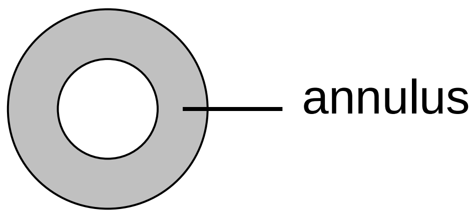
section 11.5

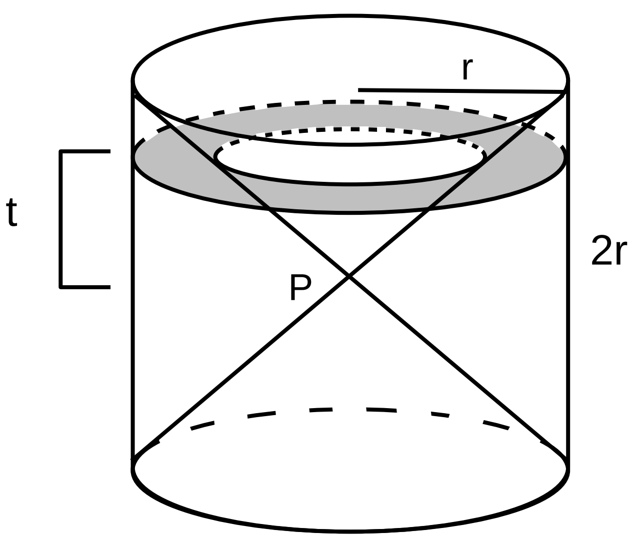
Definitions:

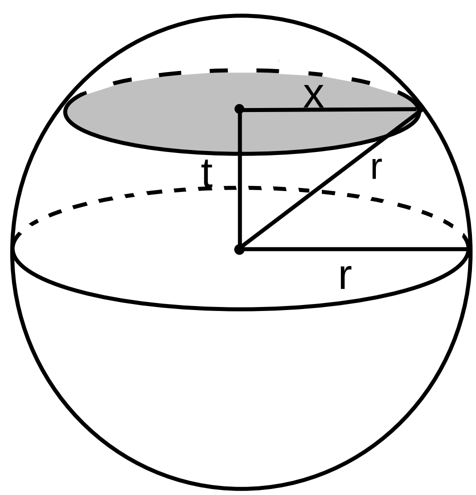
Concentric circles are circles that have the same center but radii of different lengths.

The region bound by concentric circles is called the annulus.



Volume of a Sphere:



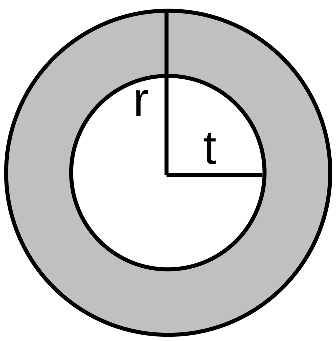


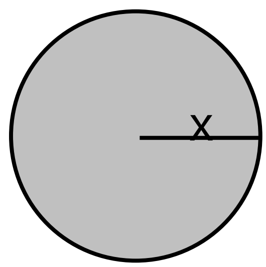
Slice both figures with a parallel plane t units above the centers.

Note the right triangle formed in the sphere with sides t, r, and x. Using Pythagorean Theorem we get:

$$x^2 = r^2 - t^2$$

Cross sections of both figures:





Area = $A_{lg} - A_{sm}$
Area = $\pi r^2 - \pi t^2$

Area = πx^2
Area = $\pi(r^2 - t^2)$ (substitute x^2 above)
Area = $\pi r^2 - \pi t^2$

Same!!

***By Cavaleri's Principle, since every horizontal plane cuts the figures into regions with equal areas, the volume of the sphere equals the volume of the solid between the cones and the cylinder.

$$V_{sphere} = V_{cylinder} - V_{2cones}$$
$$V = \pi r^2 H - 2(\tfrac{1}{3}\pi r^2 H)$$
$$V = \pi r^2(2r) - 2(\tfrac{1}{3}\pi r^2(r))$$
$$V = 2\pi r^3 - (2/3)\pi r^3$$
$$V = (4/3)\pi r^3$$

Theorem 11.7: The volume of a sphere is 4/3 π times the cube of the radius: $V = \dfrac{4\pi r^3}{3}$

Example: Find the volume of a sphere of radius 3.

$$V = (4/3)\pi(3^3)$$
$$V = (4/3)\pi(27)$$
$$V = 36\pi \text{ cubic units}$$

Sample Problem: Find the volume of a sphere whose great circle has a circumference of 16π.

$$C = 2\pi r \qquad V = (4/3)\pi(8^3)$$
$$16\pi = 2\pi r \qquad V = (4/3)\pi(512)$$
$$r = 8 \qquad V = (2048\pi/3)$$

Regular Polygon	Volume
tetrahedron	$V = \frac{\sqrt{2}}{12} e^3$
cube (hexahedron)	$V = e^3$
octahedron	$V = \frac{\sqrt{2}}{3} e^3$
dodecahedron	$V = \left(\frac{15 + 7\sqrt{5}}{3}\right) e^3$
icosahedron	$V = \left(\frac{15 + 5\sqrt{5}}{12}\right) e^3$

<u>Construction 16:</u> Segment division

Given: $\overline{AB}$

Construct: Five congruent segments with lengths that total $\overline{AB}$.

1. Draw a ray from A, forming an acute angle.
2. Place the point of the compass at A and, without changing the compass measure, mark off five equal segments on the ray. Label the points F,G,H,I, and J.
3. Draw $\overline{BJ}$.
4. Draw lines parallel to $\overleftrightarrow{BJ}$ through F,G,H, and I. You can do this by constructing congruent corresponding angles at each point. Copy $\angle AJB$ at vertices F,G,H, and I.

These parallel lines cut $\overline{AB}$ into five equal segments.

<u>Construction 17:</u> Regular Hexagon

1. Draw a circle.
2. Using the radius of the circle, mark off six consecutive arcs.
3. Connect the arc intersections with segments to form a regular hexagon.

3 Impossible Constructions:

1. Squaring a Circle: Given a circle, construct a square with the same area.
2. Doubling a Cube: Given a cube, construct a cube whose volume is twice the volume of the original cube.
3. Trisecting an Angle

Chapter 11 Vocabulary:

- annulus
- Cavalieri's principle
- concentric circles
- Congruent Solids Postulate
- cross section
- cubic unit
- oblique prism
- section
- volume
- Volume Addition Postulate
- Volume of a Cube Postulate
- Volume Postulate

Notes on test:

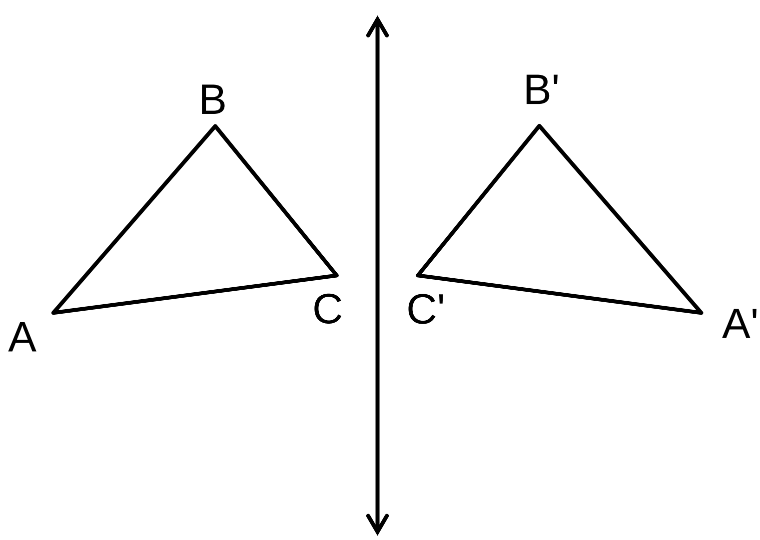
- Match formulas with figures
- Given figures, find volumes
- Short answer questions on Congruent Solids Postulate, volume definition, Cavalieri's Principle, and parallelepipeds
- Word Problems
- No Proofs

The word transformation describes a change in appearance of points in a plane. The geometric figure before a transformation is called the preimage. The resulting figure after the transformation is called the image.

Definition:

A transformation is a one-to-one function from the plane onto the plane.

Reflections



Reflection in a line is when the line acts as a mirror.

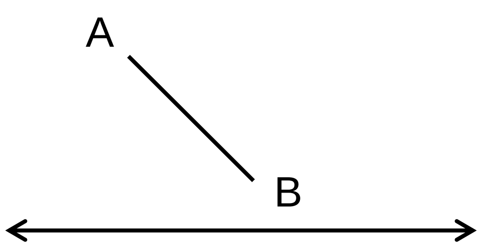
Definition:

A reflection in a line, n, is a transformation that maps each point A of a plane onto the point A' such that the following conditions are met:

1. If A is on n, then A = A'.
2. If A is not on n, then n is the perpendicular bisector of $\overline{AA'}$.

Sample Problems: Find the image of reflection.

1.



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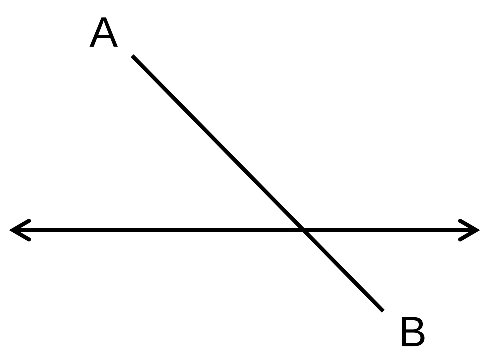
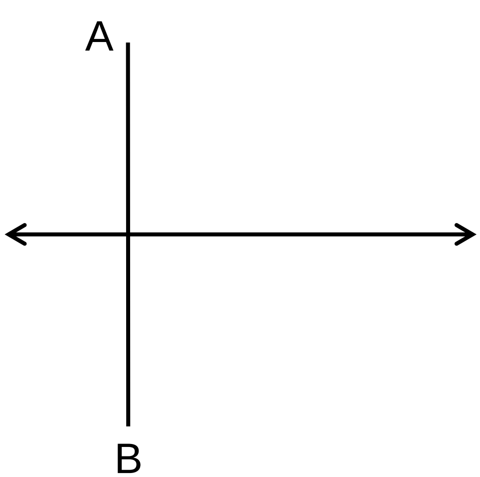


3.



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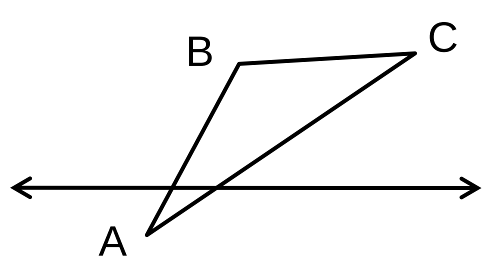
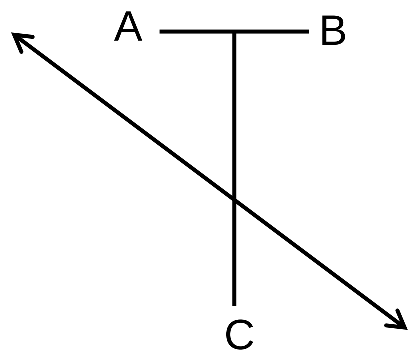
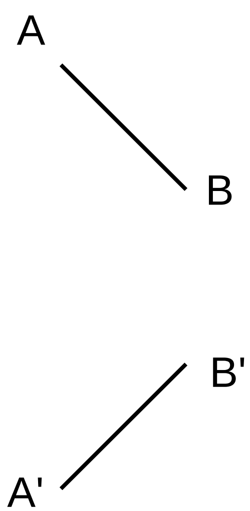


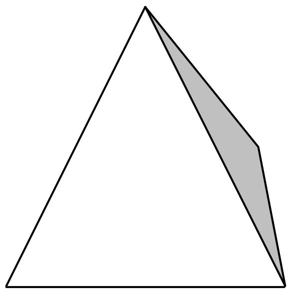
5.



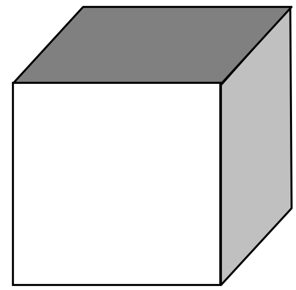
Sample Problem: Draw the line of reflection.



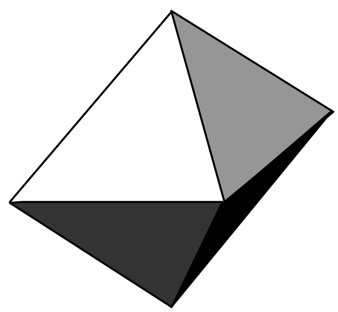
Platonic Solids



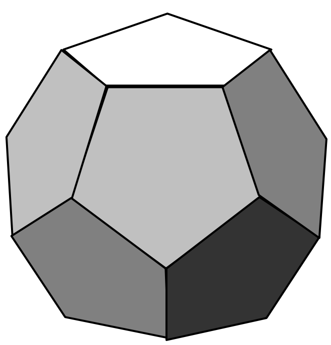
Tetrahedron



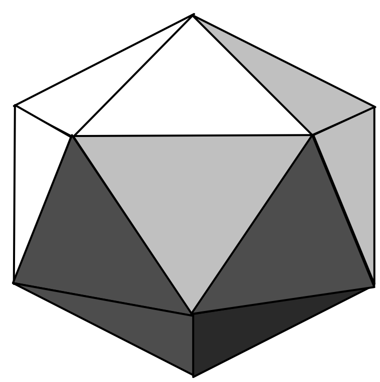
Hexahedron
(cube)



Octahedron



Dodecahedron



Icosahedron