Mathematical Modeling

Sample Problem: The chart below gives the profit for a company for the years 1990 to 1999, where 0 corresponds to 1990 and the profit is in millions of dollars.

Year	0	1	2	3	4	5	6	7	8	9
Profit	5.1	5.22	5.44	5.56	5.8	5.99	6.22	6.68	6.6	6.77

1. Make a scatter plot of the data:

[STAT] [Edit] – Make L1 correspond to the year and L2 correspond to the profit.

[STAT PLOT] [1] – Turn the statplot on.
Type: dotted
X-list: L1
Y-list: L2
Mark: +

[GRAPH] – To set the window, press [ZOOM] [ZoomStat]

2. Find the equation of the line and graph it.

[STAT] [CALC] [LinReg(ax+b)]
Now enter L1,L2,Y1 [ENTER]

> (After the 2nd comma we want to tell the calculator to store our equation in Y1 on our [Y=] screen. To do this, press [VARS] [Y-VARS] [Function] [Y1].)

You will see the numbers for your model equation.
Press [GRAPH] to see the graph of the equation.

**We can take sets of data and come up with an equation that is the best-fitting line for the data. Statisticians will find the difference between the y-values of the actual points and the y-values of the line of best fit. This is called the deviation. They will then square each of these differences and then add them all up. The smaller that sum is, the better the line fits the data.

**When you run a linear regression program, the "r-value" or correlation coefficient gives a measure of how well the model fits the data. The closer |r| is to 1, the better the fit.

Note: To turn this on, press [Catalog] [Diagnostic On] [ENTER].

Direct Variation

There are 2 basic types of linear models:

1. $y = mx + b$ where $b \neq 0$ (has a y-intercept other than 0)
2. $y = kx$ (y-intercept is the origin)

Definition: A direct variation can be defined by any of the following equivalent statements:

1. *y* **varies directly** as *x.*
2. *y* is **directly proportional** to *x.*
3. $y = kx$ for some nonzero constant *k*, called the **constant of variation** or the **constant of proportionality.**

*Note: When you read “varies directly as” or “is directly proportional to” you should translate it as “=k•”.

Example: *y* varies directly as *x. y* = 15 when *x* = 3. Find a mathematical model that gives *y* in terms of *x*.

$$y = kx$$
$$15 = k(3)$$
$$k = 5$$

So, our equation is $y = 5x$

Using the model equation, find y when x = 7.

$$y = 5x$$
$$y = 5(7) = 35$$

Example: If *y* varies directly as *x*, and *y* is 6 when *x* is 4, find the value of *y* when *x* is 20.

First, find the direct variation equation:
$y = kx$
$6 = k(4)$
$4k = 6$
$k = \frac{3}{2}$

So, $y = \frac{3}{2}x$
When $x = 20$ we get
$y = \frac{3}{2}(20) = 30$

Direct Variation as an *n*th Power

**Another type of direct variation relates one variable to the power of another.

Definition: A direct variation as an *n*th power can be defined by the following equivalent statements:

1. *y* **varies directly as the *n*th power** of *x.*
2. *y* is **directly proportional to the *n*th power** of *x.*
3. $y = kx^n$ for some nonzero constant

Example: The area of a circle is directly proportional to the square of its diameter. Find a mathematical model that gives the area of a circle in terms of its diameter if the area is 16π when the diameter is 8.

$$A = kd^2$$
$$16\pi = k \cdot 8^2$$
$$16\pi = 64k$$
$$k = \frac{\pi}{4}$$
$$A = \frac{\pi}{4} d^2$$

Example: If y is directly proportional to the third power of x, and y is 750 when x is 10, find the value of y when x is 8.

$$y = kx^3$$
$$750 = k \cdot 10^3$$
$$750 = 1000k$$
$$k = 0.75$$
$$y = 0.75\, x^3$$

For $x = 8$ we have

$$y = 0.75\,(8)^3$$
$$y = 384$$

<u>Inverse Variation</u>

Definition: An <u>inverse variation</u> can be defined by the following equivalent statements:

1. *y* **varies inversely as** *x.*
2. *y* is **inversely proportional as** *x.*
3. $y = \frac{k}{x}$ for some nonzero constant

<u>Note</u>: We can also have y vary inversely as the nth power of x. This would look like $y = \frac{k}{x^n}$.

Example: If *y* varies inversely as *x*, and *y* is 4 when *x* is 16, find the value of *y* when *x* is 10.

$$y = \frac{k}{x}$$

$$4 = \frac{k}{16}$$

$$k = 64$$

$$\text{So, } y = \frac{64}{x}$$

$$\text{For } x = 10,\ y = \frac{64}{10} = 6.4$$

Example: A gas law states that the volume of an enclosed gas varies directly as the temperature and inversely as the pressure. The pressure of a gas is 0.75 kg per square cm. When the temperature is 294 K the volume is 8000 cubic cm. Write an equation relating pressure, temperature, and volume.

$$V = \frac{kT}{p}$$

$$8000 = \frac{k(294)}{0.75}$$

$$k = \frac{8000(0.75)}{294} = \frac{1000}{49} \text{ or } 20.4$$

$$\text{So, } V = \frac{1000}{49}\left(\frac{T}{p}\right) \text{ or } V = \frac{20.4T}{p}$$

<u>Joint Variation</u>

**In our last example we had a direct variation and inverse variation occur in the same statement. They were coupled with the word “and.” To describe 2 different direct variations in the same statement we use the word “jointly.”

Definition: A joint variation can be defined by the following equivalent statements:

1. z **varies jointly** as x and y.
2. z is **jointly proportional to** x and y.
3. $z = kxy$ for some nonzero constant k.

Note: We can also have z vary jointly as the n^{th} power of x and the m^{th} power of y. This would look like:

$$z = kx^n y^m$$

Example: The volume of a right circular cylinder is jointly proportional to its height and to the square of its diameter. If the volume is 320π cm³ when the diameter is 16 cm and the height is 5 cm, what is the volume when the diameter is 10 cm and the height is 4 cm?

$V = kd^2h$

$320\pi = k \cdot 16^2 \cdot 5$

$320\pi = 1280k$

$k = \frac{\pi}{4}$

So, $V = \frac{\pi}{4}d^2h$ *and for h=4 and d=10 we get*

$V = \frac{\pi}{4}10^2 \cdot 4 = 100\pi \text{ cm}^3$