section 5.3

To reason to a correct conclusion, we must build our arguments on true statements. Sometimes it is helpful to use truth tables.

Simple Truth Table

p	**~p**
T	F
F	T

Conjunctions

Look at: A cat is a mammal and a snake is a fish.

Questions: Is this true or false?

False

Why is the statement false?

Both parts must be true for the statement to be true because of the word "and."

Definition: A <u>conjunction</u> is a statement in which two statements, p and q, are connected by "and." The notation for the conjunction "p and q" is denoted $p \wedge q$.

p: A cat is a mammal.
q: A snake is a fish.
$p \wedge q$: A cat is a mammal and a snake is a fish.

p	**q**	**$p \wedge q$**
T	T	T
T	F	F
F	T	F
F	F	F

**In order for a conjunction to be true, both p and q must be true.

<u>Disjunctions</u>

Look at: A cat is a mammal or a snake is a fish.

<u>Questions</u>: Is this true or false?

True

Why is the statement false?

Only one part must be true for the entire statement to be considered true.

Definition: A disjunction is a statement in which two statements, p and q, are connected by "or." The notation for the disjunction "p or q" is denoted $p \vee q$.

p: A cat is a mammal.
q: A snake is a fish.
$p \vee q$: A cat is a mammal or a snake is a fish.

p	**q**	**$p \vee q$**
T	T	T
T	F	T
F	T	T
F	F	F

**Only one of the statements of a disjunction needs to be true for the disjunction to be true.

Sample Problems: Determine whether the statements are true or false.

1. The president is Al Gore.

false

2. Bicycle gears require oil and a coolant.

false

3. The St. Croix or the Mississippi runs through Minnesota.

    true

4. Brown is neither a primary nor a secondary color.

    true

<u>Note</u>: "Nor" is not the same as "or" but it is the same as "and."

Sample Problems: Classify each as a conjunction or disjunction and determine the falsity.

1. Presidents Reagan and Clinton were Republicans.

    conjunction, false

2. Bret Favre or Joe Mauer is a baseball player.

    disjunction, true

3. Regular polygons are equiangular and equilateral.

    conjunction, true

4. Two lines are parallel or they intersect.

    disjunction, false

Make a truth table for (p ∧ r) ∨ (q ∧ ~r)

"Dogs and cats are mammals, or horses are mammals and cats are not mammals."

p	q	r	(p ∧ r)	~r	(q ∧ ~r)	(p ∧ r) ∨ (q ∧ ~r)
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Answer:

p	q	r	(p ∧ r)	~r	(q ∧ ~r)	(p ∧ r) ∨ (q ∧ ~r)
T	T	T	T	F	F	T
T	T	F	F	T	T	T
T	F	T	T	F	F	T
T	F	F	F	T	F	F
F	T	T	F	F	F	F
F	T	F	F	T	T	T
F	F	T	F	F	F	F
F	F	F	F	T	F	F

****Note:** If there are n statements, there are 2^n possible combinations. For 3 statements as we have here, there are $2^3 = 8$ combinations.

Make a truth table for ~ s ∨ (s ∧ t)

s	t	~ s	s ∧ t	~ s ∨ (s ∧ t)
T	T			
T	F			
F	T			
F	F			

Answer:

s	t	~ s	s ∧ t	~ s ∨ (s ∧ t)
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

Make a truth table for (~p ∧ q) ∧ r

p	q	r	~p	(~p ∧ q)	(~p ∧ q)∧r
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Answer:

p	q	r	~p	$(\sim p \wedge q)$	$(\sim p \wedge q) \wedge r$
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	T	F	F
F	F	F	T	F	F

Note: 1ST column use 4 T's and then 4 F's. 2nd column use 2 T's, and 2 F's. 3rd column use 1 T and 1 F, etc. The solutions will have the problems in this form.

section 5.4

Conditional Statements:

Look at: If the sun is shining, then it is daytime.

Let p: The sun is shining.
Let q: It is daytime.

Then this statement translates to "If p, then q."

Definition: A conditional statement is a statement of the form "If p, then q," where p and q are statements. The notation for this conditional statement is $p \rightarrow q$.

p is the "if" statement and is called the hypothesis

q is the "then" statement and is called the conclusion

Sample Problems: Write the following in if-then form:

1. There are no clouds in the sky, so it is not raining.

   If there are no clouds in the sky, then it is not raining.

2. CHAT will be cancelled if a blizzard hits.

   If a blizzard hits, then CHAT will be cancelled.

3. It will be warm tomorrow if it does not rain.

   If it does not rain, then it will be warm tomorrow.

4. When you water your plants, they grow.

   If you water your plants, then they will grow.

5. Satisfaction guaranteed or your money back.

   If you are not satisfied, then you will get your money back.

Example: TJ says, "If I ace this geometry test, then I'll take the entire class for pizza."

Let p: TJ aces the geometry test.
Let q: TJ takes the entire class out for pizza.

p → q: If TJ aces the geometry test, then he will take the entire class out for pizza.

Determine the truth table values:

Case 1: **Both p and q are true.**

TJ aces the test and he takes the class out for pizza.

∴ TJ told the truth when he made the conditional statement. **True → True is true**

Case 2: p is true and q is false

TJ aces the test but doesn't take the class for pizza.

∴ TJ lied and **True → False is false**

<u>Case 3</u>: p is false and q is true

TJ doesn't ace the test but still takes the class out for pizza.

∴ TJ didn't lie when he made his conditional statement, since nothing was said about what he would do if he didn't ace the test.
False → True is <u>true</u>

<u>Case 4</u>: Both p and q are false.

TJ doesn't ace the test and he doesn't take the class for pizza.

∴ TJ didn't lie when he made his conditional statement. **False → False is <u>true</u>**

*****In general, the only time the conditional statement is false is if the hypothesis is true, but the conclusion is false.

Truth Table for a Conditional Statement

p	**q**	**p→q**
T	T	T
T	F	F
F	T	T
F	F	T

Implications

“If p, then q” is sometimes written “p implies q”

Definition: A conditional statement that is always true is called an implication.

Example:

if-then: If it is raining, then there are clouds in the sky.
implication: The fact that it is raining implies that there are clouds in the sky.

Biconditional Statements

Look at:

p: The sun is shining.
q: There are no clouds in the sky today.

$p \rightarrow q$: If the sun is shining, then there are no clouds in the sky today.

$q \rightarrow p$: If there are no clouds in the sky today, then the sun is shining.

***If both $p \rightarrow q$ is true, and $q \rightarrow p$ is true, we call this a biconditional statement.

Definition: A <u>biconditional statement</u> is a statement of the form "p if and only if q" which means $p \rightarrow q$ and $q \rightarrow p$, and is symbolized by $p \leftrightarrow q$.

We would say:

The sun is shining if and only if there are no clouds in the sky today.

********* *When proving a biconditional statement, you must prove **both** $p \rightarrow q$ and $q \rightarrow p$.*

Truth Table for Biconditional Statements

p	**q**	**$p \rightarrow q$**	**$q \rightarrow p$**	**$(p \rightarrow q) \wedge (q \rightarrow p)$**
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

*****A biconditional statement is only true if p and q have the same truth values.**

Changing Conditionals to Disjunctions

When two statements have the same truth values, they are logically equivalent.

Theorem 5.1: The conditional statement $p \to q$ is equivalent to the disjunction $\sim p$ or q.

Truth Table showing equivalence:

p	**q**	**$p \to q$**	**$\sim p$**	**$\sim p \vee q$**	**$(p \to q) \leftrightarrow (\sim p \vee q)$**
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

The truth values match, so the 2 statements are equivalent.

If 2 statements are equivalent, then the biconditional statement will always be true.

This theorem allows us to substitute a disjunction for a conditional and vice versa. Any time we have equivalent expressions, they can be substituted for each other.

Sample Problems: Change the following conditional statements to disjunctions.

1. If a batter has 3 strikes, then he is out.

   A better doesn't have 3 strikes, or he is out.

2. If it is raining, then there are clouds in the sky.

   It is not raining, or there are clouds in the sky.

3. If a student goes to college, then he will become more educated.

   A student doesn't go to college, or he becomes more educated.

Definitions:

The <u>converse</u> of a conditional statement is obtained by switching the hypothesis and conclusion. The converse of p→q is q→p.

The <u>inverse</u> of a conditional statement is obtained by negating both the hypothesis and conclusion. The inverse of p→q is ~p→~q.

The <u>contrapositive</u> of a conditional statement is obtained by switching and negating the hypothesis and conclusion. The contrapositive of p→q is ~q→~p.

Example:

p: We have a blizzard.
q: CHAT will be cancelled.
$p \to q$: If we have a blizzard, then CHAT will be cancelled.

<u>converse</u>: $q \to p$

If CHAT is cancelled, then we had a blizzard.

<u>inverse</u>: $\sim p \to \sim q$

If we don’t have a blizzard, then CHAT won’t be cancelled.

<u>contrapositive</u>: $\sim q \to \sim p$

If CHAT is not cancelled, then we did not have a blizzard.

Sample Problem:

<u>Statement</u>: If my pet is a dog, then it is a mammal.

1. What is the converse of the statement?

    If my pet is a mammal, then it is a dog.

2. What is the inverse of the statement?

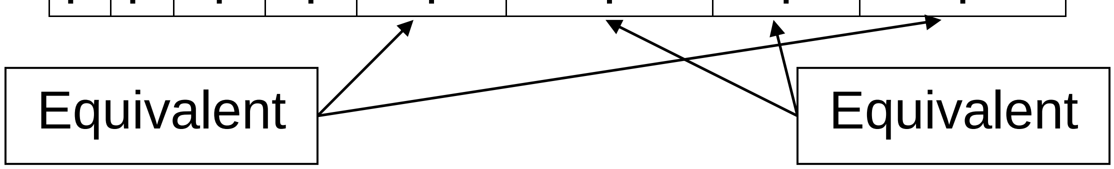
    If my pet is not a dog, then it is not a mammal.

3. What is the contrapositive of the statement?

    If my pet is not a mammal, then it is not a dog.

Truth Table for converse, inverse, & contrapositive

p	q	~p	~q	p→q	~p→~q	q→p	~q→~p
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T



*<u>Conditional statements</u> and <u>contrapositives</u> are equivalent.

*<u>Converses</u> and <u>inverses</u> are equivalent.