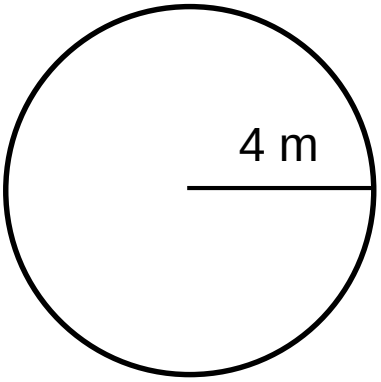
Area of Circles

The area of a circle is π times the square of the radius: $A = \pi r^2$

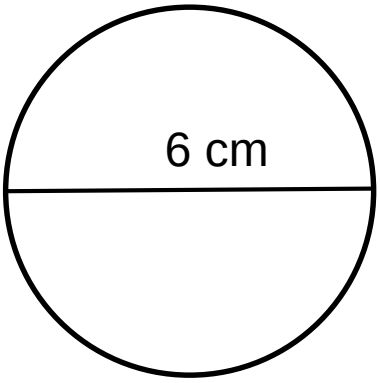
Sample Problems: Find the area.

1.



$$A = \pi(4^2) = 16\pi \approx 50.24\, m^2$$

2.

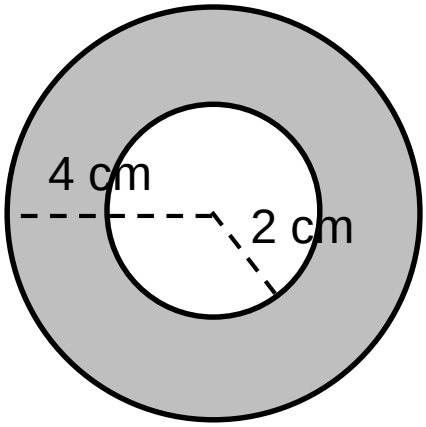


$$A = \pi(3^2) = 9\pi \approx 28.26\, m^2$$

3. Find the radius of a circle if the area is 191 un^2?

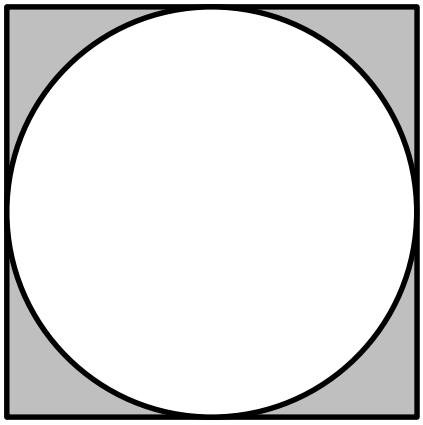
$$191 = \pi(r^2)$$
$$\frac{191}{3.14} = \frac{r^2}{3.14}$$
$$r = 7.8$$

4.



$$A = \pi(4^2) - \pi(2^2) = 12\pi\ cm^2 = 37.68\ cm^2$$

5.



$$Answer\text{:}\ A = 8^2 - \pi(4^2) \approx 13.8\ ft^2$$

Surface Areas of Prisms & Cyllinders

Definitions:

three-dimensional figure – a shape that has points in different planes

polyhedron – 3D figure whose faces are flat and shaped like polygons

face – any polygon that is part of a polyhedron

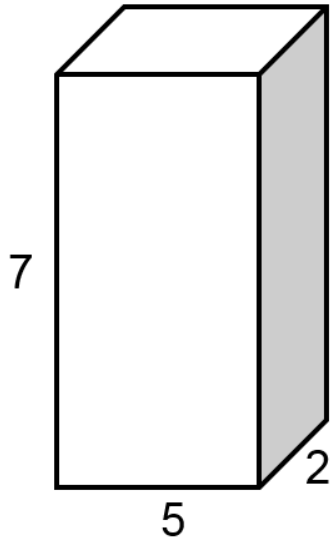
edge – the line of intersection of 2 faces of a polyhedron

vertex – any point where 3 or more faces of a polyhedron intersect

prism – a polyhedron with 2 congruent parallel faces

bases – the congruent parallel faces of a prism

lateral faces – all the faces of a prism or pyramid except the bases

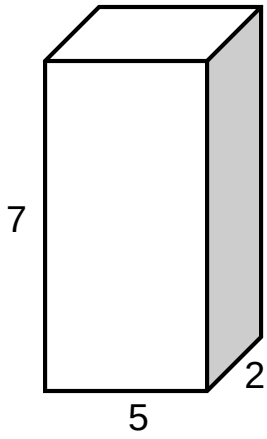


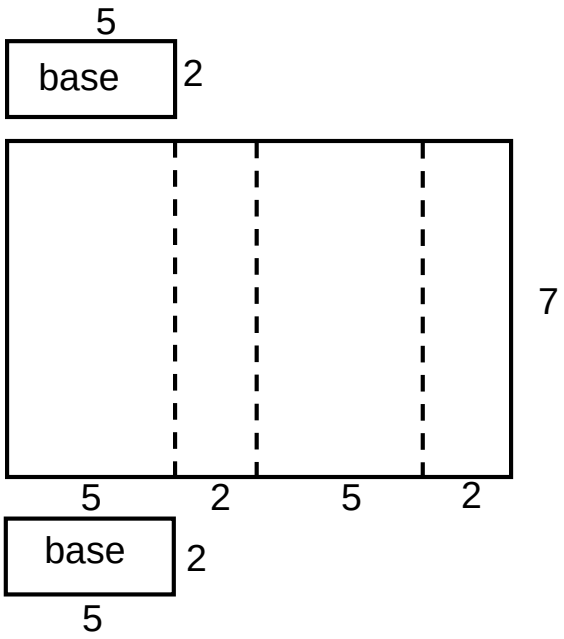
Area of front face = (5)(7) = 35
Area of base = (5)(2) = 10
Area of the side face = (7)(2) = 14

Since we have 2 of each size rectangle, we get:

$$S = 2(35) + 2(10) + 2(14) = 118 \text{ sq. units}$$

Look at:





Notice that when we unfold the sides, we get one large rectangle. The dimensions are 7 by 14. Where did that 14 come from?

14 = 5+2+5+2 (the perimeter of the base)

Formulas:

Lateral Surface Area = (perimeter of base)(height)

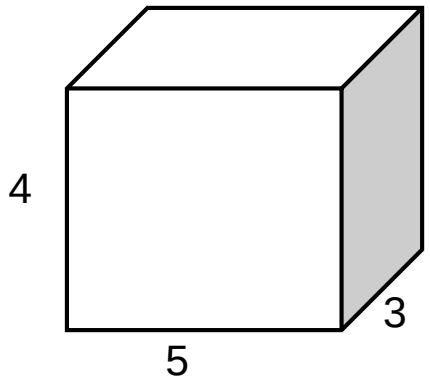
L = pH

Total Surface Area = (lateral surface area + area of 2 bases)

S = L +2B

Sample Problems: Find the lateral and total surface area of the following solid figures.

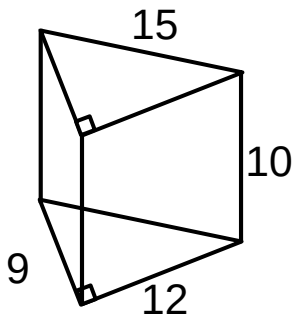
1.



Answer: $\mathrm{L} = \mathrm{pH} = (16)(4) = 64\, sq.un.$

$S = L + 2B = 64 + 30 = 94\, sq.un.$

2.

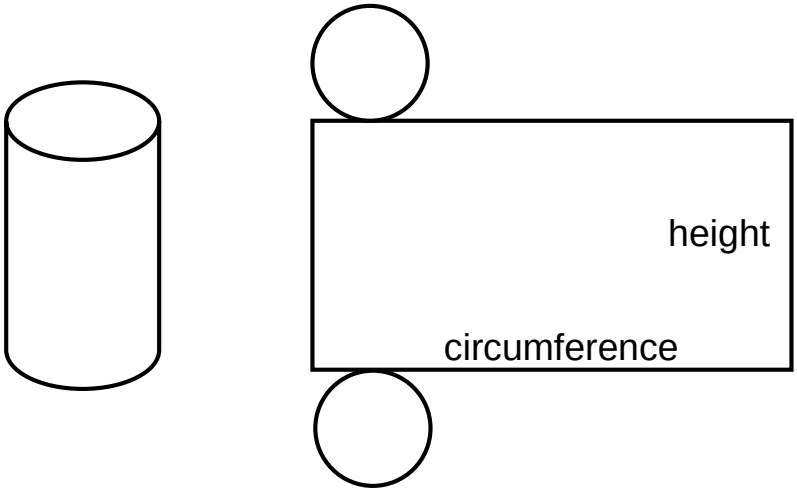


$L = pH$
$L = 36(10)$
$L = 360$

$B = \frac{1}{2}(12)(9)$
$B = 54$

$S = 360 + 2(54)$
$= 468\ sq.un.$

Circular Cylinders



Lateral Surface Area = pH

L = cH (the perimeter of the base is the circumference)

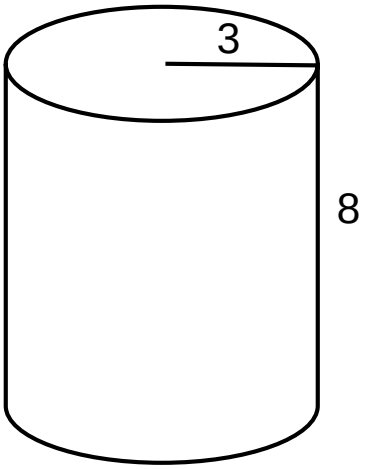
L = 2πrH

Total Surface Area = L + 2B

S = 2πrH + $2\pi r^2$

Sample Problems:

1.



Answer: $L = cH = 48\pi = 150.72\ un^2$

$S = L + 2B = 48\pi + 2(9\pi) = 66\pi = 207.24\ un^2$

2. How much aluminum sheeting is needed to make a round trash can (no lid) with a diameter of 16 inches and height of 2 feet?

$L = 2\pi(8)(24) = 384\pi$

$B = \pi(8^2) = 64\pi$

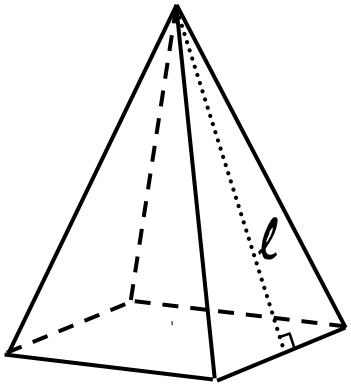
$S = \ 384\pi + \ 64\pi = 448\pi \approx 1406.7\ in^2$

$Since\ there\ are\ 144\ sq.in.in\ 1\ sq.ft,$

$$S = \frac{1406.7}{144} = 9.77\ ft^2$$

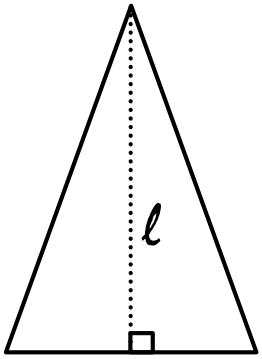
Surface Area of Pyramids, Cones, & Spheres

The lateral surface area of a regular pyramid is ½ of the perimeter times the slant height.

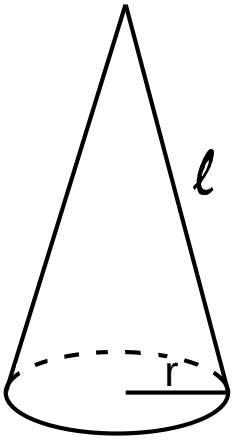


$L = \frac{1}{2} p\ell$

$S = L + B$



The lateral surface area of a circular cone is half the product of the circumference and slant height:



$L = \frac{1}{2} c\ell$

$L = \frac{1}{2} (2\pi r)\ell$

$L = \pi r\ell$

$S = L + B$

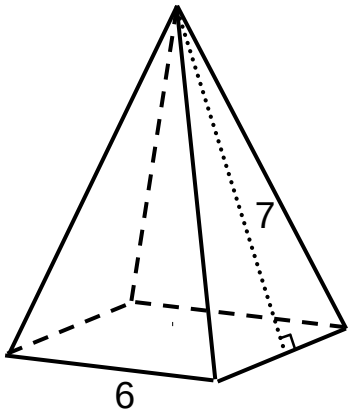
$S = \pi r\ell + \pi r^2$

The surface area of a sphere is:

$S = 4\pi r$

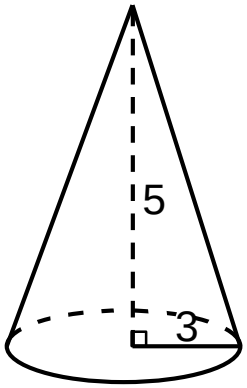
Sample Problems:

1.



$$L = \frac{1}{2}(24)(7) = 84\ un^2$$
$$B = 6^2 = 36\ un^2$$
$$S = 84 + 36 = 120\ un^2$$

2.



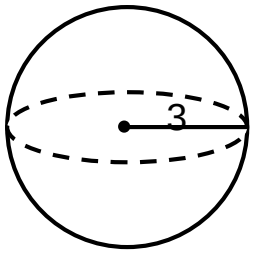
$l^2 = 3^2 + 5^2$ $\quad L = \pi(3)(5.8)$ $\quad B = \pi(3^2)$

$l^2 = 34$ $\quad L = 54.6$ $\quad B \approx 12.6$

$l = \sqrt{34} \approx 5.8$

$$S = 54.6 + 12.6 = 67.2\ un^2$$

3.



$$S = 4\pi(3^2) = 36\pi \approx 37.7\ sq.units$$

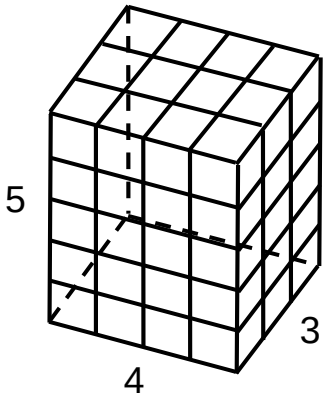
Volume

Definition:

The volume of a solid is the number of cubic units needed to fill up the interior completely.

A cubic unit is a cube whose sides measure one unit.

Prism:



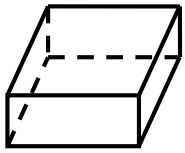
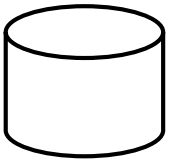
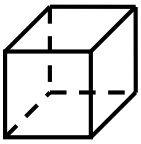
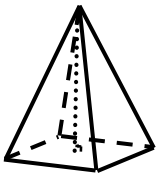
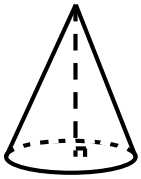
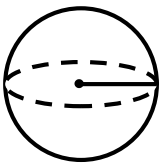
5 stacks of 12 cubes

(5)(12) = 60 cubes

Notice that 12 is also the area of the base.

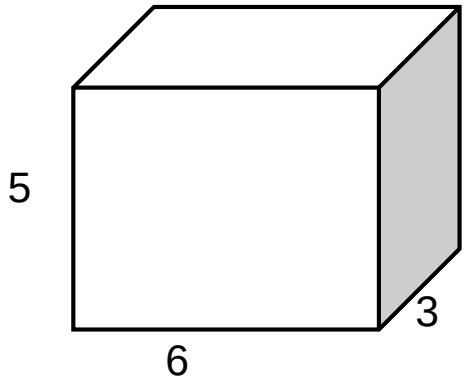
5 stacks of 12 cubes

Formulas:

Figure	Shape	Formula
Prism		$V = BH$ $V = lwh$
Cylinder		$V = BH$ $V = \pi r^2 H$
Cube		$V = s^3$
Pyramid		$V = \frac{1}{3}BH$
Cone		$V = \frac{1}{3}BH$ $V = \frac{1}{3}\pi r^2 H$
Sphere		$V = \frac{4}{3}\pi r^3$

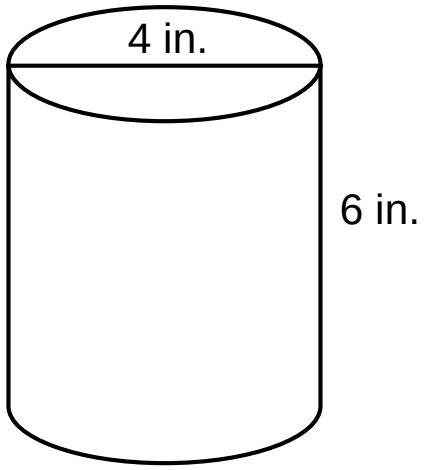
Practice Problems:

1.



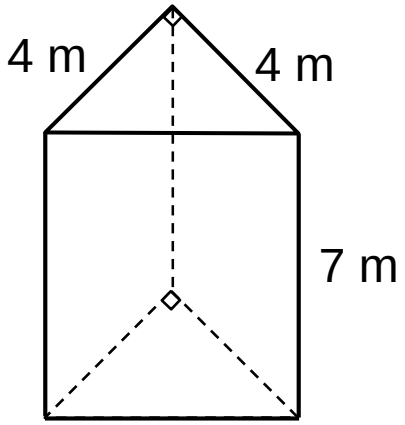
Answer: $V = (3)(6)(5) = 90\ un^3$

2.



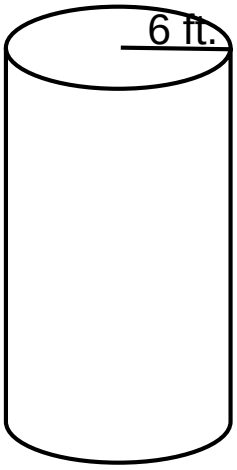
Answer: $V = (\pi)(2^2)(6) = 24\pi \approx 75.4\ in^3$

3.



$$V = \left[\frac{1}{2}(4)(4)\right](7) = 56\,m^3$$

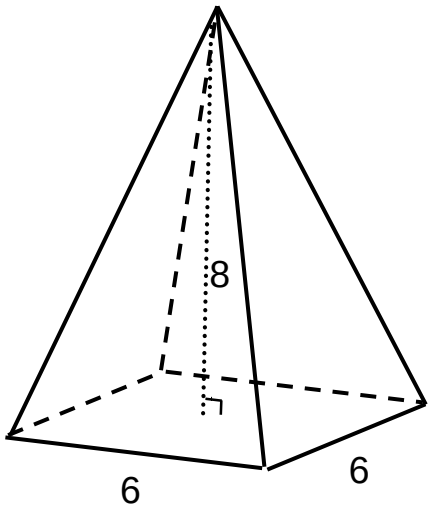
4.



If the volume is 814 square feet, find the height.

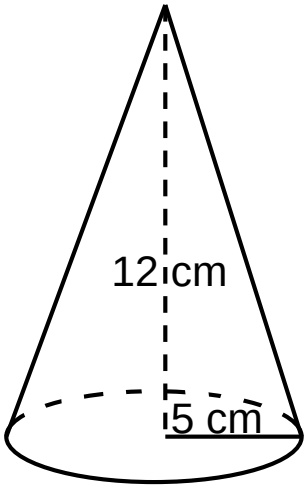
$$V = \pi(6^2)H$$
$$814 = 113.04H$$
$$H = 7.2\ ft.$$

5.



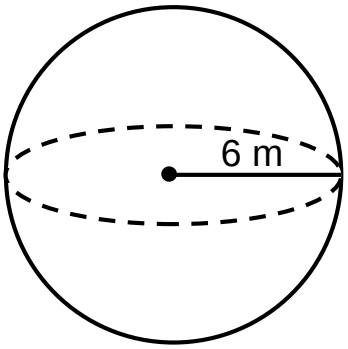
$$V = \frac{1}{3}(36)(8) = 96\ cu.units$$

6.



$$V = \frac{1}{3}\pi(5^2)(12) =$$

7.



$$V = \frac{4}{3}\pi(6^3) = 288\pi \approx 904.3\ m^3$$

8. What is the volume of a pyramid if its height is 10 units and its base is 8 units by 12 units?

$$B = (8)(12) = 96$$
$$V = \frac{1}{3}(96)(10) = 320\ un^3$$

9. Find the height of the cone if the radius is 7 cm and volume is 615.6 cm³.

$$V = \frac{1}{3}\pi r^2 H$$
$$615.6 = \frac{1}{3}(3.14)(49)H$$
$$H = 12$$