Perimeter and Circumference

Definition: The <u>perimeter</u> of a polygon is the distance around it.

Figure	Shape	Formula
Triangle		$P = a + b + c$
Rectangle		$P = 2l + 2w$
Square		$P = 4s$
Parallelogram		$P = 2a + 2b$
trapezoid		$P = a + b + c + d$

Practice Problems:

1. What is the perimeter of a square if the side length is 12 units?

$$P = 4(12) = 48 \ units$$

2. What is the length of a side of a square if its perimeter is 88 yards?

$$88 = 4s \ \rightarrow s = 22 \ yards$$

3. What is the length of a diagonal of a square whose side is 12 feet?

$$12^2 + 12^2 = d^2 \ \rightarrow d = 12\sqrt{2} \ feet$$

4. What is the perimeter of a regular decagon if the sides are each 14 inches?

$$P = 10(14) = 140 \ inches$$

5. What is the length of a rectangle whose width is 3 inches and perimeter is 30 inches?

$$2l + 2(3) = 30 \ \rightarrow l = 12$$

Definitions:

<u>circle</u> – the set of all points the same distance from a given point in a plane

<u>chord</u> – a line segment joining 2 points on a circle

<u>diameter</u> – a chord passing through the center

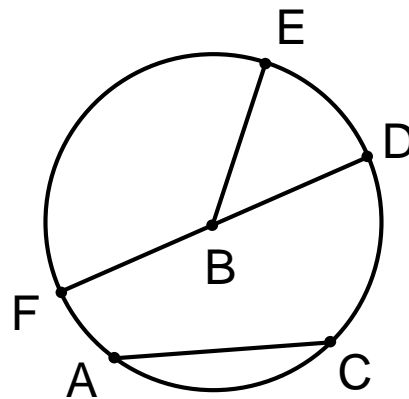<u>radius</u> – a line segment from the center to a point on a circle.

Example:

<u>center</u>: B

<u>radii</u>: $\overline{BE}$, $\overline{BD}$, $\overline{BF}$

<u>chords</u>: $\overline{FD}$, $\overline{AC}$

<u>diameter</u>: $\overline{FD}$

NOTE: In every circle, $\dfrac{circumference}{diamter}$ is always the same number. We call this number π. (≈ 3.14)

Definition: The <u>circumference</u> is the distance around the circle.

Formula: $C = \pi d$ or $C = 2\pi r$

3

Practice Problems: (Use 3.14 for π)

1. If the radius is 2 inches, find the circumference.

$$C = 2(3.14)(2) = 12.56 \; inches$$

2. If the diameter is 7 feet, find the circumference.
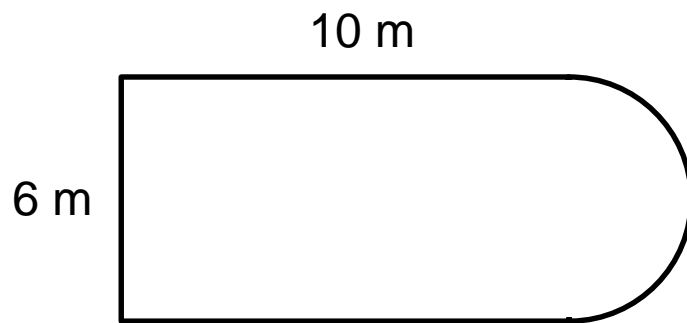
$$C = (3.14)(3.5) = 21.98 \; feet$$

3. If the circumference is 9 feet, find the radius.

$$C = 2\pi r$$
$$9 = 2(3.14)r$$
$$\frac{9}{6.28} = \frac{6.28r}{6.28}$$
$$r = 1.43 \; feet$$

4. What is the diameter if the circumference of the circle is 112 m?

$$C = \pi d$$
$$112 = 3.14d$$
$$\frac{112}{3.14} = \frac{3.14d}{3.14}$$
$$d = 35.7 \; m$$

5. Find the perimeter of this figure:

10 m

6 m

$$semicircle = \frac{1}{2}(2)(3.14)(3) = 9.42$$
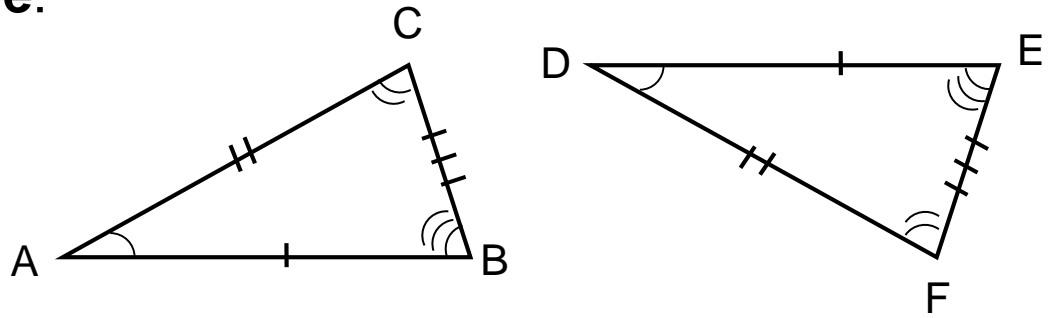$$P = 10 + 6 + 10 + 9.42 = 35.42\ m$$

Congruence and Similarity

Definition: <u>Congruent polygons</u> have the same size and shape.

Symbol: ≅

**If 2 figures are congruent, then their corresponding sides and corresponding angles are congruent.

Example:

$$\triangle\textbf{ABC} \cong \triangle\textbf{DEF}$$

$$\overline{AB} \cong \overline{DE} \qquad \angle A \cong \angle D$$
$$\overline{AC} \cong \overline{DF} \qquad \angle B \cong \angle E$$
$$\overline{BC} \cong \overline{EF} \qquad \angle C \cong \angle F$$

Practice Problem: Given $\triangle\textbf{QRS} \cong \triangle\textbf{TUV}$, find:

Answers

1. $\angle Q \cong$ _____ $\angle T$

2. $QR =$ _____ TU

3. $RS =$ _____ UV

4. $\angle U \cong$ _____ $\angle R$

5. $m\angle S =$ _____ $m\angle V$

6. $\overline{TV} \cong$ _____ $\overline{QS}$

Definition: <u>Similar polygons</u> have the same shape.

Symbol: ~

**If 2 figures are similar, then their corresponding angles are congruent and their corresponding sides are proportional.
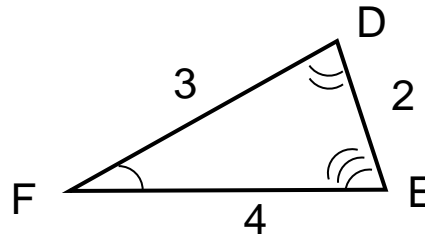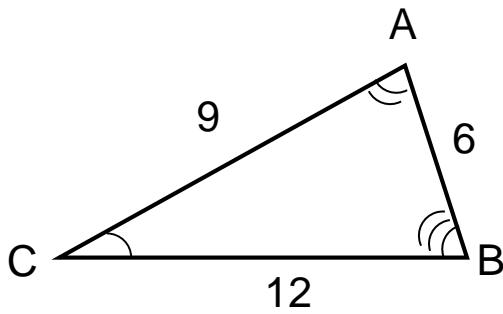
Example: $\triangle ABC \sim \triangle DEF$

From the notation we know: $\angle A \cong \angle D$
$\angle B \cong \angle E$
$\angle C \cong \angle F$

Ratios of corresponding sides: $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$

We say "AB is to DE as BC is to EF" etc.

Let's put lengths on the sides and check the ratios.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \qquad \text{or} \qquad \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC}$$

$$\frac{6}{2} = \frac{9}{3} = \frac{12}{4} \qquad\qquad\qquad \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$$
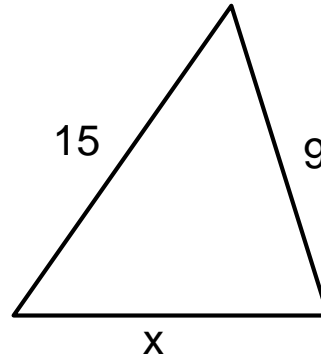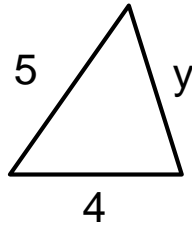
$$3 \ = \ 3 \ = 3 \qquad\qquad\qquad ⅓ \ = \ ⅓ = ⅓$$

**It does not matter which triangle you start with for your proportion.

- We say that 3 is the <u>scale factor</u> of the larger triangle to the smaller triangle.

- We say 1/3 is the <u>scale factor</u> of the smaller triangle to the larger triangle.
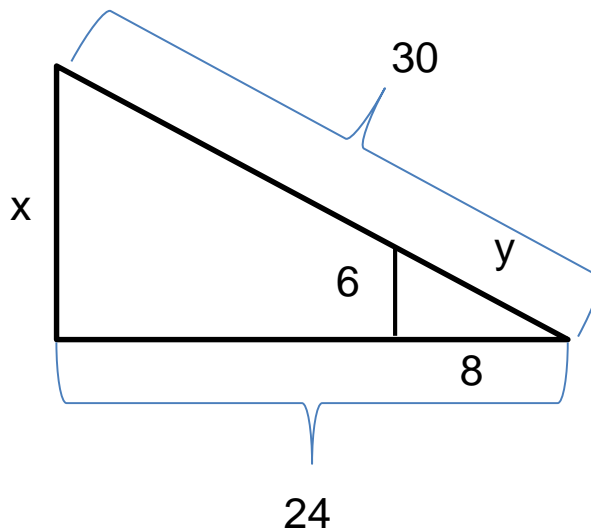
Pactice Problems: Find the unknown values.

1.
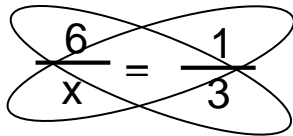
5   y

4

15   9

x

$\frac{x}{4} = \frac{15}{5}$

$(5)(x) = (4)(15)$
$5x = 60$
$x=12$

$\frac{y}{9} = \frac{5}{15}$

$(15)(y) = (9)(5)$
$15y = 45$
$y=3$

2.

30

x

6   y

8

24

$$\frac{6}{x} = \frac{8}{24}$$

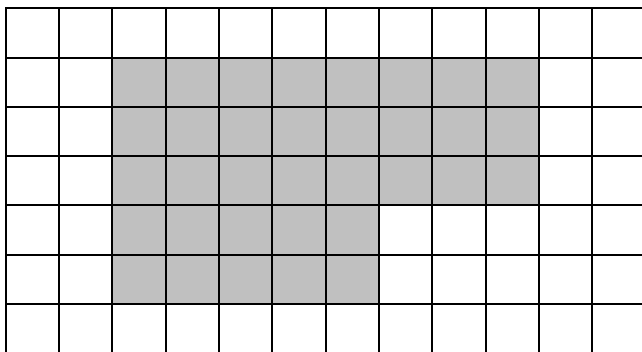$$\frac{y}{9} = \frac{8}{24}$$

$$\frac{6}{x} = \frac{1}{3}$$

$$\frac{y}{30} = \frac{1}{3}$$
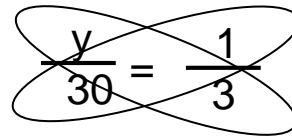
$$(3)(y) = (1)(30)$$
$$3y = 30$$
$$y = 10$$
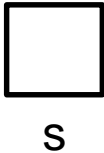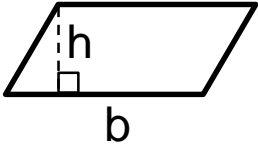
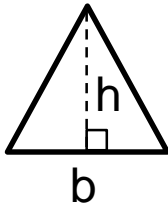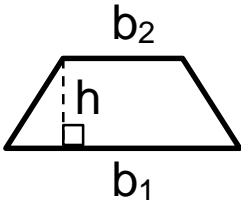$$(1)(x) = (3)(6)$$
$$x = 18$$
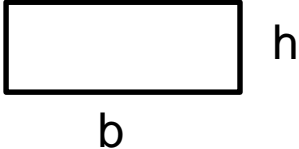
Area

Definition: <u>Area</u> is the number of square units needed to cover a region or surface.

Example:

Area = 34 square units

Area Formulas

| Figure	Shape	Formula
Rectangle	 h b	$A = bh$
Square	 s	$A = s^2$
Parallelogram	h b	$A = bh$
Triangle	h b	$A = \dfrac{1}{2}bh$
trapezoid	b_2 h b_1	$A = \dfrac{1}{2}(b_1 + b_2)h$

The <u>height</u> of a figure is the perpendicular distance between the base and the top side or vertex.

Triangles have 3 heights, called <u>altitudes</u>. The altitude is the perpendicular segment that connects a vertex to the line that contains the opposite side.
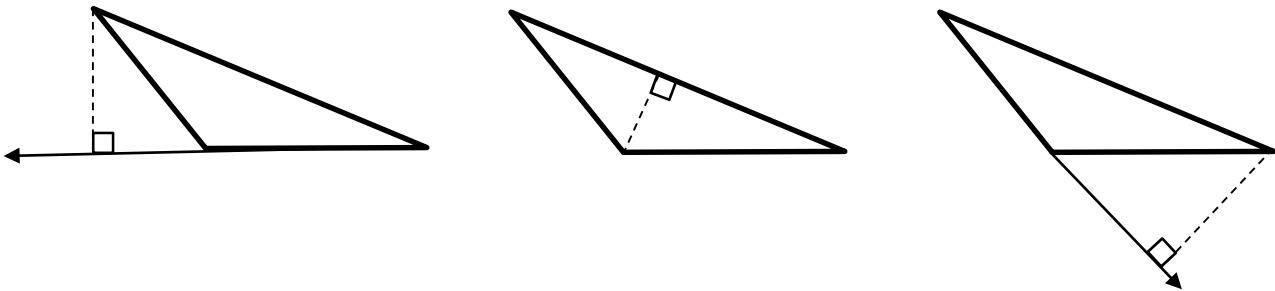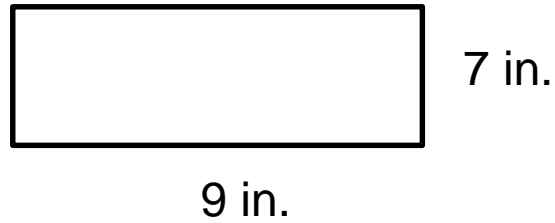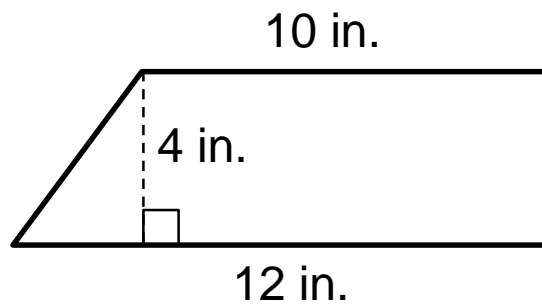
Practice Problems: Find the area.
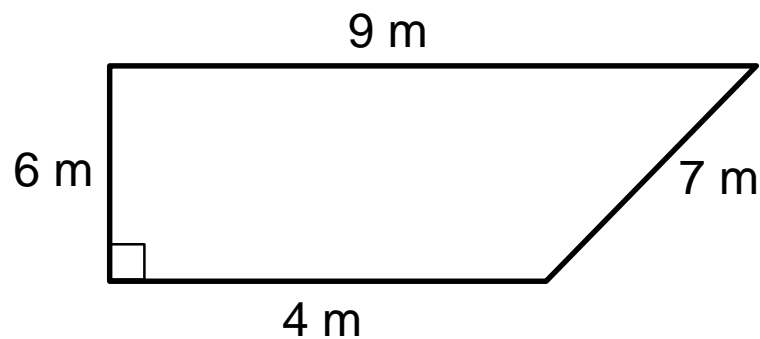
1.

5

7

10
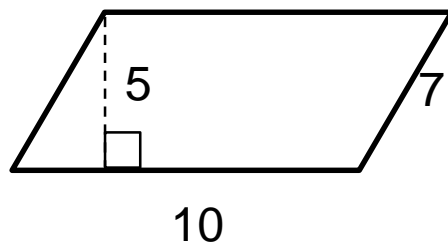
Answer: $A = (10)(5) = 50\ square\ units$

2.

7 in.

9 in.

Answer: $A = (9)(7) = 63\ in^2$

3.

10 in.

4 in.

12 in.

Answer: $A = \frac{1}{2}(12 + 10)(4) = 44\ sq.\,in.$

4.

9 m

6 m

7 m

4 m

Answer: $A = \frac{1}{2}(4 + 19)(6) = 39\ m^2$

5.

12 cm

4 cm

9 cm

3 cm

4 cm   6 cm

5 cm

14 cm
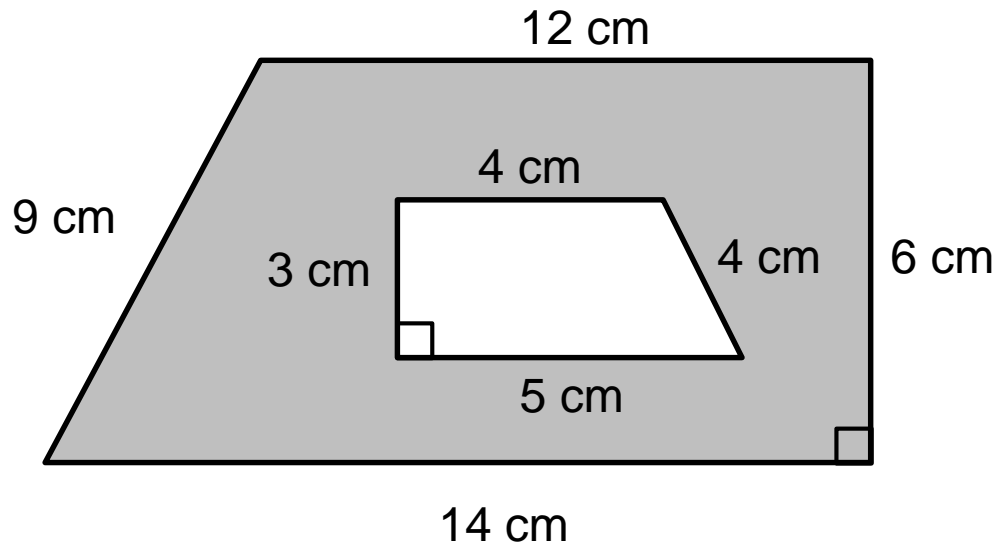
Answer: $A = \left[\frac{1}{2}(12 + 14)(6)\right] - \left[\frac{1}{2}(4 + 5)(3)\right]$

$A = 78 - 13.5 = 64.5 \ cm^2$

6.

8 m

2 m

2 m

3 m

2 m
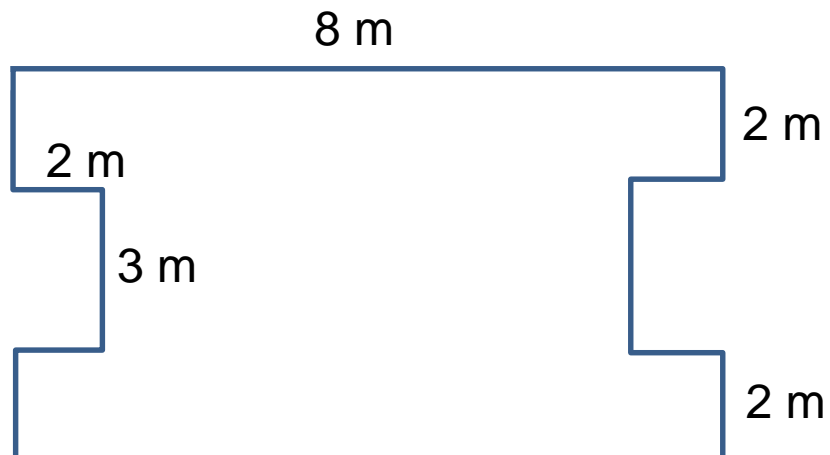
Answer: $A = (7 \cdot 8) - (2 \cdot 3) - (2 \cdot 3) = 44 \ m^2$

7. What is the length of a side of a square whose area is 225 square inches?

$$225 = s^2$$
$$s = 15 \ in.$$

8. What is the width of a rectangle if the area is 240 square feet and its length is 20 feet?
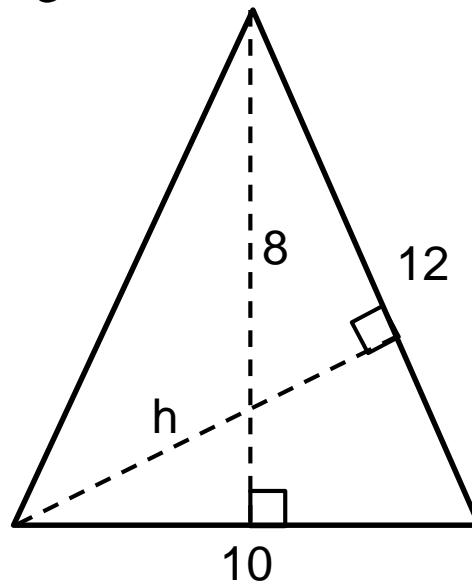
$$240 = 20w$$
$$w = 12 \ feet$$

9. What is the area of a triangle with a base of 12 inches and a height of 9 inches?

$$A = \frac{1}{2}(12)(9) = 54 \ in^2$$

10. If the base of a triangle is 10 cm and the aread is 100cm² what is its height?

$$100 = \frac{1}{2}(10h)$$
$$100 = 5h$$
$$h = 20 \ cm$$

11.   Find h in the figure.

$$A = \frac{1}{2}(10)(8) = 40$$

$$40 = \frac{1}{2}(12)h$$

$$40 = 6h$$

$$h = \frac{40}{6} = \frac{20}{3} \; units$$

12.   Take a square with a side of 10 inches. Then cut out a square with a side of 2 inches from each corner. What is the remaining area?

$$A = area\ of\ \text{large}\ square - area\ of\ 4\ small\ squares$$
$$A = 10^2 - 4(2^2) = 84\ in^2$$