Graphs of Equations

A coordinate system is a way to graphically show the relationship between 2 quantities.

Definition: A solution of an equation in two variables x and y is an ordered pair (a, b) such that when x is replaced by a and y is replaced by b, the resulting equation is a true statement. The graph of an equation of this type is the collection of all points in the rectangular coordinate system that correspond to the solution of the equation.

Sketching a Graph

To sketch the graph of an equation in two variables using the point-plotting method, construct a table of values that consists of several solution points of the equation. Plot the solution points on a rectangular coordinate system and connect the points with a smooth curve.

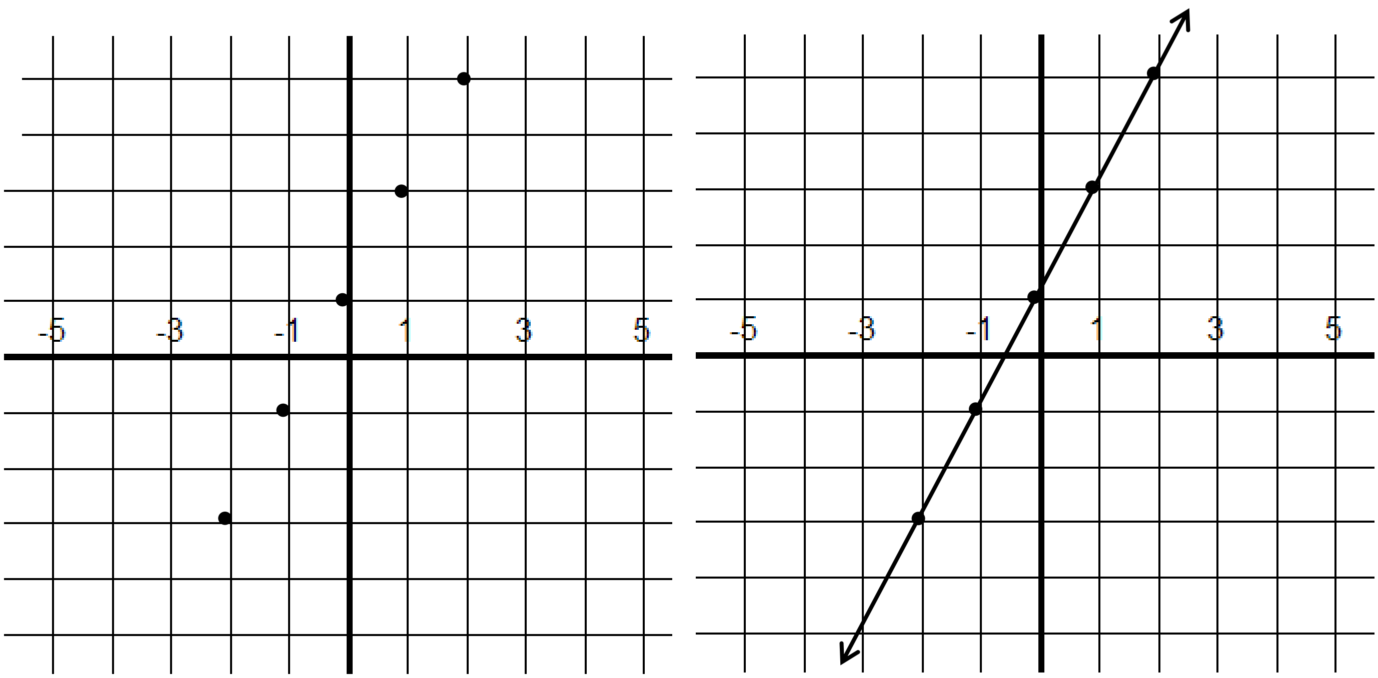
Note: A disadvantage of the point-plotting method is that with too few solution points, you can badly misrepresent the graph of an equation.

Example: Sketch the graph of $y = 2x + 1$.

Solution: Make a table of ordered pairs.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

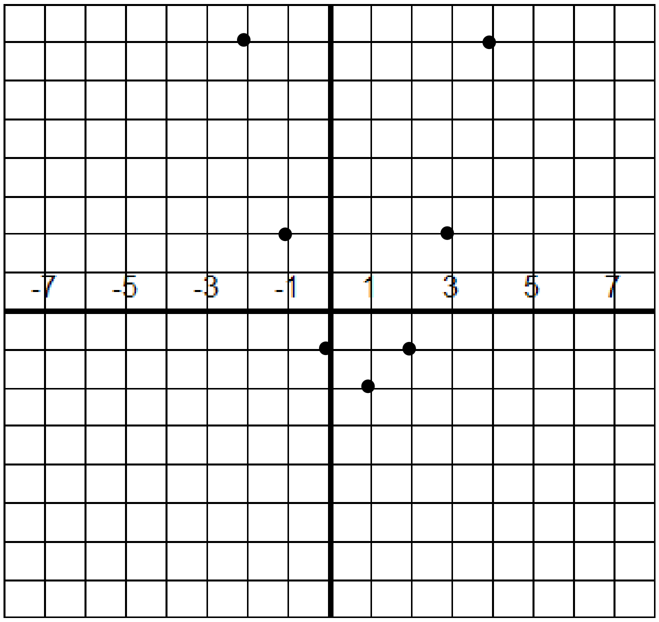
Plot the points and connect them.

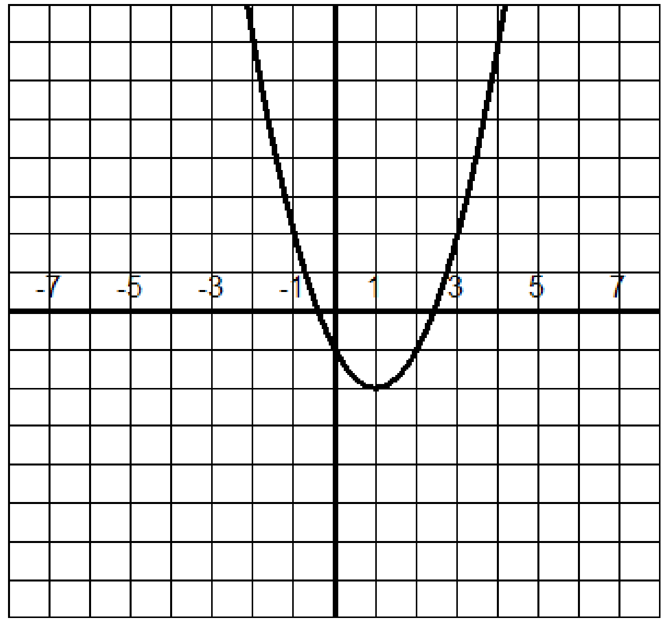


Example: Sketch the graph of $y = x^2 - 2x - 1$.

Solution:

x	-2	-1	0	1	2	3	4
y	7	2	-1	-2	-1	2	7





Note: The graphing utility does not put arrows on the ends of lines and curves, but they should be there to show that the graph goes on infinitely.

Intercepts of Graphs

Definition of Intercepts

A point at which the graph of an equation meets the x-axis is called an x-intercept. It is of the form $(x, 0)$.

To find the x-intercept: Let $y = 0$ and solve for x.

A point at which the graph of an equation meets the y-axis is called an y-intercept. It is of the form $(0, y)$.

To find the y-intercept: Let $x = 0$ and solve for y.

Example: Find the x- and y-intercepts of the graph of $2x+3y=6$

Solution: For the x-intercept, let y = 0 and solve for x.

$$2x+3y=6$$
$$2x+3(0)=6$$
$$2x=6$$
$$x=3$$

For the y-intercept, let x = 0 and solve for y.

$$2x+3y=6$$
$$2(0)+3y=6$$
$$3y=6$$
$$y=2$$

This means that the points (3, 0) and (0, 2) are points on our graph. We could find more points by making a table.

Example: Find the intercepts of the graph of

$$y = x^2 + x - 6$$

Solution: For the y-intercept, let x = 0 and solve for y.

$$y = x^2 + x - 6$$
$$y = 0^2 + 0 - 6$$
$$y = -6$$

For the x-intercept, let y = 0 and solve for x.

$$y = x^2 + x - 6$$
$$0 = x^2 + x - 6$$
$$0 = (x+3)(x-2)$$
$$x = -3 \text{ or } x = 2$$

This means that the points (0, -6), (-3, 0) and (2, 0) are points on our graph. We could find more points by making a table.

Using a Graphing Calculator to Graph Equations

To graph an equation, do the following:

1. Press [MODE] and then on the 4th line down, make sure that [Func] is highlighted.
2. Press [2nd] [QUIT] to get back to the main screen.
3. Press [Y=] to get to the equation editor. Enter your equation. Note: The equation must be of the form "y = __________". If it is not, solve it first for y and then enter it in the equation editor. Press [GRAPH].

To change the window settings, do one of the following:

1. Press [WINDOW] and manually set your ranges for x and y.
2. Press [ZOOM] and then [ZoomFit]. If your graph is not appearing on your screen, this should bring at least part of it into view. Then press [ZOOM] [Zoom Out] [ENTER] or [ZOOM] [Zoom In] [ENTER] to see your graph more accurately. (You can move the cursor to the center of the area you want to zoom before you hit [ENTER]. Use the arrow keys.)
3. Use the Zoom Box. To do this, press [ZOOM] [ZBox] [ENTER]. Use the right and left arrows to move the cursor to the corner of the box that you will draw around the area you want to include in your viewing window. Press [ENTER]. Then using the arrow keys, create your box and then press [ENTER] again.

*To get the window back to the default standard settings, press [ZOOM] [ZStandard].

Approximating Points on a Graph

To approximate a point on the graph on your calculator, first have the graph in your viewing window. Press [TRACE]. A blinking cursor will appear on your curve. Using the left and right arrows, you can move the cursor along your curve. The coordinates will appear at the bottom of the screen.

Finding Intercepts Using a Graphing Calculator

To find the x-intercept, do the following:

1. Have the graph in your viewing window.
2. Use the [CALC] feature by pressing [2^{nd}] [TRACE] and then choose [zero].
3. It will ask you for a left boundary first. Use the left and right arrow keys to close in on the area where it crosses the x-axis. Place the cursor on the curve to the left of the point where it crosses. Press [ENTER].
4. Use the right arrow key to move the cursor on the curve to the right of the point where it crosses the x-axis. Press [ENTER].
5. When you see “Guess?”, press [ENTER] again to find the value.

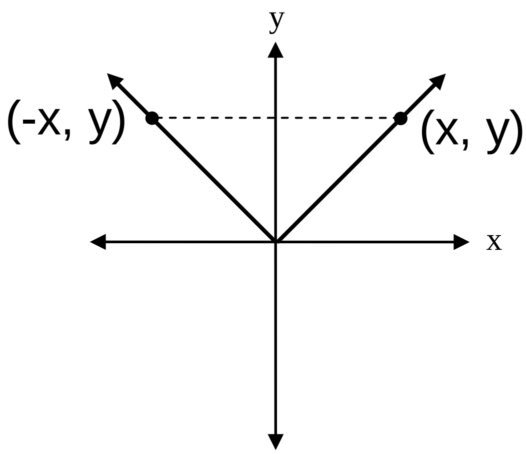
To find the y-intercept, do the following:

1. Have the graph in your viewing window.
2. Press [2nd] [TRACE] [value].
3. It will ask you for the x-coordinate. Enter 0 on your keyboard and then press [ENTER]. The y-coordinate will appear on the screen next to the x-coordinate.

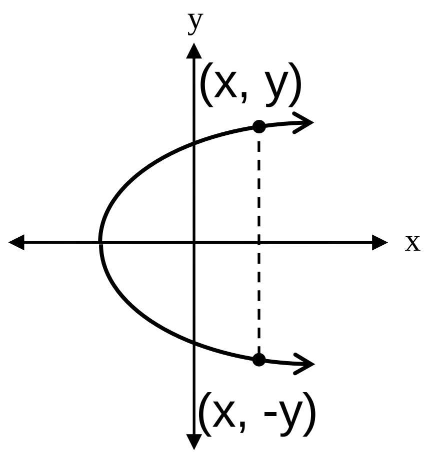
*Note: This process can be used to find any y-coordinate if you know the x-coordinate.

Symmetry

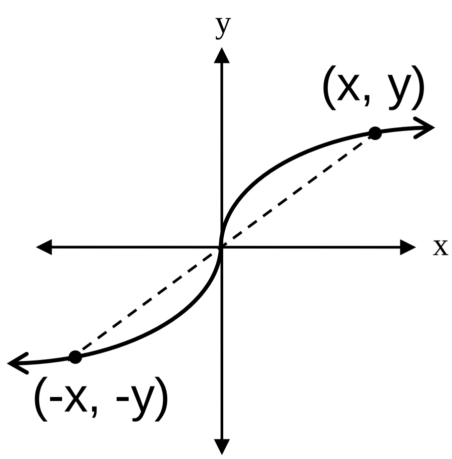
Symmetry is another tool that helps us when we graph. We will look at 3 types of symmetry:



y-axis symmetry



x-axis symmetry



origin symmetry

Graphical Tests for Symmetry

- A graph is symmetric with respect to the y-axis if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
- A graph is symmetric with respect to the x-axis if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph.
- A graph is symmetric with respect to the origin if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.

Testing for Symmetry

- The graph of an equation is symmetric with respect to the y-axis if replacing x with $-x$ yields an equivalent equation.
- The graph of an equation is symmetric with respect to the x-axis if replacing y with $-y$ yields an equivalent equation.
- The graph of an equation is symmetric with respect to the origin if replacing x with $-x$ and y with $-y$ yields an equivalent equation.

Example: What kind of symmetry does the graph of $y = x^3$ have?

<u>*Solution*</u>:

1. Test for y-axis symmetry: Substitute $-x$.

$$y = (-x)^3$$

$$y = -x^3$$

not equivalent, so <u>no</u> y-axis symmetry

2. Test for x-axis symmetry: Substitute $-y$.

$$-y = x^3$$

not equivalent, so <u>no</u> x-axis symmetry

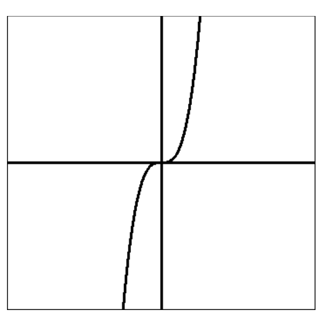
3. Test for origin symmetry: Substitute $-x$ and $-y$.

$$-y = (-x)^3$$

$$-y = -x^3$$

$$y = x^3$$

This is an equivalent equation, so there <u>is</u> origin symmetry.



Example: What kind of symmetry does the graph of $y = x^2$ have?

<u>*Solution*</u>:

1. Test for y-axis symmetry: Substitute $-x$.

$$y = (-x)^2$$

$$y = x^2$$

equivalent, so there <u>is</u> y-axis symmetry

2. Test for x-axis symmetry: Substitute $-y$.

$$-y = x^2$$

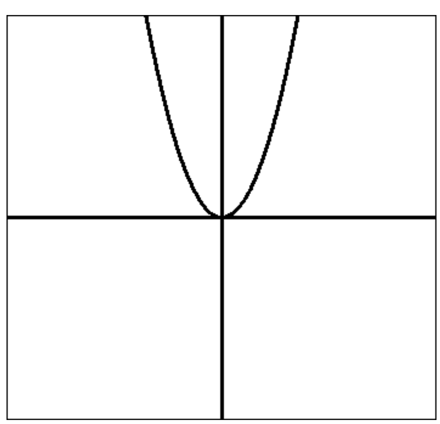
not equivalent, so <u>no</u> x-axis symmetry

3. Test for origin symmetry: Substitute $-x$ and $-y$.

$$-y = (-x)^2$$

$$-y = x^2$$

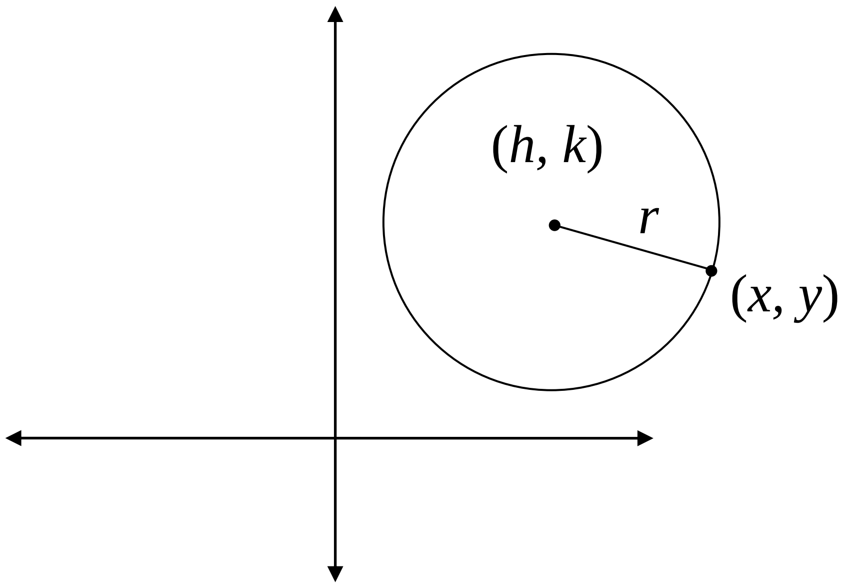
not equivalent, so <u>no</u> origin symmetry.



*We can use symmetry to reduce the amount of points we need to plot for a graph.

Circles

A circle is the set of all points that are a fixed distance from a fixed point. The fixed point is the center, and the given distance is called the radius.



Using the distance formula we get:

$$r = \sqrt{(x-h)^2 + (y-k)^2}$$

Squaring both sides gives us the standard equation for a circle.

$$r^2 = (x-h)^2 + (y-k)^2$$

Standard Equation of a Circle

The point (x, y) lies on the circle of radius r and center (h, k) if and only if

$$(x-h)^2+(y-k)^2=r^2$$

Example: Find the standard form of the equation of the circle with center at (2, -5) and radius 4.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y--5)^2=4^2$$
$$(x-2)^2+(y+5)^2=16$$

Example: Find the standard form of the equation of the circle with center at (3, -2) and which passes through the point (-1, 1).

<u>Solution</u>: First find r using the distance formula.

$$r=\sqrt{(x-h)^2+(y-k)^2}$$
$$r=\sqrt{(-1-3)^2+(1+2)^2}$$
$$r=\sqrt{16+9}$$
$$r=\sqrt{25}=5$$

Now put the radius and center into the standard equation.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-3)^2+(y+2)^2=25$$