Exponential Functions and Their Graphs

*Polynomial functions and rational functions are <u>algebraic functions</u>.

*Exponential functions and logarithmic functions are non-algebraic functions called <u>transcendental functions</u>.

<u>Definition</u> - The <u>exponential function *f* with base *a*</u> is denoted by $f(x) = a^x$, where $a > 0$, $a \neq 1$, and x is any real number.

*The reason $a \neq 1$ is because 1 raised to any power is 1, so we would have f(x) = 1, which is a horizontal line.

Look at $f(x) = 2^x$. Find:

a) f(4) answer: $2^4 = 16$

b) f(-3) answer: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

c) $f(\frac{5}{3})$ answer: $2^{\frac{5}{3}} = \sqrt[3]{2^5} = \sqrt[3]{32} \approx 3.17$

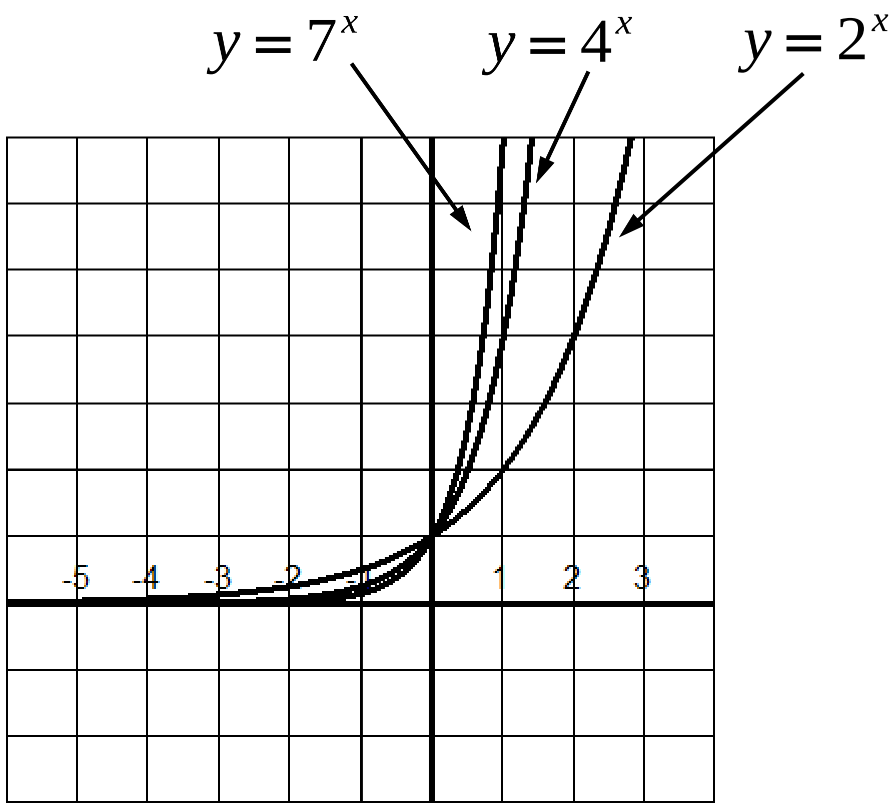
or answer: 2^(5/3) ≈3.17

d) $f(\sqrt{3})$ answer: $2^{\wedge}\sqrt{3} \approx 3.32$

On your calculator, graph: $y = 2^x$

$y = 4^x$

$y = 7^x$

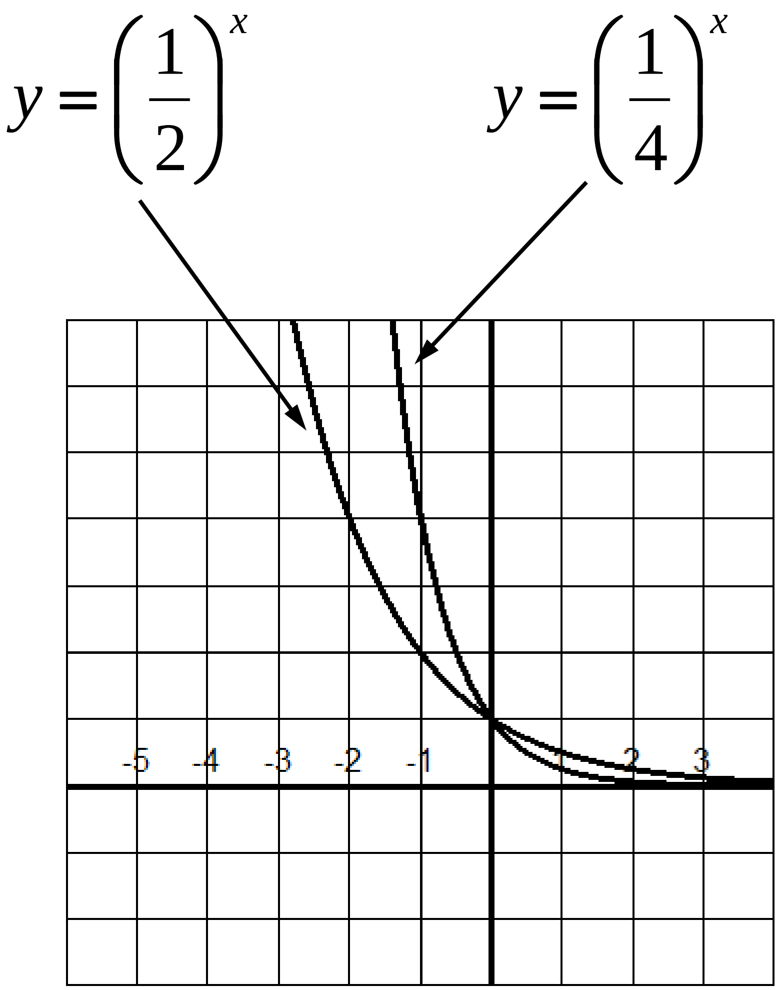


<u>Similarities</u>:

- all go through (0, 1) since $2^0 = 4^0 = 7^0 = 1$
- all are increasing
- the larger the base, the more rapidly the graph rises.

On your calculator, graph: $y = \left(\frac{1}{2}\right)^x$

$y = \left(\frac{1}{4}\right)^x$



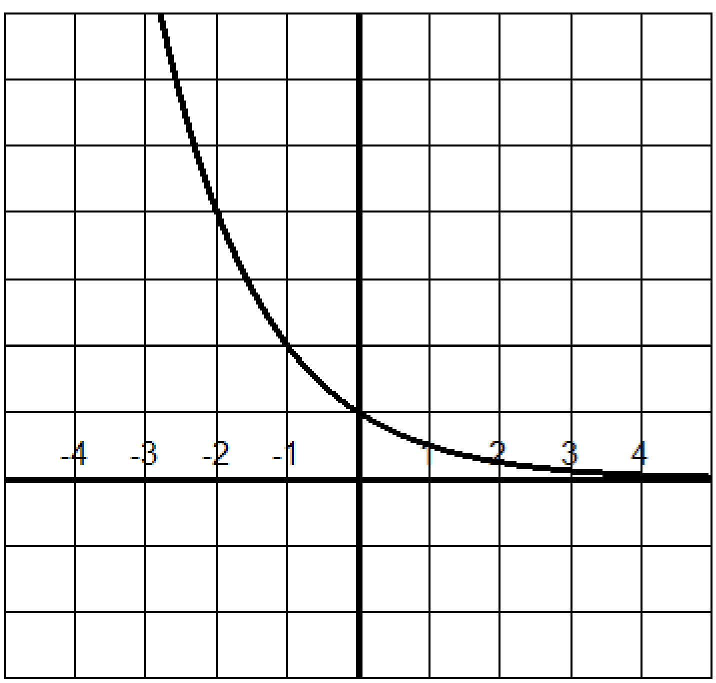
These graphs have the same basic shape but are <u>decreasing</u>.

In general, for the equation $f(x) = a^x$.

1. The domain is $(-\infty,\infty)$.
2. The range is $(0, \infty)$.
3. The y-intercept is $(0, 1)$.
4. $y = 0$ is a horizontal asymptote.
5. f is increasing if $a > 1$.
6. f is decreasing if $0 < a < 1$.
7. f is continuous.

On your calculator, graph: $y = 2^{-x}$

$$y = \left(\frac{1}{2}\right)^x$$



Both of these equations give us the same graph because

$$2^{-x} = \frac{1}{2^x} = \left(\frac{1}{2}\right)^x$$

Transformations of Graphs of Exponential Functions

Look at $f(x) = 3^x$

What will the transformation be to get $g(x) = 3^{x+1}$?

shift $f(x) = 3^x$ *1 unit to the left, since* $g(x) = f(x+1)$.

What will the transformation be to get $h(x) = 3^x - 2$?

shift $f(x) = 3^x$ *2 units down, since* $h(x) = f(x) - 2$.

What will the transformation be to get $k(x) = -3^x$?

reflect $f(x) = 3^x$ *over the x-axis, since* $k(x) = -f(x)$.

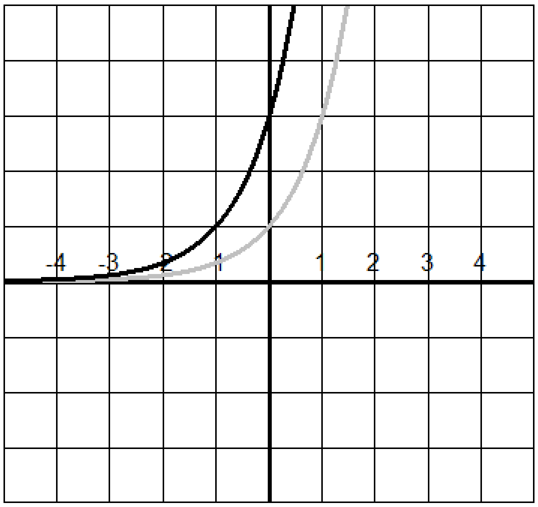
What will the transformation be to get $j(x) = 3^{-x}$?

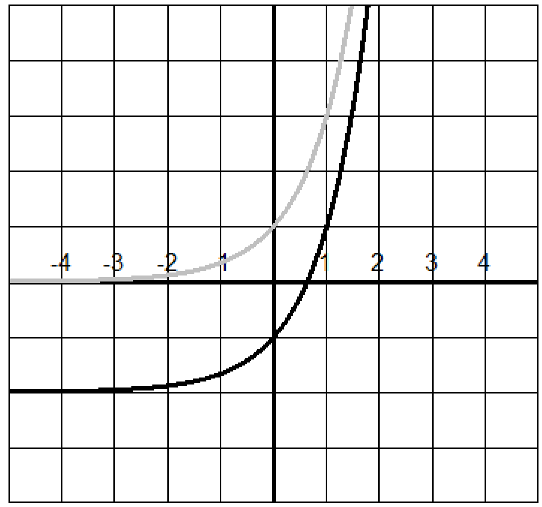
reflect $f(x) = 3^x$ *over the y-axis, since* $j(x) = f(-x)$.

*Note: A vertical shift will shift the horizontal asymptote as well.

$g(x) = 3^{x+1}$

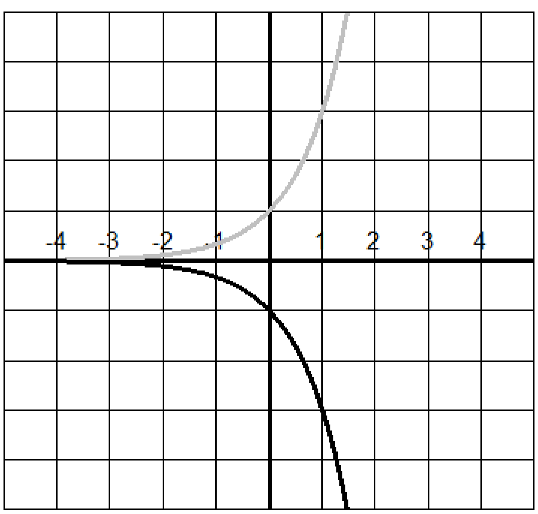
$h(x) = 3^{x} - 2$

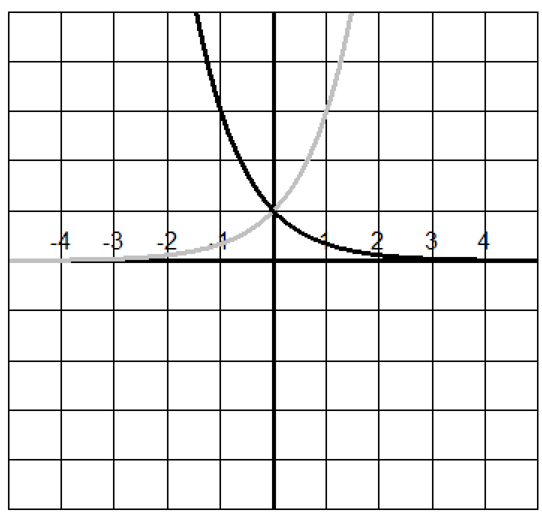




$k(x) = -3^{x}$

$j(x) = 3^{-x}$



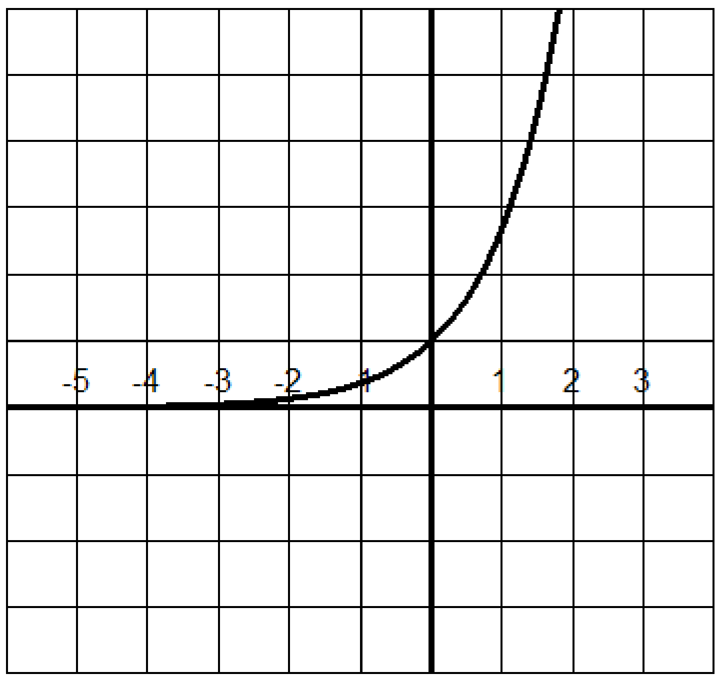


The Natural Base e

$$e \approx 2.718281828\ldots$$

This number is called the natural base.

The function $f(x) = e^{x}$ is called the natural exponential function.



The graph of $f(x) = e^{x}$ looks like our other exponential functions because e is a constant ($e \approx 2.72$).

Examples: Evaluate $f(x) = e^{x}$ for the following:

a) f(-3) answer: [2nd] [e^{x}] [-3] [ENTER]≈0.498

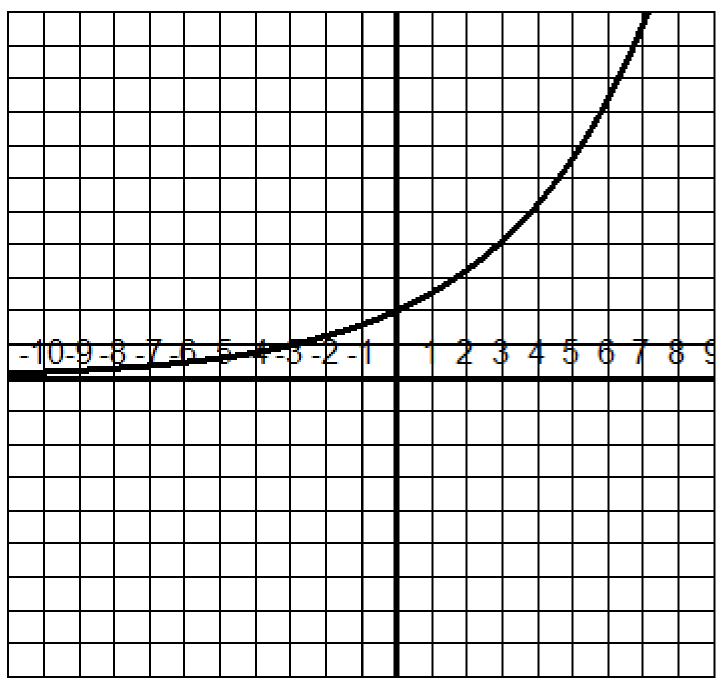
b) f(2) answer: ≈ 7.389

c) f(-0.3) answer: ≈ 0.7408

Example: Sketch $f(x) = 2e^{0.24x}$

Use the TABLE function to get a few points.

x	f(x)
-4	0.8
-2	1.2
0	2
2	3.2
4	5.2



Applications

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

This formula is used when figuring compound interest.

A = the amount in the account after t years
P = the principal invested
r = the annual interest rate as a decimal
t = the number of years the money is invested for
n = the number of times per year that the interest is compounded

Example: The amount invested in a bank account is \$4000. Calculate the amount that will be in an account after 10 years if the interest rate is 6% and the interest is compounded quarterly.

$$A = 4000\left(1 + \frac{.06}{4}\right)^{4(10)}$$

A = \$7256.07

Example: The amount invested in a bank account is \$4000. Calculate the amount that will be in an account after 10 years if the interest rate is 6% and the interest is compounded monthly.

$$A = 4000\left(1 + \frac{.06}{12}\right)^{12(10)}$$

$$A = \$7277.59$$

<u>Continuously Compounding Interest</u>

The formula for finding the amount in an account if the interest is compounded continuously is:

$$A = Pe^{rt}$$

Example: The amount invested in a bank account is \$4000. Calculate the amount that will be in an account after 10 years if the interest rate is 6% and the interest is compounded continuously.

$$A = 4000e^{.06(10)}$$

$$A = \$7288.48$$

**Continuous compounding will always yield a larger balance than compounding n times per year.

Radioactive Decay

Let Q represent a mass of radium (^{226}Ra), whose half-life is 1620 years. This means that after 1620 years, a given amount of radium will be reduced to half of the original amount. The quantity of radium present after t years is given by

$$Q = 16\left(\frac{1}{2}\right)^{t/1620}$$

(a) Determine the initial quantity (when $t = 0$).

$$Q = 16\left(\frac{1}{2}\right)^{0/1620} = 16\left(\frac{1}{2}\right)^{0} = 16(1) = 16 units$$

(b) Determine the quantity present after 1000 years.

$$Q = 16\left(\frac{1}{2}\right)^{1000/1620} \approx 10.43 units$$

(c) Use a graphing utility to graph the function over the interval $t = 0$ to $t = 5000$.