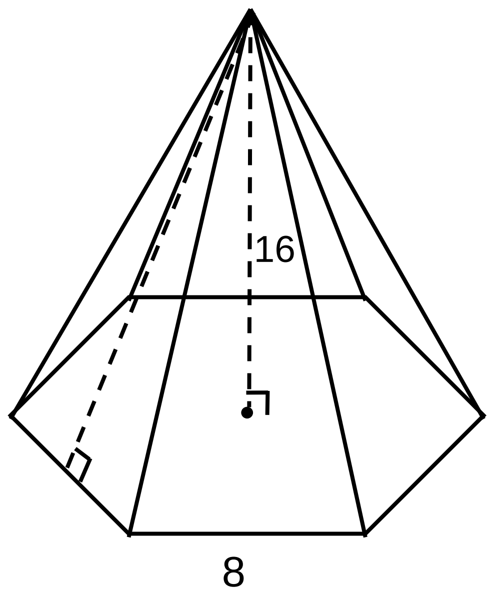
<u>Example</u>:

Find the surface area:

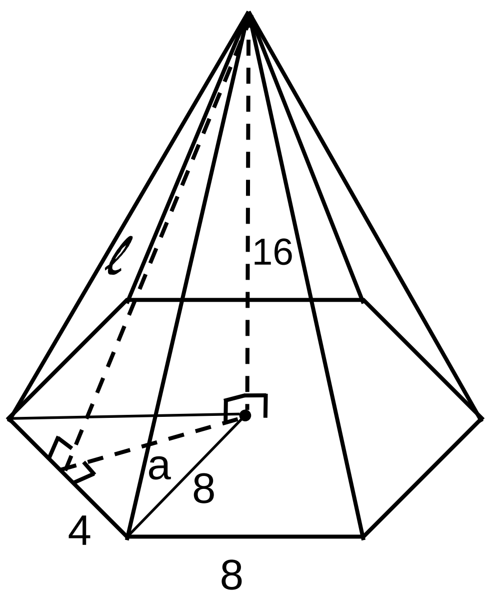


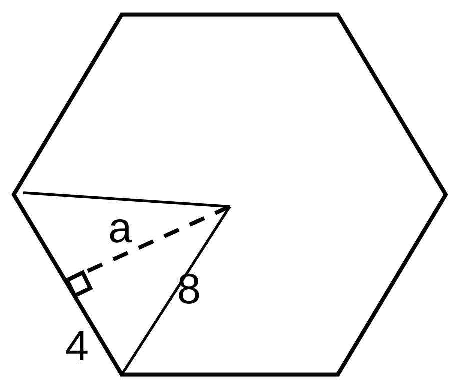
<u>Answer</u>: $S = \frac{1}{2} p(\ell + a)$
$S = \frac{1}{2} (30)(8+2) = 15(10) = 150$ sq. units

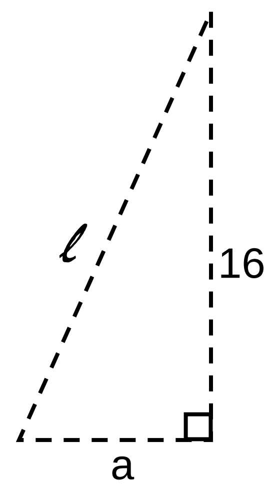
Sample Problem: Find the lateral and surface area.



Answer:







$a^2 = 8^2 - 4^2$
$a^2 = 48$
$a = 4\sqrt{3}$

$\ell^2 = a^2 + 16^2$
$\ell^2 = 48 + 256$
$\ell = 4\sqrt{19}$

$L = \frac{1}{2} p \ell \qquad B = 6\left(\frac{\sqrt{3}}{4}\right)s^2 \qquad S = L+B$

$L = \frac{1}{2}(48)(4\sqrt{19}+) \qquad B = 6\left(\frac{\sqrt{3}}{4}\right)(64) \qquad S = 96\sqrt{19}+96\sqrt{3}$

$L = 96\sqrt{19}+) \qquad B = 96\sqrt{3}$

Circular Cones

****Remember that a pyramid is actually a cone. As the number of sides gets larger, it looks more and more like a circular cone. The perimeter approaches the circumference, and the surface area approaches that of the pyramid.

Pyramid→	$L = \frac{1}{2} p\ell$	$B = \frac{1}{2} ap$	$S = \frac{1}{2} p\ell + \frac{1}{2} ap$
Cone→	$L = \frac{1}{2} c\ell$	$B = \frac{1}{2} rc$	$S = \frac{1}{2} c\ell + \frac{1}{2} rc$

Theorem 8.17: The surface area of a cone is the sum of the lateral surface area and the area of the base:

$$S = L + B$$

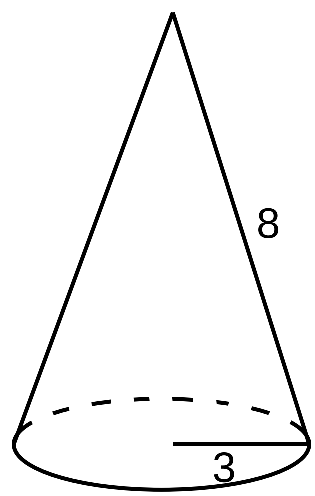
The lateral surface area of a circular cone is half the product of the circumference and slant height:

$$L = \frac{1}{2} c\ell$$

Summary

	Pyramid	Cone
Area of base	$B = \frac{1}{2}ap$	$B = \frac{1}{2}rc$ $B = \pi r^2$
Lateral area	$L = \frac{1}{2}p\ell$	$L = \frac{1}{2}c\ell$ or $\pi r\ell$ $L = \pi r\ell$
Surface area	$S = L + B$ $S = \frac{1}{2}p\ell + \frac{1}{2}ap$ $S = \frac{1}{2}p(\ell + a)$	$S = L + B$ $S = \frac{1}{2}c\ell + \frac{1}{2}rc$ $S = \frac{1}{2}c(\ell + r)$ $S = \pi r\ell + \pi r^2$

<u>Example</u>: Find the surface area.



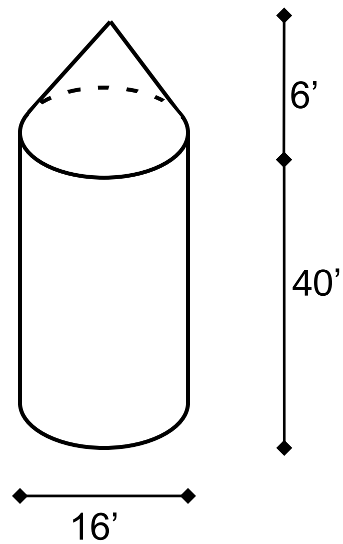
$L = \frac{1}{2}c\ell$
$L = \frac{1}{2}(2\pi r)(8)$
$L = \frac{1}{2}(2\pi)(3)(8)$
$L = 24\pi$

$B = \pi r^2$
$B = \pi(9)$
$B = 9\pi$

$S = L + B$
$S = 24\pi + 9\pi$
$S = 33\pi$ sq. un.

Sample Problems:

1. Find the surface area of a silo with a conical top as shown in the diagram.



Answer:

We need to find the slant height:

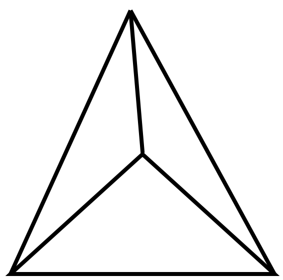
$$\ell^2 = 6^2 + 8^2$$
$$\ell^2 = 36 + 64$$
$$\ell^2 = 100$$
$$\ell = 10$$

Lateral area of cone = $\pi r \ell = \pi(8)(10) = 80\pi$

Lateral area of cylinder = $cH = \pi dH = \pi(16)(40) = 640\pi$

Surface area = $640\pi + 80\pi = 720\pi$ square feet

2. Find the total surface area of a regular tetrahedron with the length of the edge 1 meter.



Answer: Area of each triangle = $\frac{\sqrt{3}}{4}s^2 = \frac{\sqrt{3}}{4}$

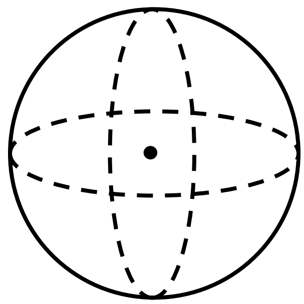
4 triangles, so S = $4 \cdot \frac{\sqrt{3}}{4} = \sqrt{3}$

section 8.8

Definition: A sphere is the set of all points in space equidistant from a given point.

Sphere Terminology:

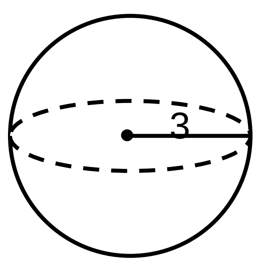
center
radius
great circle – a plane through center that intersects the sphere
lunes – when great circles separate a sphere into segments



All 4 lunes created by the great circles are of equal size and are each ¼ of the sphere. The surface area of one of these lunes is πr^2. This makes the surface area of the entire sphere = $4\pi r^2$.

Theorem 8.18: The surface area of a sphere is 4π times the square of the radius: $S = 4\pi r^2$.

Example: Find the surface area of a sphere with radius = 3.



$$S = 4\pi r^2 = 4\pi(3)^2 = 4\pi(9) = 36\pi$$

Definition: A regular polyhedron is a polyhedron with faces bounded by congruent regular polygons and with the same number of faces intersecting at each vertex.

Historical Note: The 5 regular polyhedra pictured on page 356 are the only possible regular polyhedra. They are called Platonic solids because they were discovered by Plato as the only regular polyhedra possible.

Theorem 8.19: The surface area of a regular polyhedron is the product of the number of faces and the area of one face: $S = nA$.

Sample Problems:

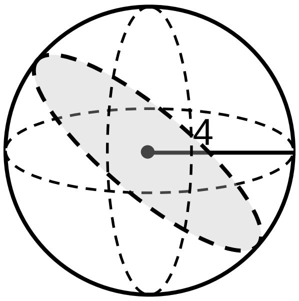
1. Find the surface area of a sphere whose diameter is 16 meters.

   Answer: $S = 4\pi r^2 = 4\pi(8^2) = 256\pi$

2. Find the surface area of a sphere whose circumference is 60π units.

$c = 60\pi$	$S = 4\pi r^2$
$2\pi r = 60\pi$	$S = 4\pi(30^2)$
$r = 30$	$S = 4\pi(900)$
	$S = 3600\pi$

3. Find the surface area of a lune with a central angle of 45° on a sphere of radius 4 inches.



The 2 perpendicular lunes have an angle of 90° and a surface area of $\pi r^2 = 16\pi$. Our lune splits the 90° in half, so the surface area would be ½ of 16π, which is 8π.

4. What is the surface area of a regular octahedron with edge 5 mm. Each face is an equilateral triangle.

8 faces → each Δ has area = $(\sqrt{3}/4)s^{2\ =}=25(\sqrt{3}/4)$

$$S = 8\ \mathrm{C}\ 25(\sqrt{3}/4) = 50\sqrt{3}$$

Chapter 8 Review

Terms:

- altitude (cone, pyramid)
- apothem of a regular polygon
- Area Addition Postulate
- area of a region
- Area of a Square Postulate
- Area Postulate
- center of a regular polygon
- central angle of a regular polygon
- Congruent Regions Postulate
- great circle
- lateral surface area
- lune
- Platonic solid
- Pythagorean Theorem
- radius (circle, regular polygon, sphere)
- regular (polygon, polyhedron, prism, pyramid)
- slant height
- surface area