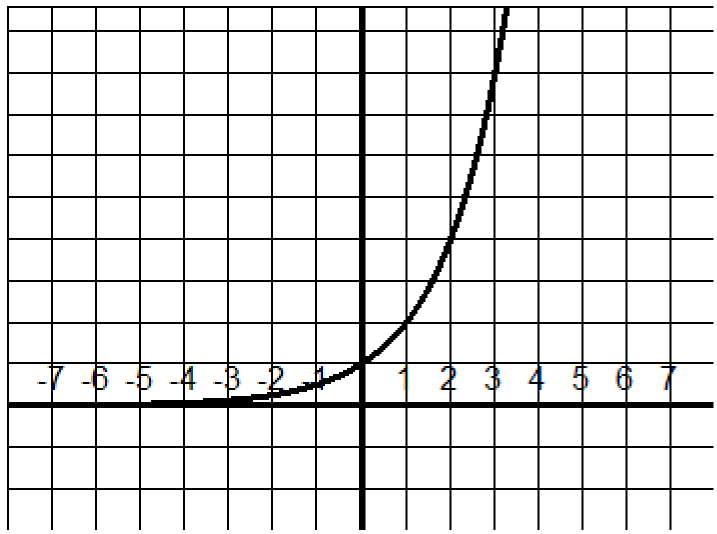
Logarithmic Functions and Their Graphs

Look at the graph of $f(x) = 2^x$



Does this graph pass the Horizontal Line Test?

yes

What does this mean?

that its inverse is a function

Find the inverse of $y = a^x$. (switch x and y and solve for y)

$y = a^x$
$x = a^y$

*We don't know how to solve for y!!!

Definition: The function given by $f(x) = \log_a x$, where $x > 0$, $a > 0$, and $a \neq 1$, is called the logarithmic function with base *a.* It is the inverse of the exponential function $f(x) = a^x$.

*Thus, $y = \log_a x$ is equivalent $x = a^y$.

(They both name the inverse of $y = a^x$.)

Working definition: The log of a number is the exponent that you put on the base to get that number. *(Memorize this!)*

**Remember: The logarithm is an exponent.

Examples: Solve.

(a) $\log_2 32 = y$

$$2^y = 32$$
$$y = 5$$

(b) $y = \log_3 1$

$$3^y = 1$$
$$y = 0$$

(c) $y = \log_{10} \frac{1}{100}$

$$10^y = \frac{1}{100} \quad \rightarrow \quad 10^y = 10^{-2} \quad \rightarrow \quad y = -2$$

(d) $y = \log_4 2$

$4^y = 2$

$(2^2)^y = 2$

$2^{2y} = 2$

$2y = 1$

$y = \frac{1}{2}$

(e) Evaluate $f(x) = \log_2 x$ for x=8

$f(8) = \log_2 8$

$y = \log_2 8$

$2^y = 8$

$y = 3$

(f) Evaluate $\log_2 0.25$

$2^y = 0.25$

$2^y = \frac{1}{4}$

$2^y = \frac{1}{2^2}$

$2^y = 2^{-2}$

$y = -2$

(g) Evaluate $\log_3 81$

$3^y = 81$

$3^y = 3^4$

$y = 4$

**To find the log of a number, write it in exponential form, get the bases the same, and the set the exponents equal and solve.

Common Logarithms

Because we are working in a base 10 number system, we call the logarithmic function with base 10 the common logarithmic function. This is the function that corresponds to the LOG button on our calculators. The common logarithmic function is one function for which we need not write the base.

Examples: Find the following:

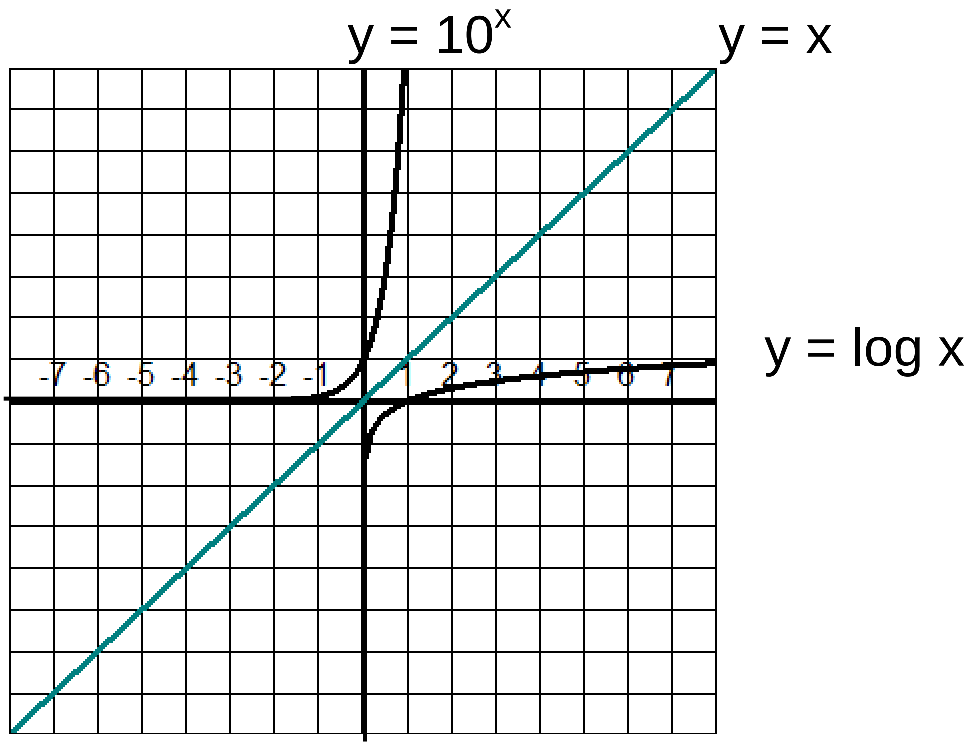
(a) $\log 10$ (b) $\log \frac{1}{4}$ (c) $\log 3.5$ (d) $\log(-2)$

1 *≈-0.602* *≈0.544* *Error “ NONREAL ANS”*

Note: $10^y = -2$ will **never** happen.

Graphing Logarithmic Functions

On your calculator, graph $y = 10^x$
$y = \log x$
$y = x$



*Notice that for y = log x, we don't use any x-values to the left of zero. That is why we could not put in -2 for x in the example, because -2 is not in the domain of y = log x.

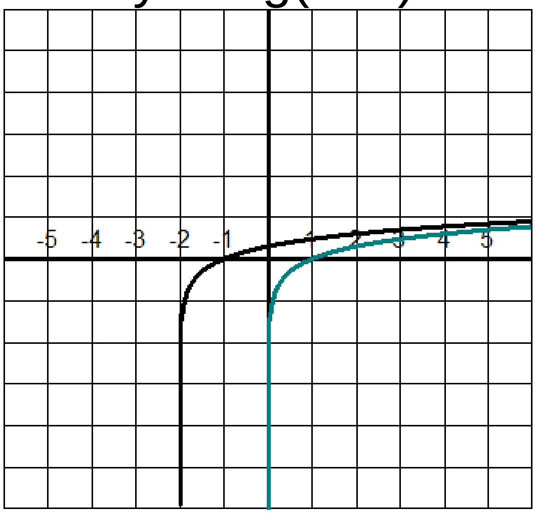
Basic Characteristics of Logarithmic Graphs $f(x) = \log_a x$

1. The domain is $(0, \infty)$.
2. The range is $(-\infty,\infty)$.
3. The *x*-intercept is (1, 0).
4. The *y*-axis is a vertical asymptote.
5. The function is increasing ($a > 0$).
6. The function is continuous.
7. The function passes the Horizontal Line test (ie. it is one-to-one) so it has an inverse function.
8. The function is a reflection of $y = a^x$ over the line y=x.

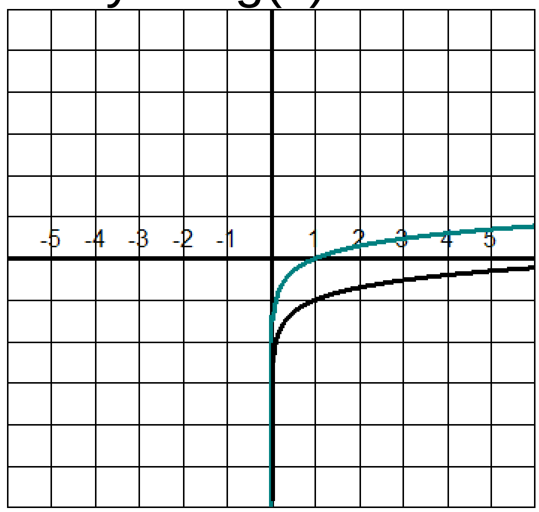
Rigid Transformations

Graph the following: $y = \log x$
$y = \log(x+2)$
$y = \log(x) - 1$

$y = \log(x+2)$



$y = \log(x) - 1$



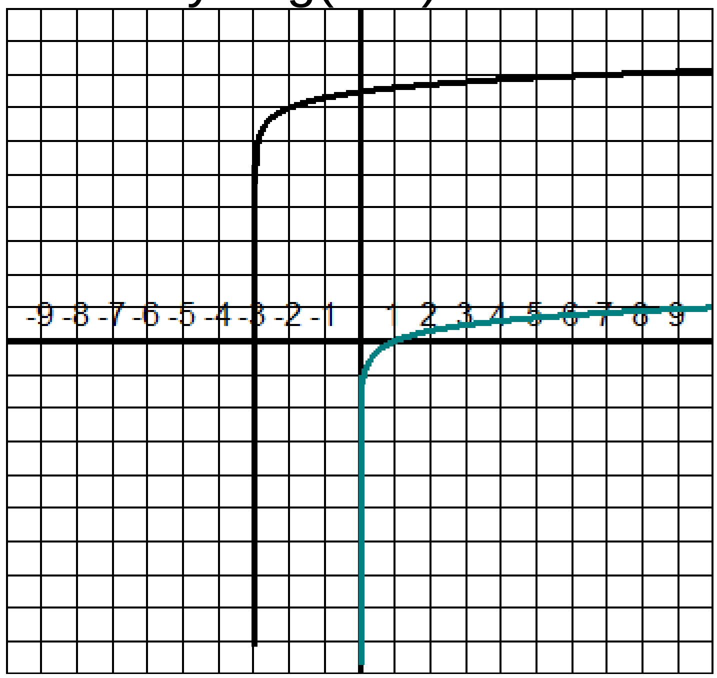
What kind of transformations do we have?

$y = \log(x+2)$ is $y=\log x$ shifted 2 units to the left.
$y = \log(x) - 1$ is $y = \log x$ shifted 1 unit down.

What would $y=\log(x+3) + 7$ look like?

$y=\log(x+3) + 7$ would be $y = \log x$ shifted 3 units left and 7 units up.

$y=\log(x+3) + 7$



Properties of Logarithms

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$ and $a^{\log_a x} = x$
4. If $\log_a x = \log_a y$, then $x = y$.

Examples: Solve the following equations:

(a) $\log_5 x = \log_5 8$

$x = 8$

(b) $\log_5 1 = x$

$5^x = 1$

$x = 0$

c) $\log_7 x = 1$

$7^1 = x$
$x = 7$

Examples: Simplify the following:

(a) $\log_6 6^x$

$6^n = 6^x$
$n = x$

(b) $5^{\log_5 20}$

$5^{\log_5 20} = 20$

<u>The Natural Logarithmic Function</u>

Remember $f(x) = e^{x}$ (where $e \approx 2.72$)

The inverse would be $f(x) = \log_e x$.

Since this is used a great deal, we have a notation (and button on our calculator dedicated to it.

Definition: The <u>logarithmic function with base e</u> is denoted

$$f(x) = \ln x$$

*Remember that ln x is just $\log_e x$.

Properties of Natural Logarithms

5. $\ln 1 = 0$
6. $\ln e = 1$
7. $\ln e^x = x$ and $e^{\ln x} = x$
8. If $\ln x = \ln y$, then $x = y$.

Examples: Evaluate the following: e

(a) $\ln e^5$ **(b)** $e^{\ln 3}$ **(c)** $\ln \frac{1}{e^2}$

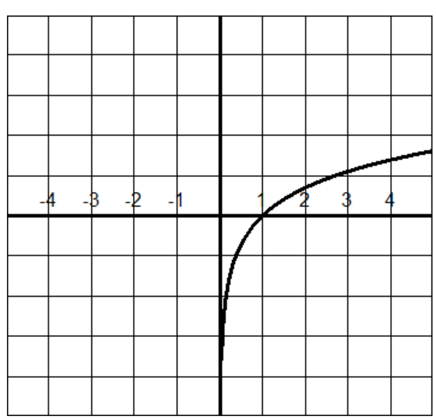
5 *3* $\ln e^{-2} = -2$

Examples: Use your calculator to find the following.

(a) $\ln 3$ (b) $\ln 0.2$ (c) $\ln(-1)$ (d) $\ln(1+\sqrt{5})$

≈ 1.099 ≈ -1.609 *Error* ≈ 1.174

*Look at the graph of $y = \ln x$. Just as with $y = \log_x a$, we cannot take the log of a negative number.



y = ln x

<u>Finding the Domain of the Logarithmic Function</u>

The domain of f(x) = log x and f(x) = ln x is (0, ∞)

(a) What is the domain of f(x) = log(x+2) ?

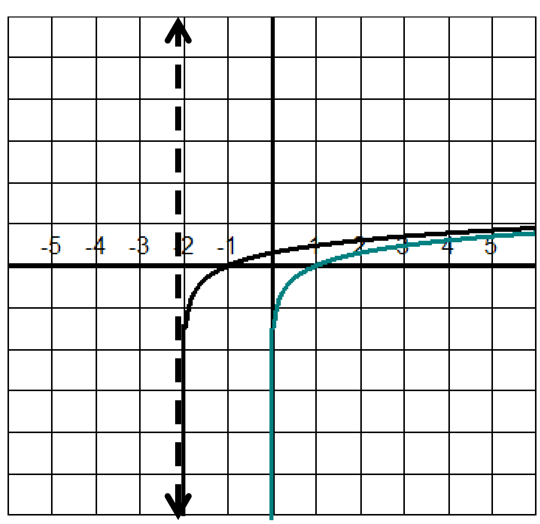
We must have x+2 > 0 since what we take the log of must <u>not</u> be negative.

Solving x+2 > 0 we get x > -2, so the domain is (-2, ∞).

*<u>Note</u>: This makes sense, because we know that
f(x) = log (x+2) is f(x) = log x shifted 2 units left.

The vertical asymptote of f(x) = log x is the y-axis (x=0).
The vertical asymptote of f(x) = log(x+2) is x= -2.

y = log(x+2)



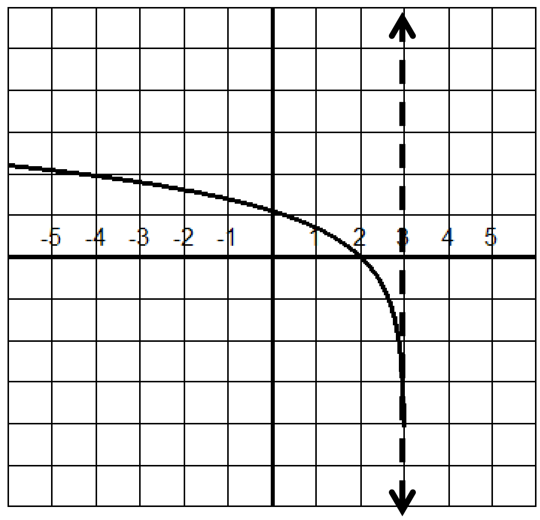
(b) What is the domain of ln(3 - x) ?

We must have $3 - x > 0$

$-x > -3$

$x < 3$*, so the domain is* $(-\infty, 3)$

ln(3 – x)



(c) What is the domain of ln x^2 ?

$x^2 > 0$*, which means*

$x > 0$ *or* $x < 0$ *(ie. x can be anything but 0)*

So the domain is $(-\infty, 0) \cup (0, \infty)$

ln x^2