Section 1.5

Mathematicians desire 3 qualities for a system of postulates:

1. consistent – postulates don't contradict each other
2. independent – postulates can't be proved from other postulates
3. complete – all statements in the system can be proved or disproved from the postulates

Example 1:

Postulates:
1. 2 distinct lines intersect at exactly one point.
2. 2 distinct lines intersect in exactly 2 points.

This system is not consistent because these 2 postulates contradict each other.

Example 2:

Postulates:

1. All dogs are carnivores.
2. Smokey is a carnivore.
3. Smokey is a dog.

Which postulate is unnecessary?

Answer: #2, which can be proved as a theorem given #1 and #3.

Could #3 be deduced from #1 and #2?

Answer: No, there are other carnivores besides dogs. You could not assume Smokey is s dog. It could be a line, for example.

**A system that is not independent is not wrong, it is just not *efficient.*

Incidence Postulates

Expansion Postulate: A line contains at least two points. A plane contains at least three noncollinear points. Space contains at least 4 noncoplanar points.

Line Postulate: Any two points in space lie in exactly one line.

Plane Postulate: Three distinct noncollinear points lie in exactly one plane.

Flat Plane Postulate: If two points lie ina plane, then the line containing these two points lies in the same plane.

Plane Intersection Postulate: If two planes intersect, then their intersection is exactly one line.

True/False (based on the Incidence Postulates)

1. 2 planes always intersect.

   False, they could be parallel

2. 4 noncoplanar points determine space.

   True, by the Expansion Postulate

3. A plane always contains at least 2 lines.

   True, by the Expansion Postulate and Line Postulate

4. A plane must contain infinitely many points.

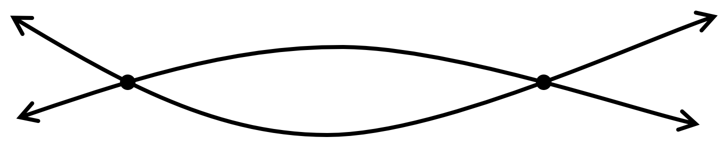
   False, not guaranteed by given postulates

5. A plane must contain at least 5 points.

   False, not guaranteed by given postulates. Only 3 points are guaranteed.

Theorem 1.1: If two distinct lines intersect, they intersect in one and only one point.

Reasoning Through: Suppose 2 lines intersect at 2 points?



The Line Postulate (Postulate 1.2) says that 2 points lie on exactly one line.

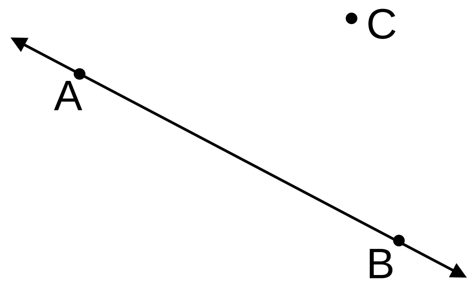
How can 2 points lie on exactly one line and yet form 2 distinct line?

Answer: They can't!

In Euclidean plane geometry, lines do not "bend."

(Note: By definition, a line and a curve are two different things.)

Questions:



- How many lines pass through both A and B?

answer: 1

- How many planes can pass through line AB?

answer: infinitely many

- How many planes can pass through points A, B, and C?

answer: 1

- How many planes pass through any three given noncollinear points?

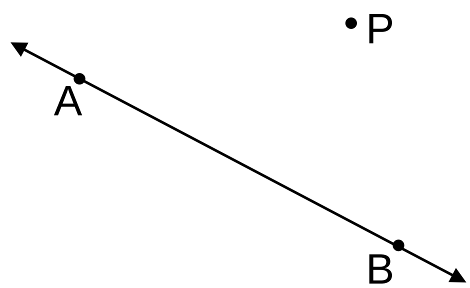
answer: 1

- How many planes pass through point C and line AB?

answer: 1

Theorem 1.2: A line and a point not on that line are contained in one and only one plane.

Proof:



Supply the correct postulate or definition for each step:

1. The line contains 2 points

   Expansion Postulate

2. A, B, and P are noncollinear.

   Because P is not on the line.
   (definition of collinear)

3. Exactly 1 plane passes through A, B, and P.

   Plane Postulate

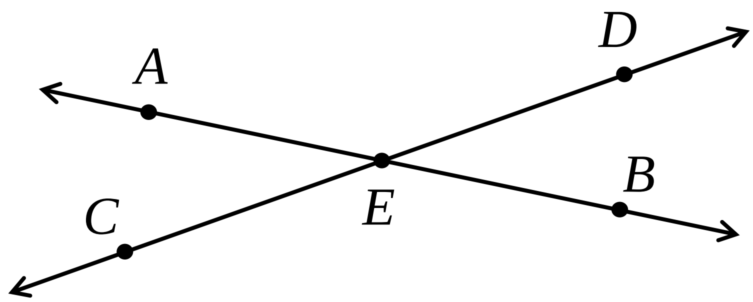
4. Since A and B are in the plane, the line is also in the plane.

   Flat Plane Postulate

Conclusion: There is exactly one plane that contains the line and the point not on the line.

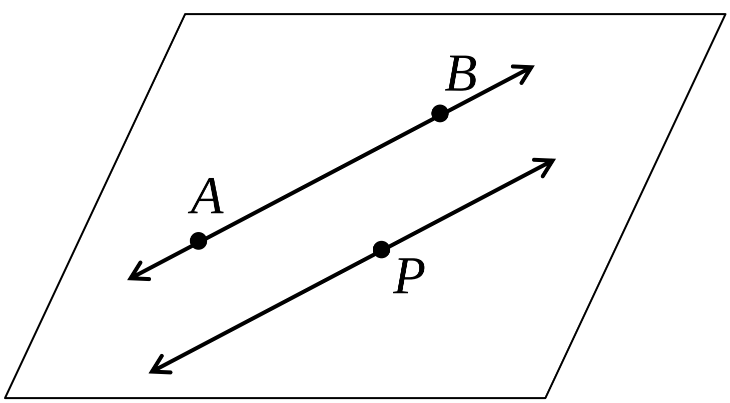
Theorem 1.3: Two intersecting lines are contained in one and only one plane.

Proof:



$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point E. Using points D, E, and A (3 non-collinear points) and the Plane Postulate, the points determine a plane (one and only one plane).

Theorem 1.4: Two parallel lines are contained in one and only one plane.



By definition of parallel, P is not on $\overleftrightarrow{AB}$. A, B, & P are then non-collinear points. By Thm. 1.2 we have one plane.

Definitions:

Sketch – permits you to draw a neat freehand picture

Draw – allows you to use any tools you desire but freehand is forbidden

Construct – requires a drawing using only a straightedge and a compass

Construction 1: Circle

Given: Two points *A* and *B*.
Construct: A circle with radius having a length equal to the distance between *A* and *B*.

1. Place the point of the compass on point *A*.

2. Adjust the pencil to rest on point *B*.

3. Without changing the compass width, rotate the compass completely around point *A*

Ch. 1 Vocabulary:

collinear
points
complement
complete
concurrent
lines
consistent
construction
coplanar
definition
disjoint sets
drawing
element
empty set

equal sets
equivalent
sets
Incidence
Postulates
independent
intersection
line
member
noncollinear
points
parallel
plane
point

postulate
proper subset
set
set braces
set-builder
notation
sketch
skew lines
space
subset
theorem
union
universal set
Venn diagram

Symbols:

$\in$	is an element of
$\notin$	is not an element of
$\cup$	union
$\cap$	intersection
$\subset$	is a proper subset of
$\subseteq$	is a subset of
$\not\subset$	is not a subset of
$\varnothing$ or $\{\ \}$	empty set
A'	complement