ch. 12 review

Chapter 12 Vocabulary:

angle of rotation
axis of symmetry
center of rotation
composition
dilation
enlargement
fixed point
identity transformation
image
invariance
isometry
line of reflection
line symmetry
magnitude of a rotation
mapping
orientation
point symmetry
preimage
preserved
reduction
reflection
rotation
rotational symmetry
scale factor
similar figures
transformation
translation

Notes on the test:

- 25 true/false
- Draw reflections, rotations, dilations
- Symmetry regarding a regular polygon

<u>Definition</u>:

<u>Similar polygons</u> are polygons having corresponding angles that are congruent and corresponding sides that are proportional. If ΔABC and ΔDEF are similar, the proper notation is $\Delta ABC \sim \Delta DEF$.

<u>Review Proportions:</u>

Remember that a proportion is what we get when we set 2 ratios (fractions) equal to each other.

Example:

$$\frac{1}{2} = \frac{3}{6}$$

1 and 6 are called the <u>extremes</u>
2 and 3 are called the <u>means</u>

**The product of the means = the product of the extremes

(i.e the cross products are equal)

$$\frac{1}{2} = \frac{3}{6}$$

$$(1)(6) = (2)(3)$$
$$6 = 6$$

Important: The means can exchange positions with each other or the extremes can exchange positions with each other and the proportion remains true.

$\frac{1}{2} = \frac{3}{6}$ or $\frac{1}{3} = \frac{2}{6}$ or $\frac{6}{2} = \frac{3}{1}$

6 = 6 6 = 6 6 = 6

Sample Problems: Solve for x.

1. $\frac{x}{8} = \frac{9}{24}$

$24x = 72$
$x = 3$

2. $\frac{x}{4} = \frac{5}{6}$

$6x = 20$
$x = 10/3$

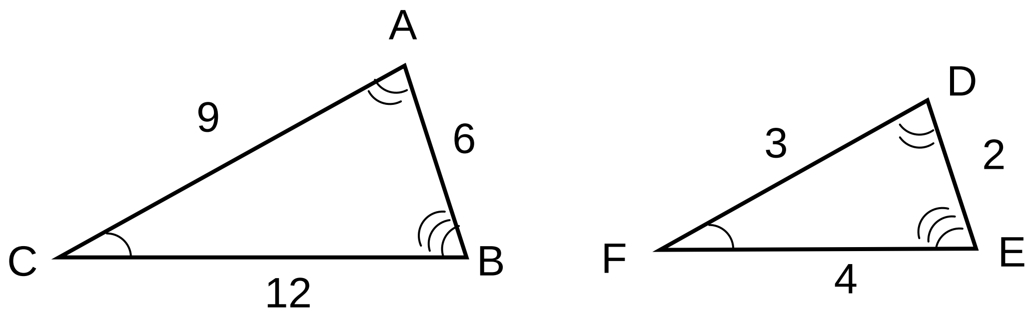
Consider: $\Delta ABC \sim \Delta DEF$

From the notation we know: $\angle A \cong \angle D$
$\angle B \cong \angle E$
$\angle C \cong \angle F$

Ratios of corresponding sides: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

We say “AB is to DE as BC is to EF” etc.

Let’s put lengths on the sides and check the ratios.



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad \text{or} \quad \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$$

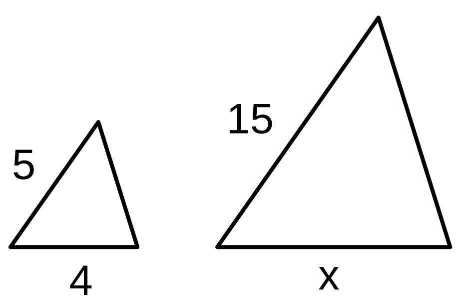
$$\frac{6}{2} = \frac{9}{3} = \frac{12}{4} \qquad \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$$

$$3 = 3 = 3 \qquad \tfrac{1}{3} = \tfrac{1}{3} = \tfrac{1}{3}$$

**Unless you are finding the scale factor of a dilation, it does not matter which triangle you start with for your proportion.

Sample Problem: If the pairs of figures are similar, find the unknown values.

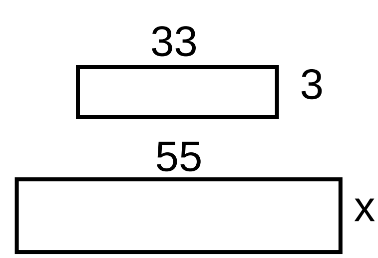
1.



$$\frac{x}{4} = \frac{15}{5}$$

$x = 12$

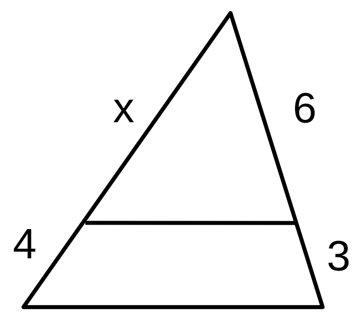
2.



$$\frac{x}{3} = \frac{55}{33}$$

$x = 5$

3.



$$\frac{x}{x+4} = \frac{6}{9}$$

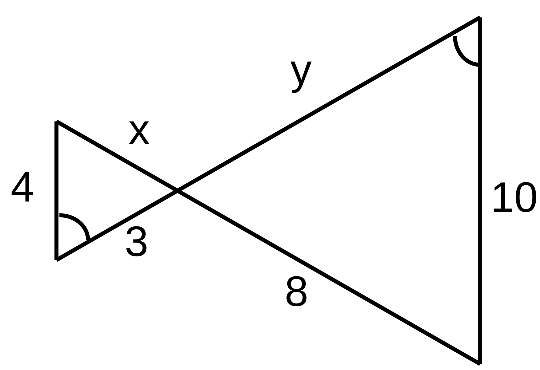
9x = 6(x+4)

9x = 6x + 24

3x = 24

x = 8

4.



$$\frac{4}{10} = \frac{x}{8}$$

10x = 32

x = 32/10

x = 16/5

$$\frac{4}{10} = \frac{3}{y}$$

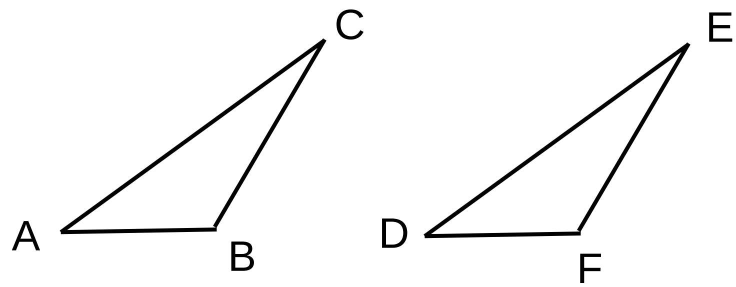
4y = 30

y = 30/4

y = 15/2

<u>Sample Problem</u>: Prove that if two triangles are congruent, then they are similar.

Given: ΔABC ≅ ΔDEF
Prove: ΔABC ~ ΔDEF



Statement	Reason
1. ΔABC ≅ ΔDEF	1. Given
2. ∠A ≅ ∠ D, ∠B ≅ ∠E, C ≅ ∠F $\overline{AB} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DE}$	2. Def. of congruent Δ's
3. AB = DF, BC = EF, AC = DE	3. Def. of congruent
4. $\frac{AB}{DF} = \frac{DF}{DF} = 1$ $\frac{BC}{EF} = \frac{EF}{EF} = 1$ $\frac{AC}{DE} = \frac{DE}{DE} = 1$	4. Multiplication Property of Equality
5. $\frac{AB}{DF} = \frac{BC}{EF} = \frac{AC}{DE}$	5. Transitive
6. ΔABC ~ ΔDEF	6. Def. of similar

AA Similarity Postulate (13.1): If 2 angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

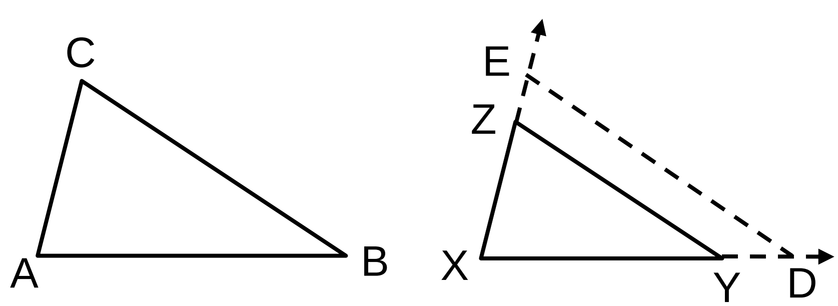
Remember: We know from an earlier theorem that if 2 angles of a triangle are congruent to 2 angles of another triangle, then the 3rd pair must also be congruent.

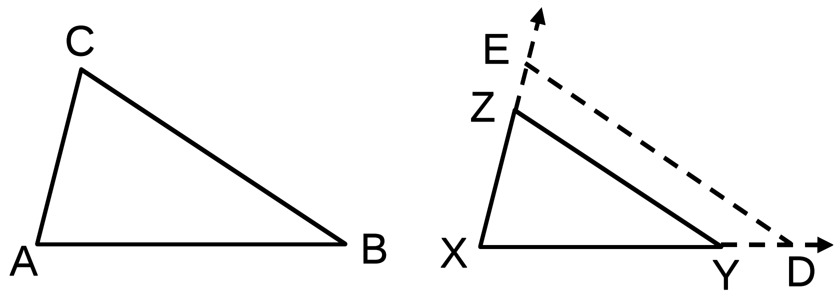
SSS Similarity Theorem (13.1): If three sides of one triangle are proportional to the corresponding sides of another triangle, then the triangles are similar.

Given: In ΔABC and ΔXYZ,

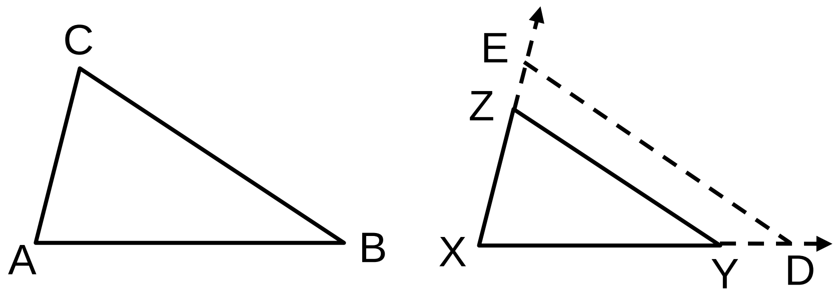
$$\frac{XY}{AB} = \frac{YZ}{BC} = \frac{XZ}{AC}$$

Prove: $\Delta ABC \sim \Delta XYZ$





Statement	Reason
1. $\frac{XY}{AB} = \frac{YZ}{BC} = \frac{XZ}{AC}$	1. Given
2. Draw segment congruent to $\overline{AB}$ by extending $\overline{XY}$ and call it $\overline{XD}$; $\overline{AB} \cong \overline{XD}$	2. Auxiliary lines
3. AB = XD	3. Def. of congruent seg
4. $\frac{XY}{XD} = \frac{YZ}{BC}$	4. Substitution (step 3 into 1
5. Construct $\overleftrightarrow{DE}$ parallel to $\overleftrightarrow{YZ}$	5. Auxiliary lines
6. $\angle XYZ \cong \angle XDE$ $\angle XZY \cong \angle XED$	6. Corresponding Angle Theorem
7. $\Delta XDE \sim \Delta XYZ$	7. AA
8. $\frac{XY}{XD} = \frac{YZ}{DE} = \frac{XZ}{XE}$	8. Def. of similar Δ's
9. $\frac{YZ}{DE} = \frac{YZ}{BC}$	9. Transitive (steps 4 and 8
10. $\frac{XZ}{XE} = \frac{XZ}{AC}$	10. Substitution (steps 8 and 1 into 9
11. (YZ)(BC) = (YZ)(DE) (XZ)(AC) = (XZ)(XE)	11. Mult. Prop. of Eq. (cross mult. steps 9 & 10)



12. BC = DE, AC = XE	12. Mult. Prop. of Eq.
13. $\overline{BC} \cong \overline{DE}$, $\overline{AC} \cong \overline{XE}$	13. Def. of congruent seg
14. $\Delta ABC \cong \Delta XDE$	14. SSS
15. $\angle B \cong \angle XDE$ $\angle C \cong \angle XED$	15. Def. of congruent Δ's
16. $\angle B \cong \angle XYZ$ $\angle C \cong \angle XZY$	16. Transitive (see step 6)
17. $\Delta ABC \sim \Delta XYZ$	17. AA
18. If three sides of one triangle are proportional to the corresponding sides of another triangle, then the triangles are similar.	18. Law of Deduction

SAS Similarity Theorem (13.2): If two sides of a triangle are proportional to the corresponding two sides of another triangle and the included angles between the sides are congruent, then the triangles are similar.

Theorem 13.3: Similarity of triangles is an equivalence relation.

(reflexive, symmetric, and transitive)

Summary:

3 Ways to Prove Similar Triangles:

1. AA
2. SSS
3. SAS

Note: ASA and SAA are not needed because they are covered by AA.

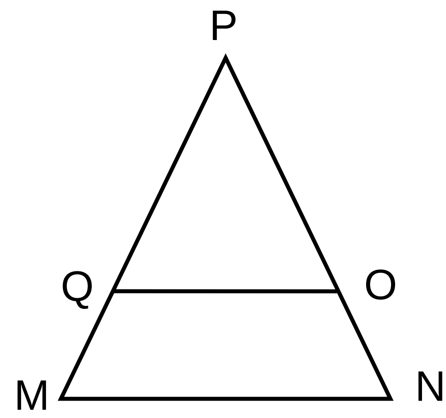
Sample Problem:

Given: $\overleftrightarrow{MN} \parallel \overleftrightarrow{OQ}$
Prove: $\Delta MNP \sim \Delta QOP$

Statement	Reason
1. $\overleftrightarrow{MN} \parallel \overleftrightarrow{OQ}$	1. Given
2. $\angle PQO \cong \angle PMN$ $\angle POQ \cong \angle PNM$	2. Corresponding Angle Theorem
3. $\Delta MNP \sim \Delta QOP$	3. AA

Sample Problem:

Given: $\overleftrightarrow{MN} \parallel \overleftrightarrow{OQ}$
Prove: $\Delta MNP \sim \Delta QOP$



Statement	Reason
1.	1. Given
2.	2.
3.	3.
4.	4.
5.	5.

Sample Problem: Prove Similarity of triangles is transitive (If $\Delta ABC \sim \Delta LMN$ and $\Delta LMN \sim \Delta PQR$, then $\Delta ABC \sim \Delta PQR$.)

Given: $\Delta ABC \sim \Delta LMN$
$\Delta LMN \sim \Delta PQR$

Prove: $\Delta ABC \sim \Delta PQR$

Statement	**Reason**
1.	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Solution:

Sample Problem: Prove Similarity of triangles is transitive (If ΔABC ~ ΔLMN and ΔLMN ~ ΔPQR, then ΔABC ~ ΔPQR.)

Given: ΔABC ~ ΔLMN
ΔLMN ~ ΔPQR

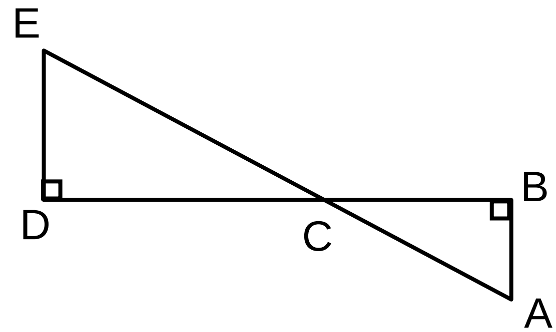
Prove: ΔABC ~ ΔPQR

Statement	Reason
1. ΔABC ~ ΔLMN ΔLMN ~ ΔPQR	1. Given
2. ∠A ≅ ∠L ∠L ≅ ∠P ∠B ≅ ∠M ∠M ≅ ∠Q	2. Def. of similar Δ's
3. ∠A ≅ ∠P ∠B ≅ ∠Q	3. Transitive property of congruent angles
4. ΔABC ~ ΔPQR	4. AA

Sample Problem:

Given: $\overleftrightarrow{DB} \perp \overleftrightarrow{DE}$
$\overleftrightarrow{DB} \perp \overleftrightarrow{AB}$

Prove: $\Delta ABC \sim \Delta EDC$



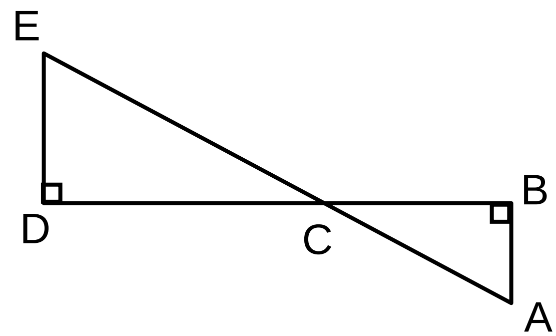
Statement	Reason
1.	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Solution:

Sample Problem:

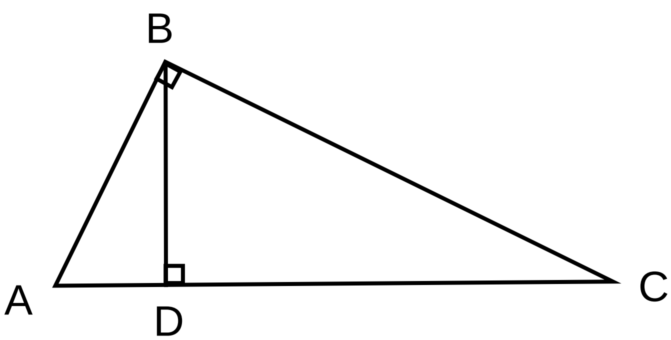
Given: $\overleftrightarrow{DB} \perp \overleftrightarrow{DE}$
$\overleftrightarrow{DB} \perp \overleftrightarrow{AB}$

Prove: $\Delta ABC \sim \Delta EDC$



Statement	Reason
1. $\overleftrightarrow{DB} \perp \overleftrightarrow{DE}$ $\overleftrightarrow{DB} \perp \overleftrightarrow{AB}$	1. Given
2. $\angle CDE$ and $\angle CBA$ are right angles	2. Def. of perpendicular
3. $\angle CDE \cong \angle CBA$	3. All right angles are congruent
4. $\angle DCE \cong \angle ACB$	4. Vertical Angle Thm.
5. $\Delta ABC \sim \Delta EDC$	5. AA

Theorem 13.4: An altitude drawn from the right angle to the hypotenuse of a right triangle separates the original triangle into two similar triangles, each of which is similar to the original triangle.



$\Delta ADB \sim \Delta BDC$
$\Delta ADB \sim \Delta ABC$
$\Delta BDC \sim \Delta ABC$

Proof of the 2nd case: ***Given***: $\overline{BD}$ is altitude of ΔABC
Prove: $\Delta ADB \sim \Delta ABC$

Statement	Reason
1. $\overline{BD}$ is altitude of ΔABC	1. Given
2. $\overline{BD} \perp \overline{AC}$	2. Def. of altitude
3. $\angle BDA$ is a right angle	3. Def. of perpendicular
4. $\angle ABC \cong \angle BDA$	4. All rt. angles are $\cong$
5. $\angle A \cong \angle A$	5. Reflexive
6. $\Delta ADB \sim \Delta ABC$	6. AA

Look at: $\frac{16}{8} = \frac{8}{4}$

When the denominator of one fraction of a proportion is the same as the numerator of the other fraction, that number is called the geometric mean.

Example: Find the geometric mean between 3 and 27

$$\frac{3}{x} = \frac{x}{27}$$

$x^2 = 3(27)$
$x^2 = 81$
$x = \pm\sqrt{81}$
$x = \pm 9$

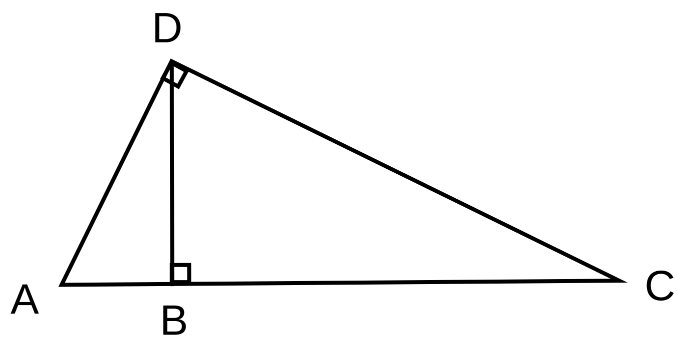
Since we want a number between 3 and 27, we will choose 9 instead of -9.

Sample Problem: Find the geometric mean between 12 and 20.

$$\frac{12}{x} = \frac{x}{20}$$

$x^2 = 240$
$x = \pm\sqrt{240} = \pm 4\sqrt{15}$

Theorem 13.5: In a right triangle, the altitude to the hypotenuse cuts the hypotenuse into two segments. The length of the altitude is the geometric mean between the lengths of the 2 segments of the hypotenuse.



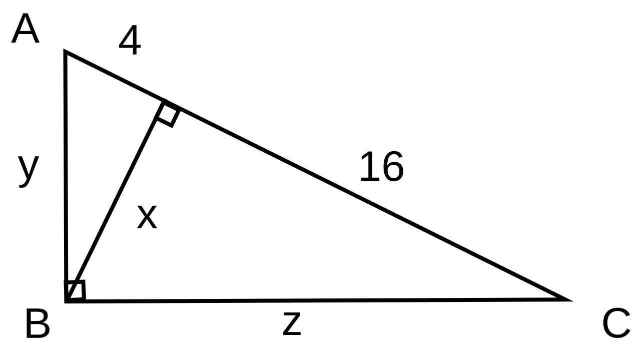
Given: Right ΔACD
DB is an altitude of ΔACD

Prove: $\frac{AB}{DB} = \frac{DB}{BC}$

Statement	Reason
1. Right ΔACD DB is an altitude of ΔACD	1. Given
2. ΔABD ~ ΔDBC	2. The altitude divides a rt. Δ into 3 ~ Δ's
3. $\frac{AB}{DB} = \frac{DB}{BC}$	3. Def. of similar Δ's

<u>Theorem 13.6</u>: In a right triangle, the altitude to the hypotenuse divides the hypotenuse into 2 segments such that the length of a leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to the leg.

<u>Example</u>: Given ΔABC, find x, y, and z.



From Thm.13.5 we get:

$$\frac{x}{4} = \frac{16}{x}$$

$x^2 = 64 \Rightarrow x = 8$

From Thm.13.6 we get:

$$\frac{y}{4} = \frac{20}{y}$$

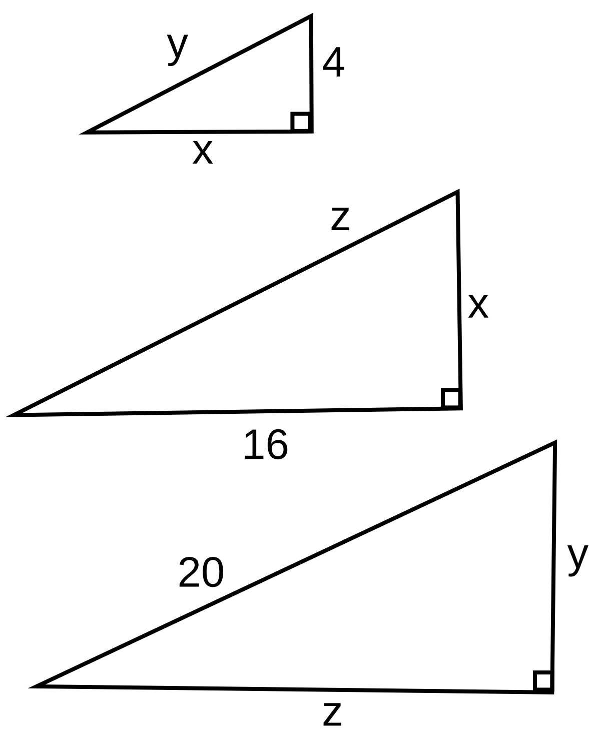
$y^2 = 80 \Rightarrow y = 4\sqrt{5}$

From Thm.13.6 we get:

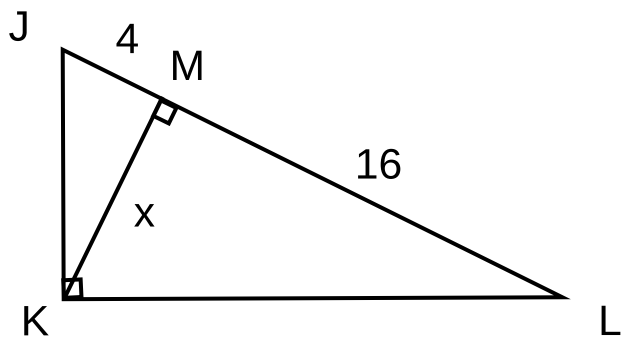
$$\frac{z}{20} = \frac{16}{z}$$

$z^2 = 320 \Rightarrow z = 8\sqrt{5}$

Sometimes it helps to separate the triangles.



Sample Problem: Given right ΔJKL with altitude to the hypotenuse, $\overline{MK}$, find KM if LJ = 20 and MJ = 4.



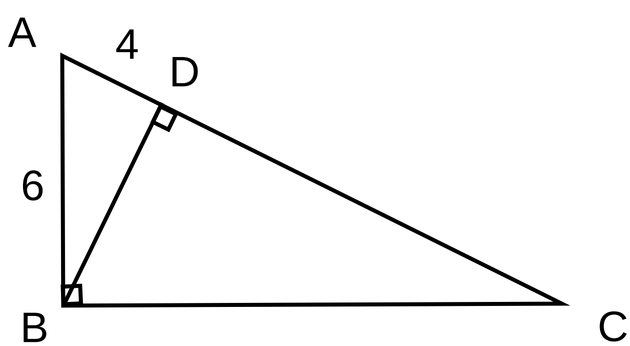
$\overline{MK}$ is the geometric mean of MJ and ML

$$\frac{x}{4} = \frac{16}{x}$$

$$x^2 = 64$$

$$x = 8$$

Sample Problem: Given right ΔABC with altitude to the hypotenuse, $\overline{DB}$, find AC if AD = 4 and AB = 6.



$\overline{AB}$ is the geometric mean of AD and AC

$$\frac{x}{6} = \frac{6}{4}$$

$$4x = 36 \Rightarrow x = 9$$