section 10.3

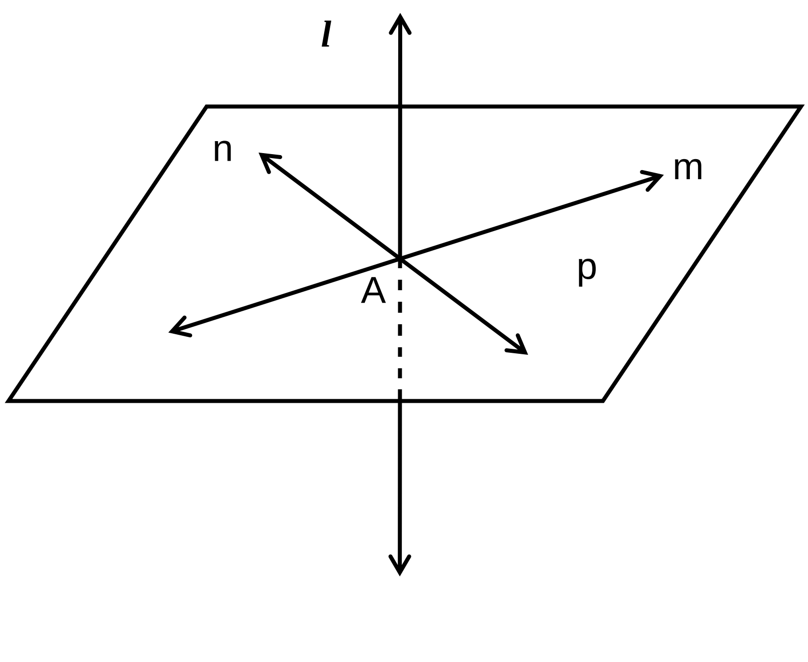
Definitions:

Perpendicular planes are two planes that form right dihedral angles.

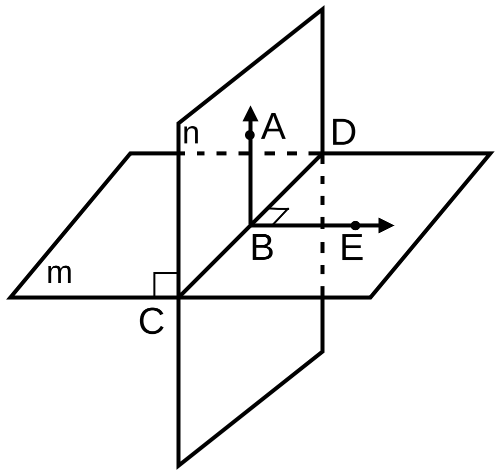
A line perpendicular to a plane is a line that intersects a plane and is perpendicular to every line in the plane that passes through the point of intersection.

A perpendicular bisecting plane of a segment is a plane that bisects a segment and is perpendicular to the line containing the segment.

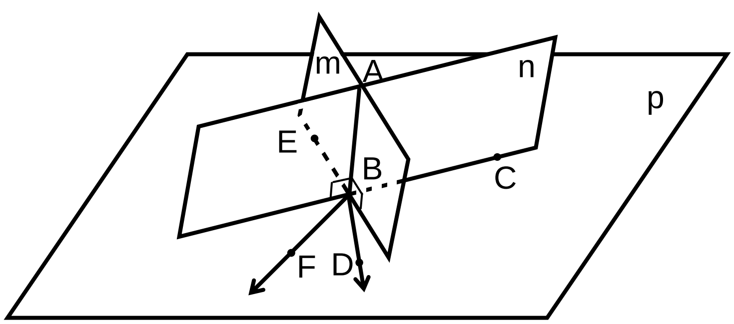
Theorem 10.2: A line perpendicular to two intersecting lines in a plane is perpendicular to the plane containing them.



Theorem 10.3: If a plane contains a line perpendicular to another plane, then the planes are perpendicular.



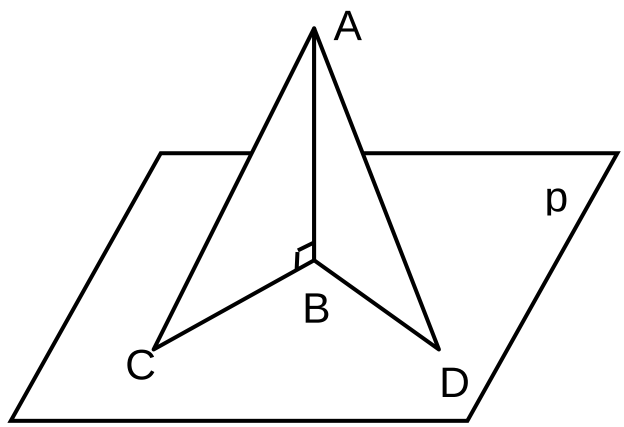
Theorem 10.4: If intersecting planes are each perpendicular to a third plane, then the line of intersection of the first two is perpendicular to the third plane.



Theorem 10.5: If AB is perpendicular to plane p at B, and $\overline{BC} \cong \overline{BD}$ in plane p, then $\overline{AC} \cong \overline{AD}$.

Given: $\overleftrightarrow{AB} \perp p$ at B
$\overline{BC} \cong \overline{BD}$
C, D $\in$ p

Prove: $\overline{AC} \cong \overline{AD}$

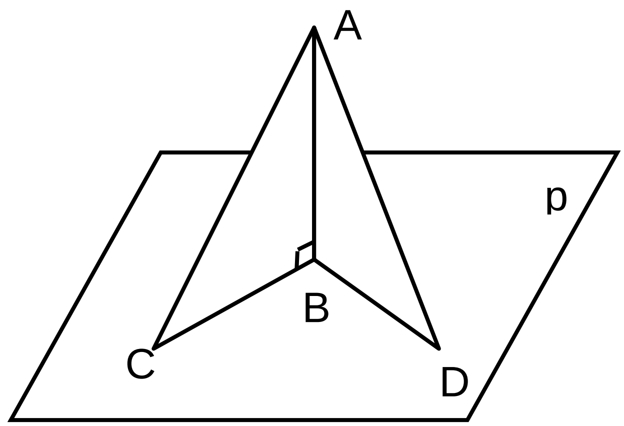


Statement	Reason
1. $\overleftrightarrow{AB} \perp p$ at B, $\overline{BC} \cong \overline{BD}$, C, D $\in$ p	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.

Theorem 10.5: If AB is perpendicular to plane p at B, and $\overline{BC} \cong \overline{BD}$ in plane p, then $\overline{AC} \cong \overline{AD}$.

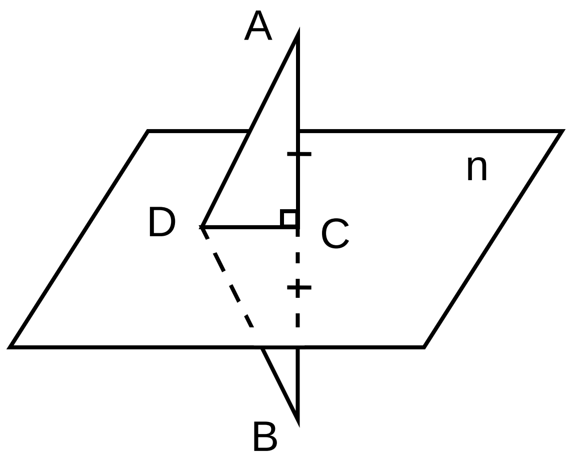
Given: $\overleftrightarrow{AB} \perp p$ at B
$\overline{BC} \cong \overline{BD}$
C, D ∈ p

Prove: $\overline{AC} \cong \overline{AD}$



Statement	Reason
1. $\overleftrightarrow{AB} \perp p$ at B, $\overline{BC} \cong \overline{BD}$, C, D ∈ p	1. Given
2. $\overline{AB} \perp \overline{BC}$, $\overline{AB} \perp \overline{BD}$	2. Def. of a line ⊥ to a plane
3. ∠ABC & ∠ABD are right angles	3. Def. of perpendicular
4. ∠ABC ≅ ∠ABD	4. Rt. ∠'s are congruent
5. $\overline{AB} \cong \overline{AB}$	5. Reflexive
6. ΔABC ≅ ΔABD	6. SAS (or LL)
7. $\overline{AC} \cong \overline{AC}$	7. CTCTC

Theorem 10.6: Every point in the perpendicular bisecting plane of segment AB is equidistant from A and B.



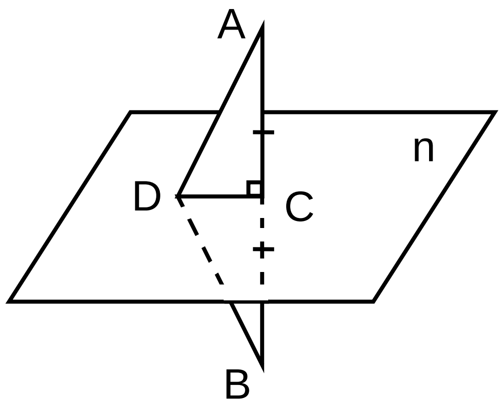
Given: ⊥ bisecting plane n of $\overline{AB}$ contains C & D
$C \in \overline{AB}$

Prove: AD = BD

Statement	Reason
1. ⊥ bisecting plane n of $\overline{AB}$ contains C & D, $C \in \overline{AB}$	1. Given
2. Draw $\overline{CD}$, $\overline{AD}$, and $\overline{BD}$	2. auxiliary lines
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.
11.	11.

Theorem 10.6: Every point in the perpendicular bisecting plane of segment AB is equidistant from A and B.

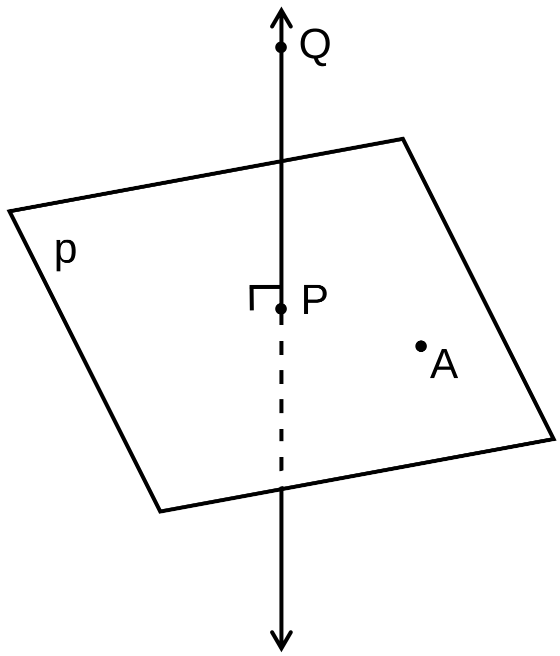
Given: ⊥ bisecting plane n of $\overline{AB}$ contains C & D
C ∈ $\overline{AB}$



Prove: AD = BD

Statement		Reason	
1.	⊥ bisecting plane n of $\overline{AB}$ contains C & D, C ∈ AB	1.	Given
2.	Draw $\overline{CD}$, $\overline{AD}$, and $\overline{BD}$	2.	auxiliary lines
3.	C is midpt. of $\overline{AB}$, $\overline{AB}$ ⊥ n	3.	Def. ⊥ bisecting plane
4.	AC = BC	4.	Def. of midpoint
5.	$\overline{AC} \cong \overline{BC}$	5.	Def. of ≅
6.	$\overline{CD} \perp \overline{AB}$	6.	Def. of line ⊥ to plane
7.	∠ACD & ∠BCD are rt. ∠'s	7.	Def. of rt. ∠
8.	ΔACD & ΔBCD are rt. Δ's	8.	Def. of rt Δ
9.	∠ACD ≅ ∠BCD	7.	Right ∠'s are ≅
10.	$\overline{CD} \cong \overline{CD}$	8.	Reflexive
11.	ΔACD ≅ ΔBCD	9.	SAS (or LL)
12.	$\overline{AD} \cong \overline{BD}$	10.	CPCTC
11.	AD = BD	11.	Def. of ≅

Theorem 10.7: The perpendicular is the shortest segment from a point to a plane.



Sample Problems: Answer each True/False question and draw a picture to illustrate your answer.

1. Two planes perpendicular to the same plane are parallel.

   False

2. Two lines perpendicular to the same plane are parallel.

   True

3. Two planes perpendicular to the same line are parallel.

   True

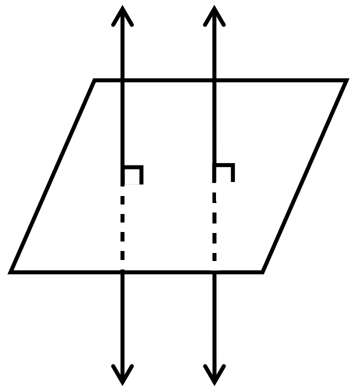
section 10.4

Definitions:

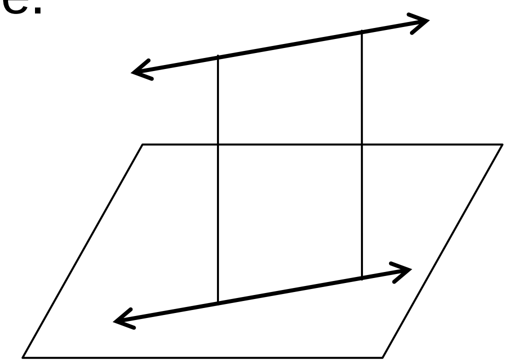
Parallel planes are two planes that do not intersect.

A line parallel to a plane is a line that does not intersect the plane.

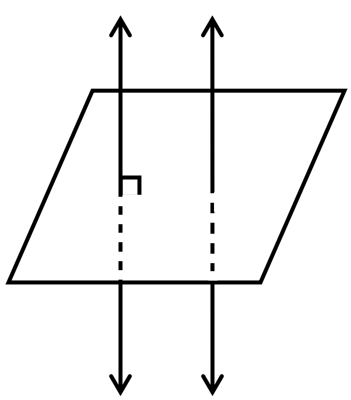
Theorem 10.8: Two lines perpendicular to the same plane are parallel.



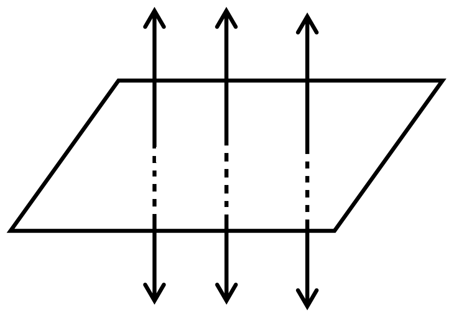
Theorem 10.9: If two lines are parallel, then any plane containing exactly one of the two lines is parallel to the other line.



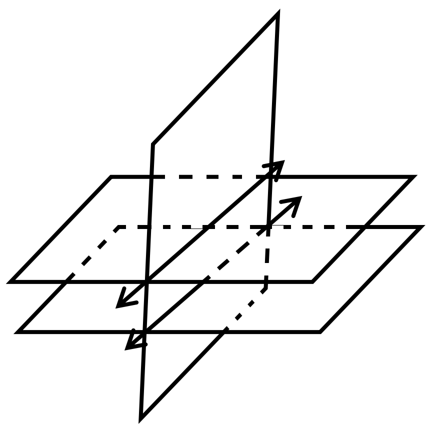
Theorem 10.10: A plane perpendicular to one of two parallel lines is perpendicular to the other line also.



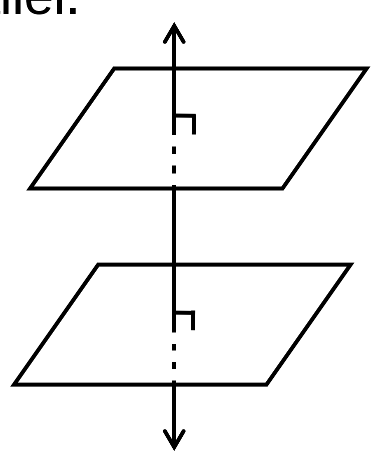
Theorem 10.11: Two parallel lines parallel to the same line are parallel.



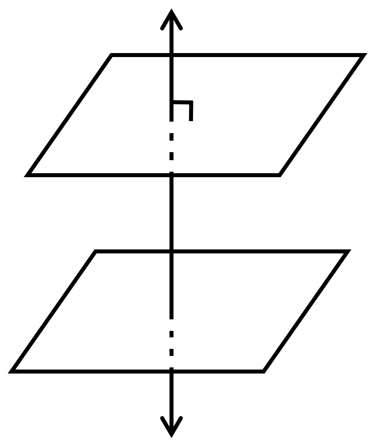
Theorem 10.12: A plane intersects two parallel planes in parallel lines.



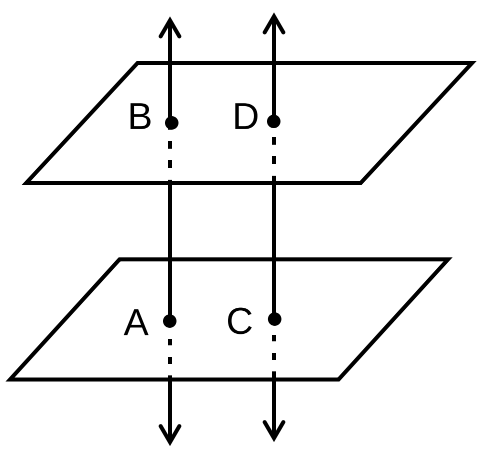
Theorem 10.13: Two planes perpendicular to the same line are parallel.



Theorem 10.14: A line perpendicular to one of two parallel planes is perpendicular to the other also.



Theorem 10.15: Two parallel planes are everywhere equidistant.



BA = DC for all lines perpendicular to the planes

Sample Problems:

1. Show how two lines can be perpendicular to the same line but not parallel to each other?

skew lines

2. Given line n and two planes p and q, suppose n ∥ p. If n ⊥ q, is p ⊥ q?

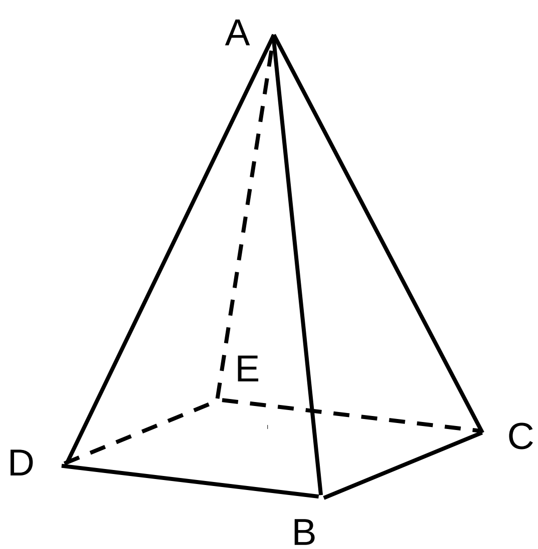
yes

3. Given a line n and two planes p and q, suppose n ∥ p. If p ⊥ q, is n ⊥ q?

no

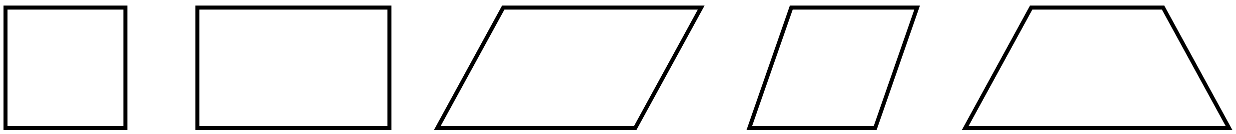
4. Does the phrase skew planes make sense?

no, planes are either parallel or they intersect



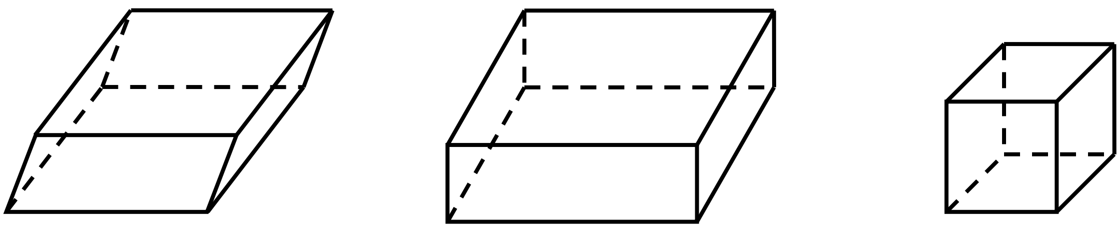
∠A-BC-D or ∠A-BC-E name the same dihedral angle. The dihedral angle is not a subset of the polyhedron but the polyhedron *determines* the dihedral angle.

Classifications of Quadrilaterals:



square rectangle parallelogram rhombus trapezoid

Classifications of Hexahedra (6-sided polyhedra):



rectangular prism	right rectangular prism	regular right rectangular prism (cube)
(base is a rectangle)	(base is a rectangle and sides are $\perp$ to base)	(right rectangular prism with all sides congruent)

Definitions:

A parallelepiped is a hexahedron in which all faces are parallelograms. (*This includes the 3 figures above.)

A diagonal of a hexahedron is any segment joining vertices that do not lie on the same face.

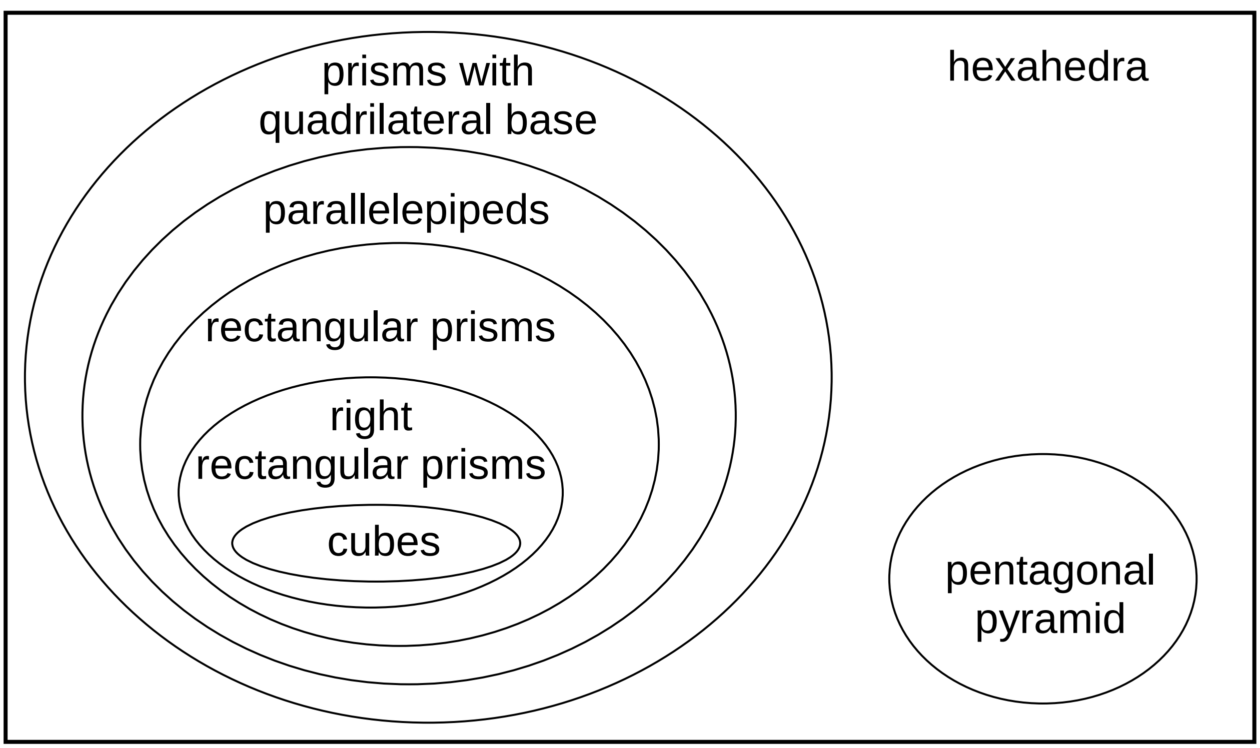
Opposite faces of a hexahedron are two faces with no common vertices.

Opposite edges of a hexahedron are two edges of opposite faces that are joined by a diagonal of the parallelepiped.

Theorem 10.16: Opposite edges of a parallelepiped are parallel and congruent.

Theorem 10.17: Diagonals of a parallelepiped bisect each other.

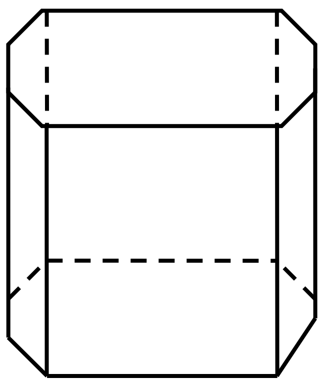
Theorem 10.18: Diagonals of a right rectangular prism are congruent.



<u>Euler's Formula</u>: V – E + F = 2 where V, E, and F represent the number of vertices, edges, and faces of a convex polyhedron respectively.

*Euler's Formula works for <u>any</u> convex polyhedra.

<u>Sample Problem</u>: Find V, E, F for an octagonal prism and verify Euler's Formula.



V = 16
E = 24
F = 10

V – E + F = 2
16-24+10 = 2

***Note:**

V = 16 = 2(8) = 2(number of sides in the base)
E = 24 = 3(8) = 3(number of sides in the base)
F = 10 = 8+2 = (number of sides in the base)+2

In general, for a prism where the base is an n-gon,

V = 2n
E = 3n
F = n+2