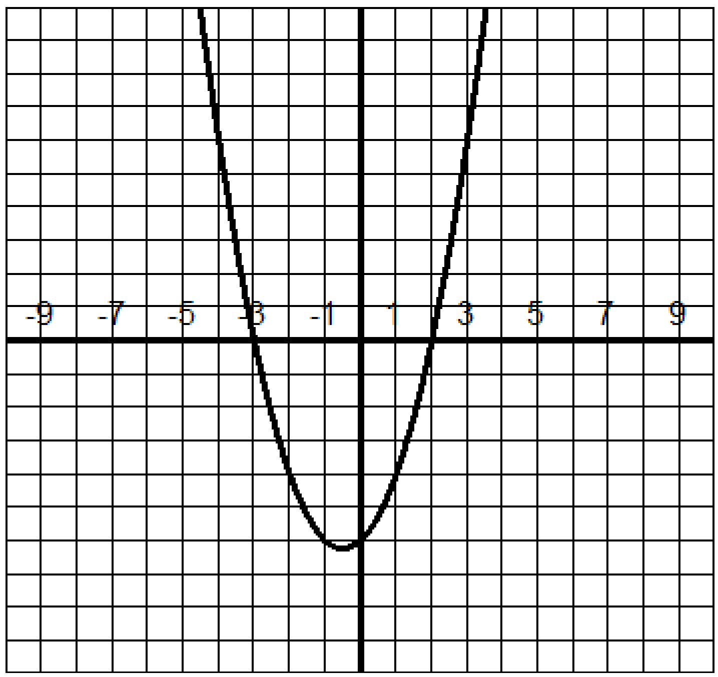
Functions and Their Graphs

Zeros of Functions

On a graphing calculator, graph $y = x^2 + x - 6$



Question: Where does the graph intersect the x-axis?

(-3, 0) and (2, 0)

We call the numbers -3 and 2 the zeros of this function, because it is the value at which the function equals zero.

Definition: If the graph of a function of x has an x-intercept at (a, 0), then a is a <u>zero</u> of the function and $f(a) = 0$.

*The x-intercepts give us the zeros.

Example: Find the zeros of $f(x) = x^2 - 3x - 10$.

Solution: We need to find the x-intercepts, which means we need to let $f(x) = 0$. (Remember that $f(x)$ is the same thing as y.)

$$f(x) = x^2 - 3x - 10$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x - 5 = 0 \quad \text{or} \quad x + 2 = 0$$

$$x = 5 \quad \text{or} \quad x = -2$$

The zeros are 5 and -2.

Example: Find the zeros of $f(x) = \sqrt[3]{x-1}$.

$\sqrt[3]{x-1} = 0$ when $x = 1$, so the zero is 1.

Example: Find the zeros of $h(x)=\dfrac{x-5}{2x-1}$.

<u>*Solution*</u>: Set $h(x)=0$ and solve.

$$0=\frac{x-5}{2x-1}$$

Multiply both sides by (2x-1)

$$0=\frac{x-5}{2x-1}$$

$$0=x-5$$

$$x=5$$

The zero is 5.

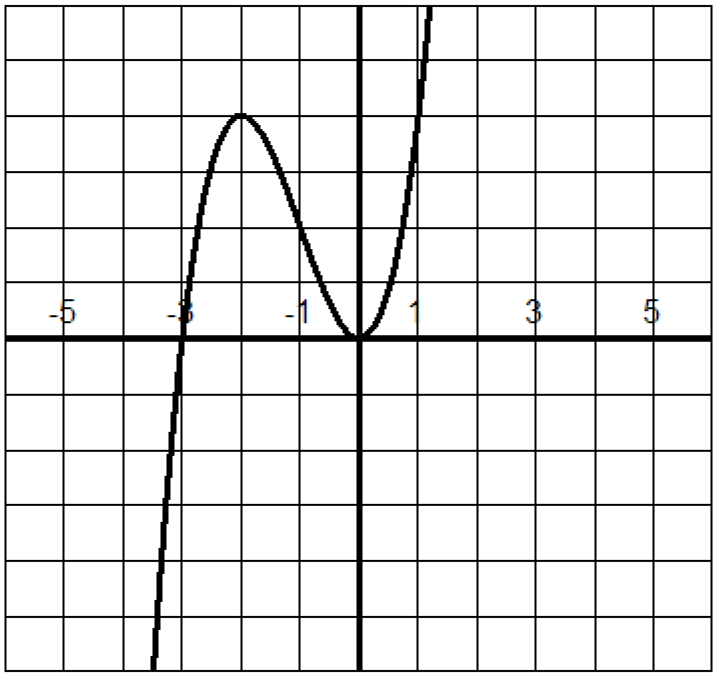
*<u>Note</u>: The zero occurs when the numerator = 0.

<u>Increasing and Decreasing Functions</u>

On a graphing calculator, graph $f(x)=x^3+3x^2$.

Use your graphing calculators to find the high points and low points. These are called the <u>relative minimum</u> and <u>relative maximum</u>. (The true minimum is $-\infty$ and the true maximum is ∞, but we can find the *relative* minimums and maximums.)

The graph of $f(x) = x^3 + 3x^2$ is



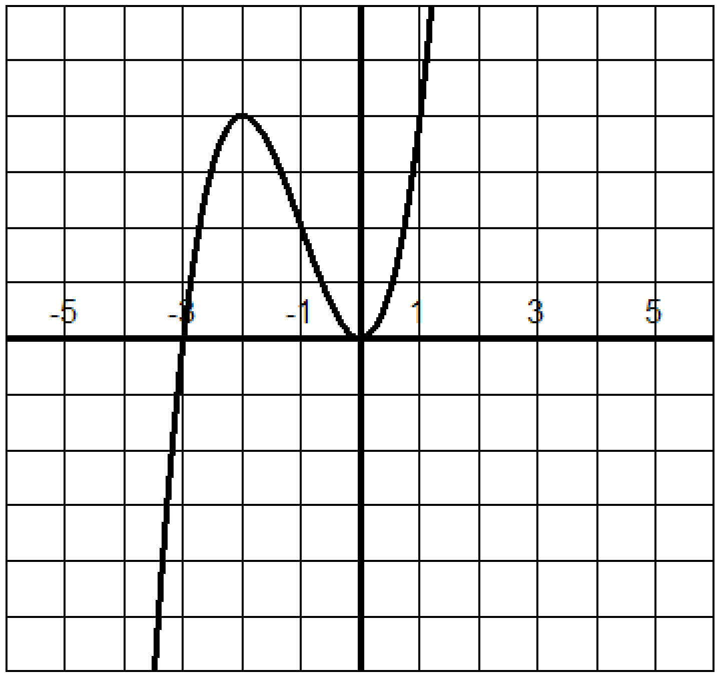
The graph has a relative maximum at (-2, 4) and a relative minimum at (0, 0).

Finding Minimums and Maximum Values of a Graph Using a Graphing Calculator

To find the highest or lowest point on a graph:

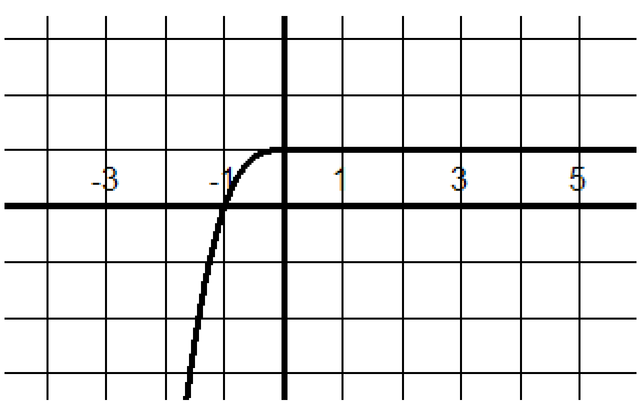
1. Have the graph in your viewing window.
2. Press [2nd] [TRACE] [maximum] (or [minimum]).
3. Use the left and right arrows to set the left boundary just to the left of the highest (or lowest) point on your graph. Press [ENTER].
4. Set your right boundary just to the right of the highest (or lowest) point. Press [ENTER]
5. When you see “Guess?”, press [ENTER] again to find the value.

Look at the graph again.



- The function is <u>increasing</u> on the interval $(-\infty,-2)$.
- The function is <u>decreasing</u> on the interval $(-2,0)$.
- The function is <u>increasing</u> on the interval $(0,\infty)$.

A function can also be <u>constant</u>.



- The function is <u>increasing</u> on the interval $(-\infty,0)$.
- The function is <u>constant</u> on the interval $(0,\infty)$.

Definitions

- A function f is <u>increasing</u> on an interval if, for any x_1 and x_2 in the interval such that $x_1 < x_2$ we have $f(x_1) < f(x_2)$.
- A function f is <u>decreasing</u> on an interval if, for any x_1 and x_2 in the interval such that $x_1 < x_2$ we have $f(x_1) > f(x_2)$.
- A function f is <u>constant</u> on an interval if, for any x_1 and x_2, we have $f(x_1) = f(x_2)$.

Note: We do not include the relative minimum or maximum in the interval notation for increasing, decreasing, and constant functions.

You can also find the relative minimums and maximums by looking at the x-y table for a function.

Example: Find the relative maximum of the function $f(x) = -3x^2 - 2x + 1$ using the Table feature on a graphing calculator.

Finding Minimums and Maximum Values of a Graph using the Table Feature

1. First graph the equation.
2. Look at the graph and pick an x-value to the left of the maximum (or minimum).
3. Press [TBLSET] ([2nd][WINDOW]). Use that x-value that you just picked as your TblStart number.
4. For the ∆Tbl use 0.1.
5. Press [TABLE] ([2nd] [GRAPH]).
6. Scroll up or down, watching how the y-values change. This is how you can determine the intervals in which the function is increasing or decreasing. If the y values are increasing as the x-values increase, then the function is increasing there. If the y values are decreasing as the x-values increase, then the function is decreasing there. Where they change from increasing to decreasing would be your relative maximum. Reverse that if you scroll up and make the x-values decrease.
7. To narrow down the value for the minimum or maximum, change the TblStart number to be the x-value from your table just before the function changes from increasing to decreasing (or decreasing to increasing, etc.).
8. For the ∆Tbl use 0.01 instead of 0.1.
9. Press [TABLE] ([2nd] [GRAPH]). Approximate the value of the min or max based on the new table.

Example: Find the relative maximum of the function $f(x) = -3x^2 - 2x + 1$ using the TABLE feature on a graphing calculator.

Solution: When using the instruction above, you find that the maximum is between -0.4 and -0.2. Change the TblStart to -0.4 and ΔTbl to 0.01. You will find that the max is between -0.34 and -0.32. Both numbers round to -0.3, so use that as your answer.

Even and Odd Functions

Definition:

Even functions are symmetric with respect to the y-axis.
Odd functions are symmetric with respect to the origin.

- For an even function, if the point (x, y) is on the graph, so is the point $(-x, y)$.
- For an odd function, if the point (x, y) is on the graph, so is the point $(-x, -y)$.

Determining Even and Odd Functions

To determine whether a function is even or odd, put $-x$ into the function (i.e. find $f(-x)$).

- If the resulting equation is equivalent to the original equation, then the function is even. That is,

$$f(-x) = f(x)$$

- If the resulting equation is the opposite of the original equation, then the function is odd. That is,

$$f(-x) = -f(x)$$

Example: Is the function $f(x) = x^4 - |x|$ even or odd?

Solution:

$$\begin{aligned} f(x) &= x^4 - |x| \\ f(-x) &= (-x)^4 - |-x| \\ &= x^4 - |x| \\ &= f(x) \end{aligned}$$

This function is even.

Example: Is the function $g(x) = \dfrac{x}{x^2 + 1}$ even or odd?

Solution:

$$g(x) = \frac{x}{x^2 + 1}$$

$$g(-x) = \frac{-x}{(-x)^2 + 1}$$

$$= \frac{-x}{x^2 + 1}$$

$$= -\frac{x}{x^2 + 1}$$

$$= -g(x)$$

The function is odd.

Example: Is the function $h(x) = x + 6$ even or odd?

Solution:

$$h(x) = x + 6$$

$$h(-x) = -x + 6$$

Since this does not give us $h(x)$ or $-h(x)$, the function is neither even nor odd.

Using a Graphing Calculator to Determine if a Function is Odd or Even.

1. Enter the equation for Y_1 in the [y=] screen.
2. Enter the equation $Y_2 = Y_1$ (-X). To enter Y_1, press [VARS] [Y-VARS] [function] [Y_1]. Then enter the –X in the parentheses. (Remember to us (-) for the negative and not the subtraction symbol.)
3. Enter the equation $Y_3 = - Y_1$ (X).

To determine if the function is even:

Graph Y_1 and Y_2 and see if they are the same. If so, the function is even. You can also compare the values using the [Table] feature instead to see if they are the same. This tests $f(-x) = f(x)$.

To determine if the function is odd:

Graph Y_2 and Y_3 and see if they are the same. If so, the function is odd. Again, you can also compare the values using the [Table] feature instead to see if they are the same. This tests $f(-x) = -f(x)$.

Note: To turn an equation off or on in the [y=] screen, highlight the = sign and press [ENTER].