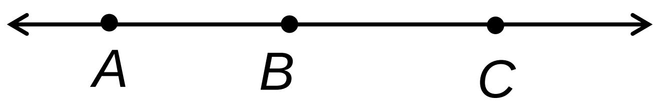
section 2.1

Definition:

<u>half-line</u> – the set of all points on a line on a given side of a given point of the line



<u>notation</u>: $\overset{\circ\!\!\longrightarrow}{AC}$ is the half-line that contains all points on the same side of *A* as point *C* is.

Point *A* is called the <u>origin</u> of the half-line.

<u>Important</u>: Point *A* is ***not*** part of the half-line.

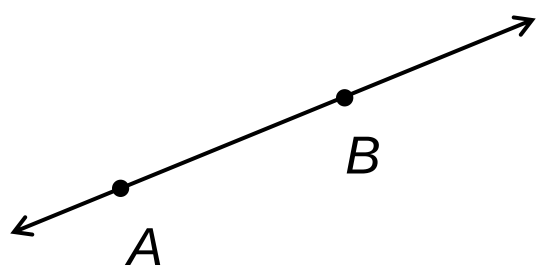
Also, $\overset{\circ\!\!\longrightarrow}{AB}$ names the same half-line as $\overset{\circ\!\!\longrightarrow}{AC}$.

Postulate:

Line Separation Postulate: Every point divides any line through that point into 3 disjoint sets: the point and each of the 2 half-lines.

Definition:

ray – the union of a half-line and its origin. It extends infinitely in one direction from a point.

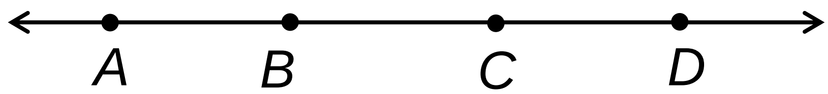


notation: $\overrightarrow{AB}$ is the ray having *A* as its origin.

$\overrightarrow{BA}$ is the ray having *B* as its origin.

*Note: The difference between half-lines and rays is that rays contain the origin and half-lines do not.

Sample Problem: Find the following.



1. $\overrightarrow{AD} \cap \overset{\circ\!\!\longrightarrow}{BC}$

   $\overset{\circ\!\!\longrightarrow}{BC}$

2. $\overset{\circ\!\!\longrightarrow}{BC} \cap \overrightarrow{CB}$

   $\overset{\circ\!\!-\!\!-}{BC}$

3. $\overset{\circ\!\!\longrightarrow}{BA} \cap \overset{\circ\!\!\longrightarrow}{CD}$

   $\varnothing$

4. $\overrightarrow{BC} \cap \overrightarrow{CB}$

   $\overline{BC}$

5. $\overset{\circ\!\!\longrightarrow}{BC} \cap \overset{\circ\!\!\longrightarrow}{CB}$

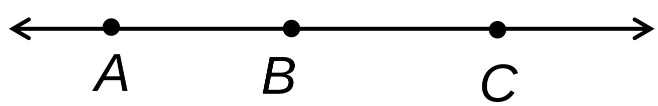
   $\overset{\circ\!\!-\!\!\circ}{BC}$

Definitions

between – B is between A and C if $\overrightarrow{BC} \cap \overrightarrow{BA} = \{B\}$ when A,B, and C are collinear.

notation: A-B-C or C-B-A

opposite rays – $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays if and only if B is between A and C.



segment – the set consisting of 2 points A and B, and all the points in between them.

$\overline{AB} = \{A,B\} \cup \{X | \text{A-X-B}\}$

notation: $\overline{AB}$

*A note about vectors:

vector – directed line segment $\overrightharpoonup{AB}$

ray – directed, but infinite length $\overrightarrow{AB}$

Subsets of Planes

*Just as a point divides a line into 3 sets, a line divides a plane into 3 sets.

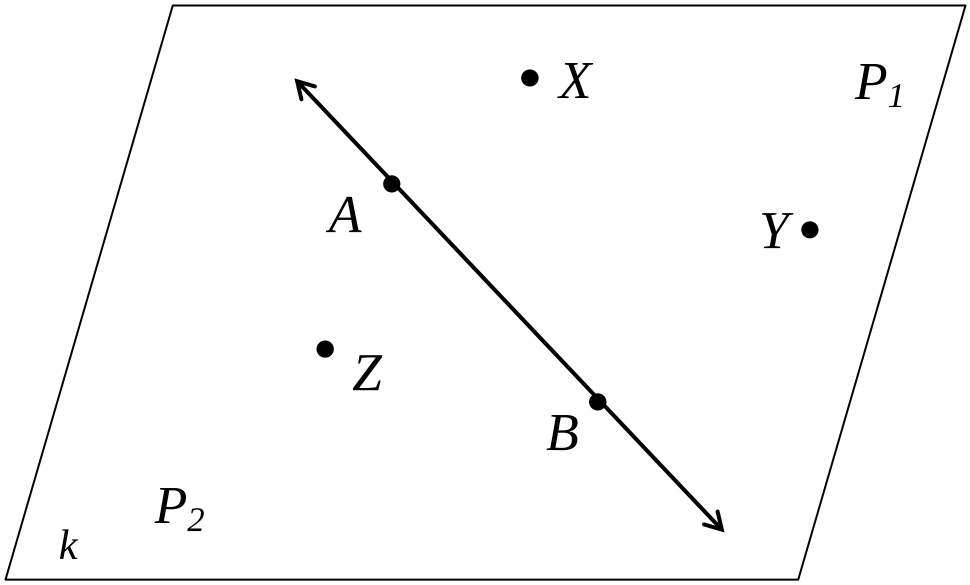
<u>Plane Separation Postulate</u>: Every line divides any plane containing the line into 3 disjoint sets: the line and the two half-planes

Definitions:

<u>half-plane</u> – a subset of a plane consisting of all points on a given side of a line in the plane. If points *P* and *Q* are in the same half-plane, then so is the segment joining them.

<u>edge of a half-plane</u> – the line that separates the plane into two half-planes. The line is not part of either half-plane.

<u>opposite half-planes</u> – the two half-planes that are separated by a particular line of the plane. If points *Y* and *Z* are in opposite half-planes, the segment joining them must intersect the edge.



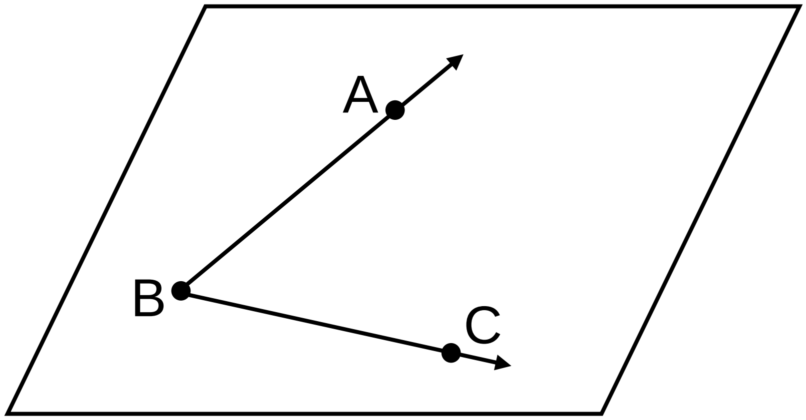
- $\overleftrightarrow{AB}$ is the edge of the half-plane.
- P_1 and P_2 are half-planes
- P_1 and P_2 are opposite half-planes

Definitions:

angle – the union of two distinct rays with a common endpoint

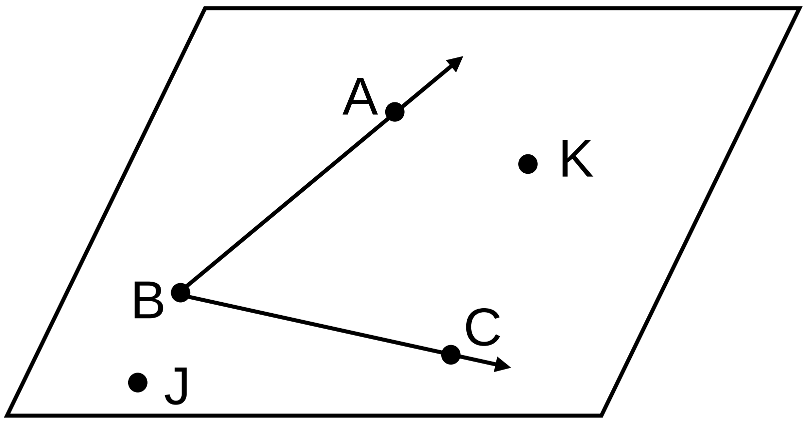
sides of an angle – two rays that form the angle

vertex of an angle – the common endpoint (origin) of the two rays



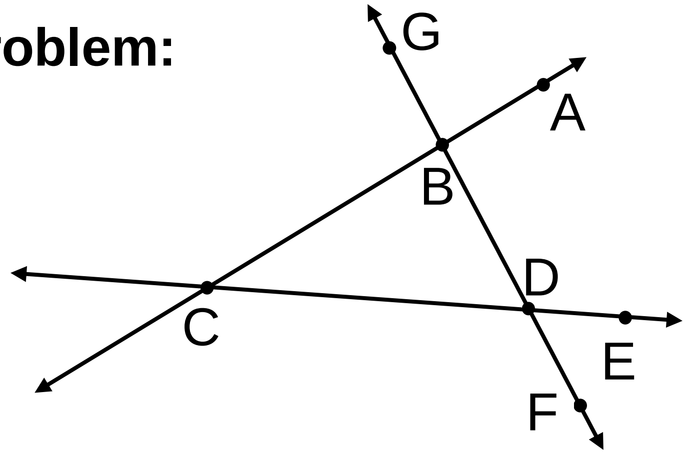
interior of an angle – the intersection of the two half-planes each determined by a side of the angle and containing the other side (except for the vertex).

exterior of an angle – the complement of the union of the angle and its interior



- Point J is in the exterior of the ∠ABC.
- Point K is in the interior of ∠ABC.

Practice Problem:



1. Using the edge $\overleftrightarrow{FG}$ classify all points according to the Plane Separation Postulate.

   B,D,F,G *are points on the line* $\overleftrightarrow{FG}$
   C *is in one half-plane*
   A *and* E *are in the other half-plane*

2. E is in the interior of what angle?

   $\angle$ABF

3. $\overrightarrow{BC}$ is a side of what angles?

   $\angle$CBG *and* $\angle$FBC

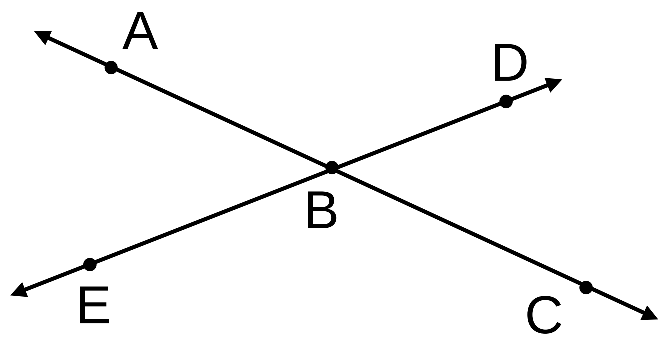
4. Name 4 angles with vertex D.

   $\angle$GDE, $\angle$EDF, $\angle$FDC, $\angle$GDC

5. Name points in the exterior of $\angle$BCD.

   G *and* F

Practice Problem:



True/False:

1. $\overline{AB} \cup \overline{BD} = \angle ABD$

   False, no rays

2. $\angle CBE \cap \overleftrightarrow{DE} = \overrightarrow{BE}$

   True

3. $\angle ABD \cap \overrightarrow{BE} = \{B\}$

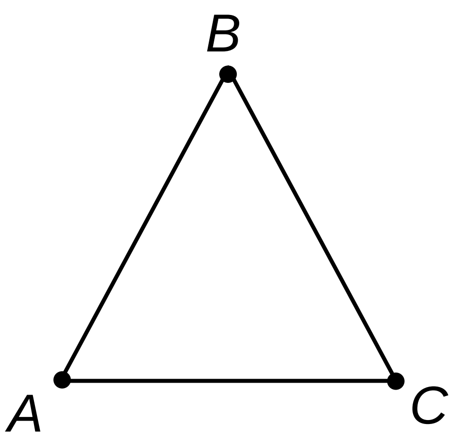
   True

4. $\overset{\circ\!\!\longrightarrow}{BE} \cup$ interior of $\angle ABE \cup$ interior of $\angle CBE =$ half-plane determined by $\overleftrightarrow{AC}$

   True

Definitions:

<u>triangle</u> – the union of segments that connect 3 noncollinear points



<u>Notation</u>:

ΔABC, ΔBCA, ΔCAB, etc.

- The segments are called <u>sides</u>.
- The corners are called <u>vertices</u> (plural for <u>vertex</u>).

Some facts about triangles:

- $\angle$A is the angle that *contains* the sides $\overline{AB}$ and $\overline{BC}$.
- Angles are subsets of a plane, but not of a triangle.
- A triangle does <u>not</u> contain any angles.
- A triangle *determines* 3 angles.
- Two sides of a triangle are subsets of the rays of each angle.

Definitions

curve – a continuous set of points

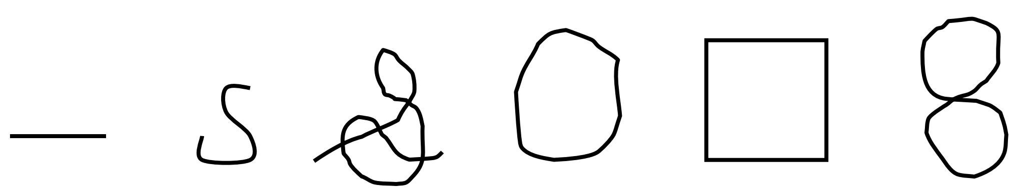
* Note: A curve can be “straight.”

closed curve – begins and ends at the same point

simple curve – doesn’t intersect itself (unless starting and ending points coincide)

simple closed curve – a simple curve that is also a closed curve

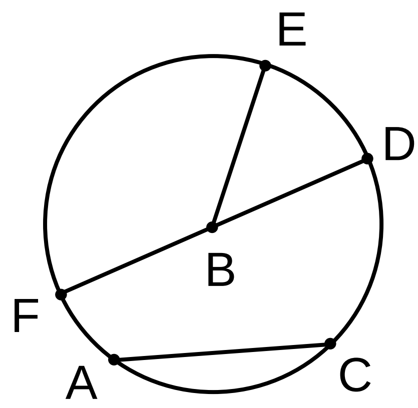
Examples of curves:



Simple curve | Simple curve | Curve | Simple closed curve | Simple closed curve | Closed curve

Definitions:

circle – the set of all points that are given distance from a given point in a given plane



Notation: $\odot$ B

center – the given point in the plane

radius of a circle – a segment that connects a point on the circle with the center (plural = radii)

chord of a circle – a segment having both endpoints on the circle

diameter – a chord that passes through the center of a circle

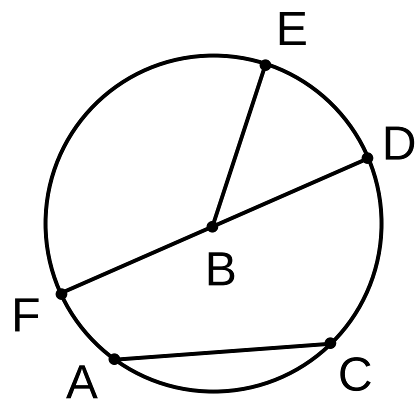
arc – a curve that is a subset of a circle center: B

radii: $\overline{BE}$, $\overline{BD}$, $\overline{BF}$

chords: $\overline{FD}$, $\overline{AC}$

diameter: $\overline{FD}$

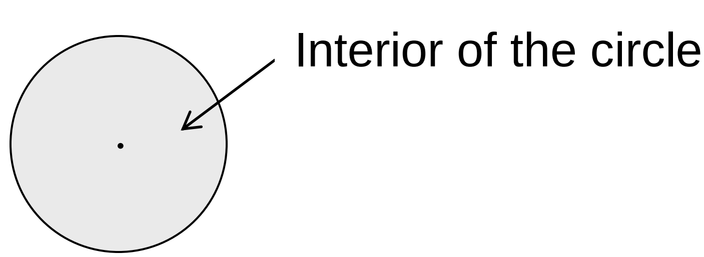
arc: $\overset{\frown}{ED}$, $\overset{\frown}{DC}$, $\overset{\frown}{FC}$, etc.



Definitions:

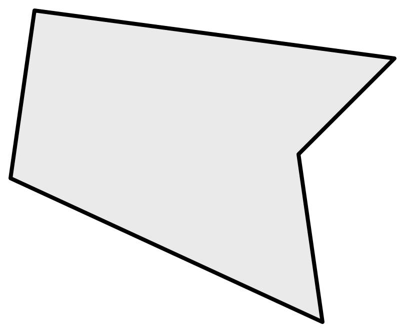
interior of a circle - the set of all planar points whose distance from the center of the circle is less than the lenth of the radius

exterior of a circle – the set of all planar points whose distance from the center is greater than the length of the radius



Theorem 2.1: ***Jordon Curve Theorem***

Any simple closed curve divides a plane into three disjoint sets: the curve itself, its interior, and its exterior.



Definition:

region – the union of a simple closed curve and its interior. The curve is the *boundary* of the region.

Question: What is the union of a region and its exterior?

Answer: the plane

Why?

Answer: because of the Jordon Curve Theorem