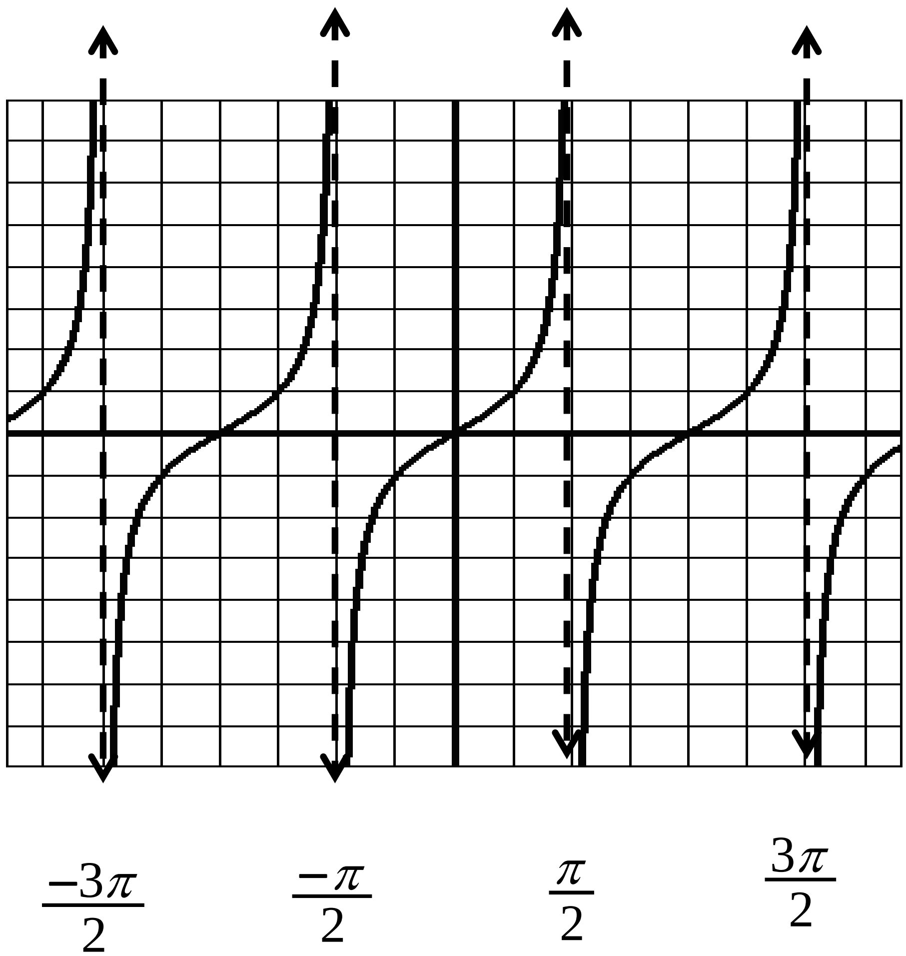
Graphs of Other Trig Functions

Graph $y = \tan x$.

x	y
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx .58$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.7$
$\frac{\pi}{2}$	undefined
$\frac{2\pi}{3}$	$-\sqrt{3} \approx -1.7$
$\frac{3\pi}{4}$	-1
$\frac{5\pi}{6}$	$\frac{-\sqrt{3}}{3} \approx -.58$
π	0

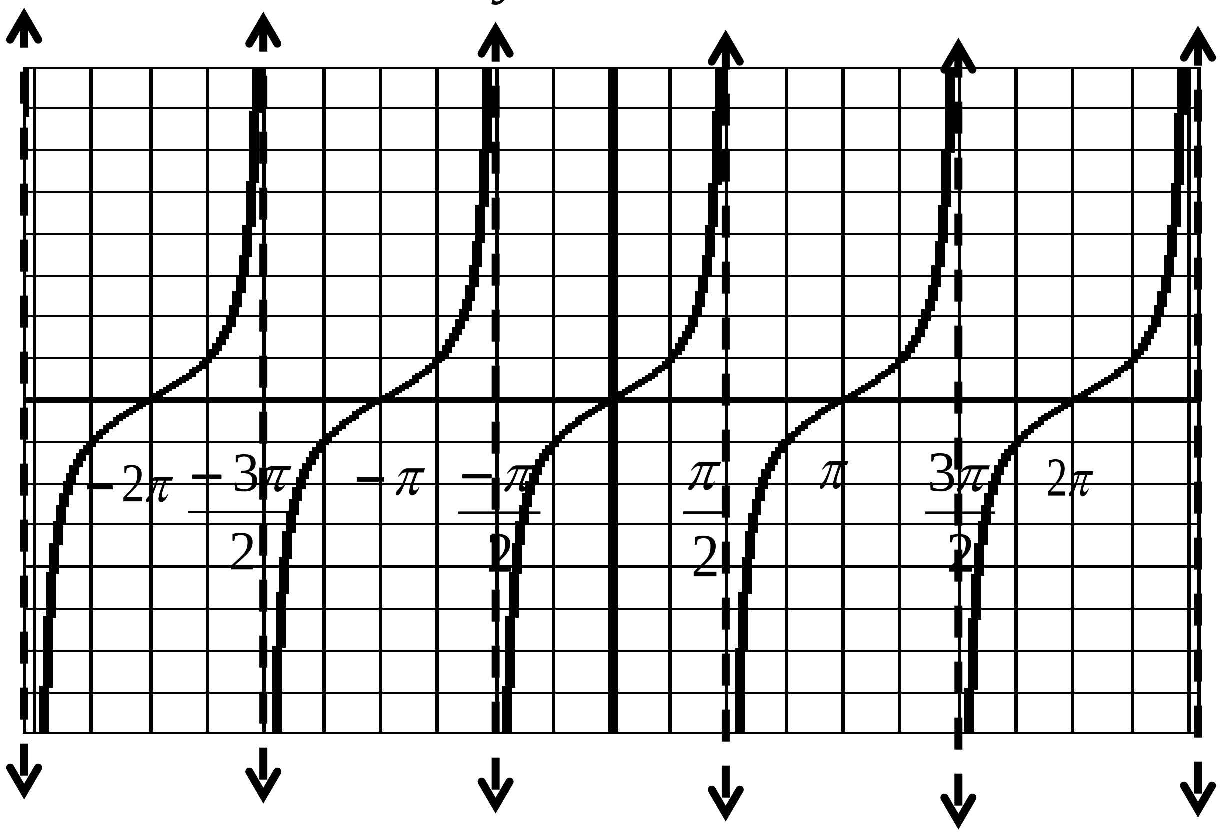


The <u>Domain</u> is all real numbers except multiples of $\frac{\pi}{2}$.

(We say the domain is all x ≠ $\frac{\pi}{2}$ + nπ)

The <u>Range</u> is the set of all real numbers.

$y = \tan x$



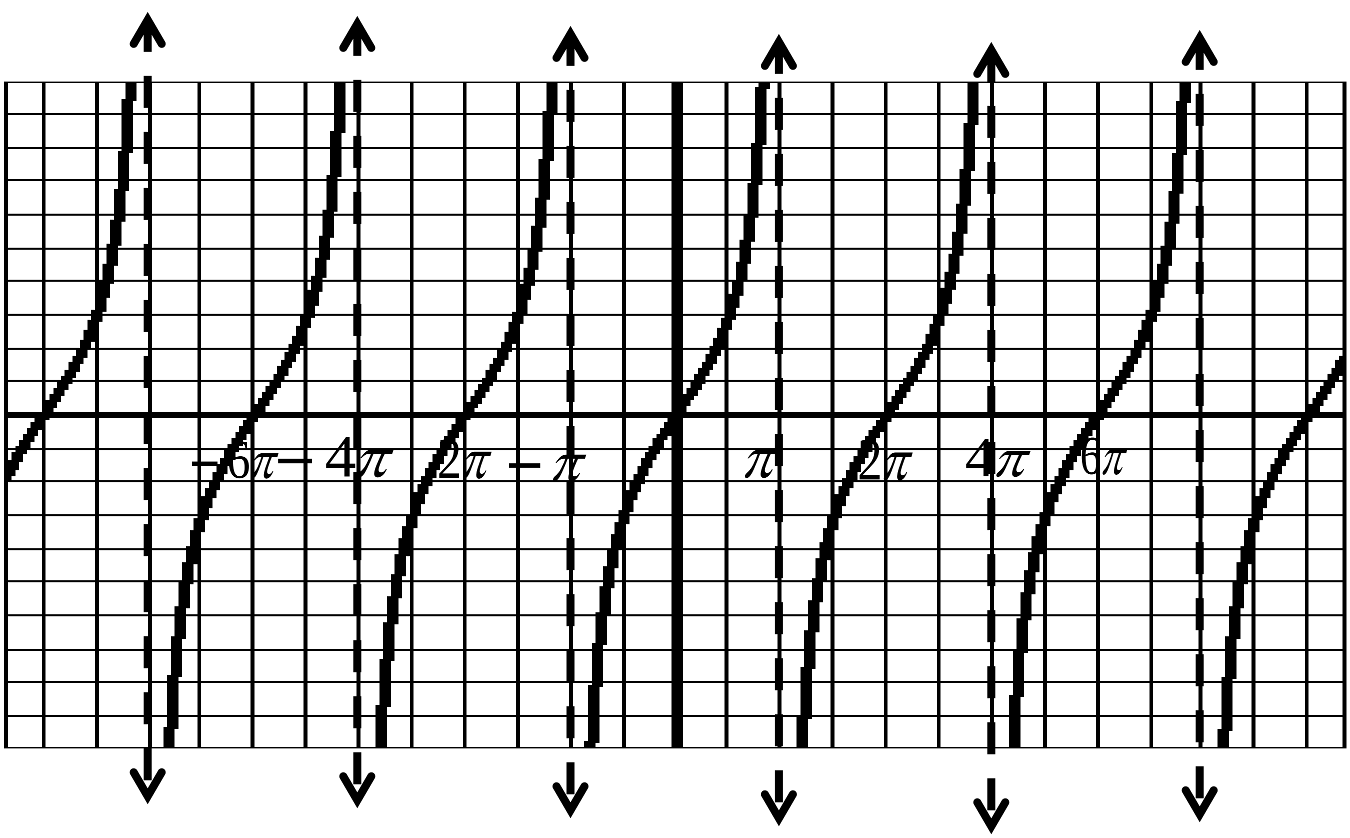
- The period for tangent is π.
- One cycle is $\frac{-\pi}{2}$ < x < $\frac{\pi}{2}$. (Note that it's not $\leq$)
- There is no amplitude.
- One cycle goes from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.
- There is a vertical asymptote at $x = \frac{\pi}{2} + n\pi$
  (at every x-value for which the tangent is undefined.)
- The Domain is all $x \neq \frac{\pi}{2} + n\pi$
- The Range is all real numbers.

Example : Graph $y = 3\tan\frac{x}{2}$

The period goes from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$ so we have:

$$\frac{-\pi}{2} < \frac{x}{2} < \frac{\pi}{2}$$
$$-\pi < x < \pi$$

One period will go from $-\pi$ to π, with asymptotes at $-\pi$ and π. There will be an x-intercept at 0 (half-way between $-\pi$ and π.)



With tangent, we need to put in at least one point between the x-intercept and asymptote to see how quickly the curve rises or falls.

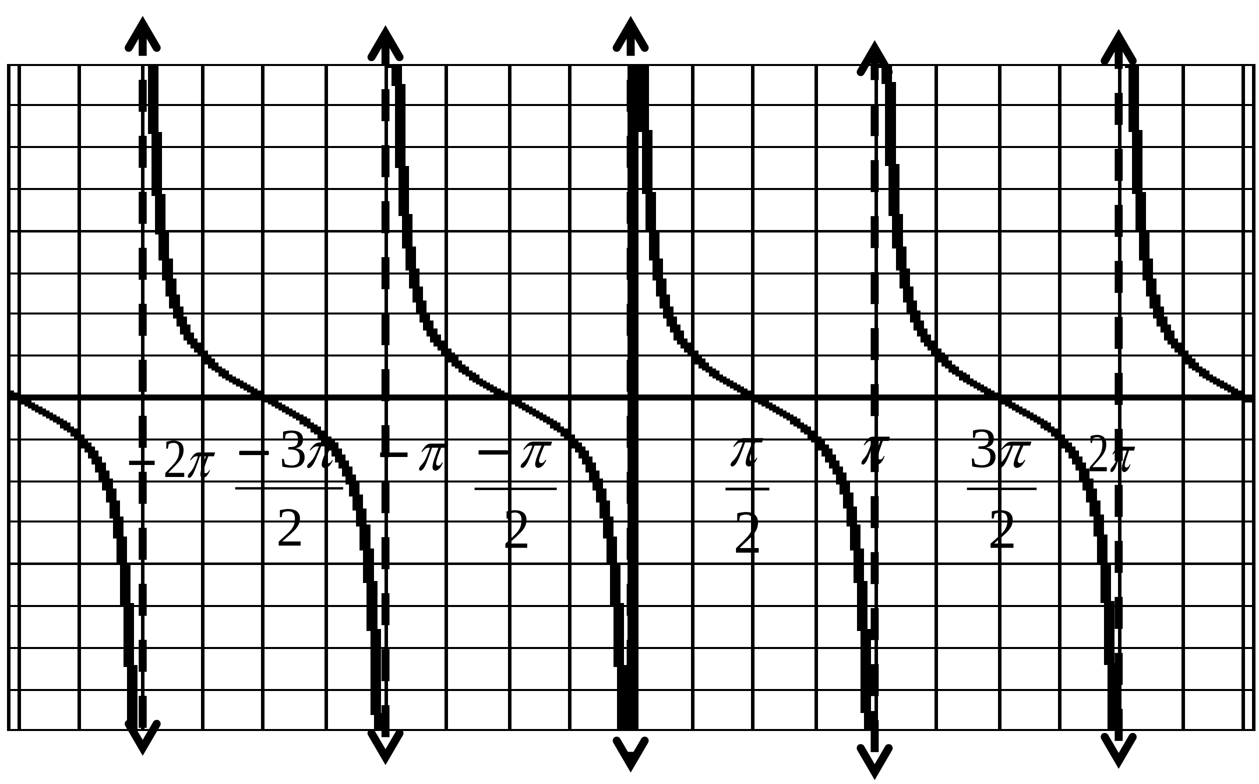
When x = $\frac{\pi}{2}$, y = 3, so we have the point ($\frac{\pi}{2}$, 3).

Graph of Cotangent

$$\cot x = \frac{\cos x}{\sin x} \qquad \text{or} \qquad \cot x = \frac{1}{\tan x}$$

Cotangent will be undefined when the sin x is 0, and this is where the asymptotes will be. Also, since it is the reciprocal function of tangent, every y coordinate in our table for tangent will become the reciprocal of that number for cotangent.

Example: If the point (π, 4/7) is on a tangent graph, then (π, 7/4) will be on the cotangent graph. If the point (3π, 5) is on the tangent graph, then (3π, 1/5) will be on the cotangent graph.



For $y = \cot x$

- The period for cotangent is π.
- One cycle is $0 < x < \pi$. (Note that it's not #)
- There is no amplitude.
- One cycle goes from 0 to π.
- There is a vertical asymptote at $x = n\pi$
  (at every x-value for which the cotangent is undefined.)
- The Domain is all $x \neq n\pi$
- The Range is all real numbers.

Graphs of Secant and Cosecant

Sine and Cosecant functions are reciprocals.
Cosine and Secant functions are reciprocals.

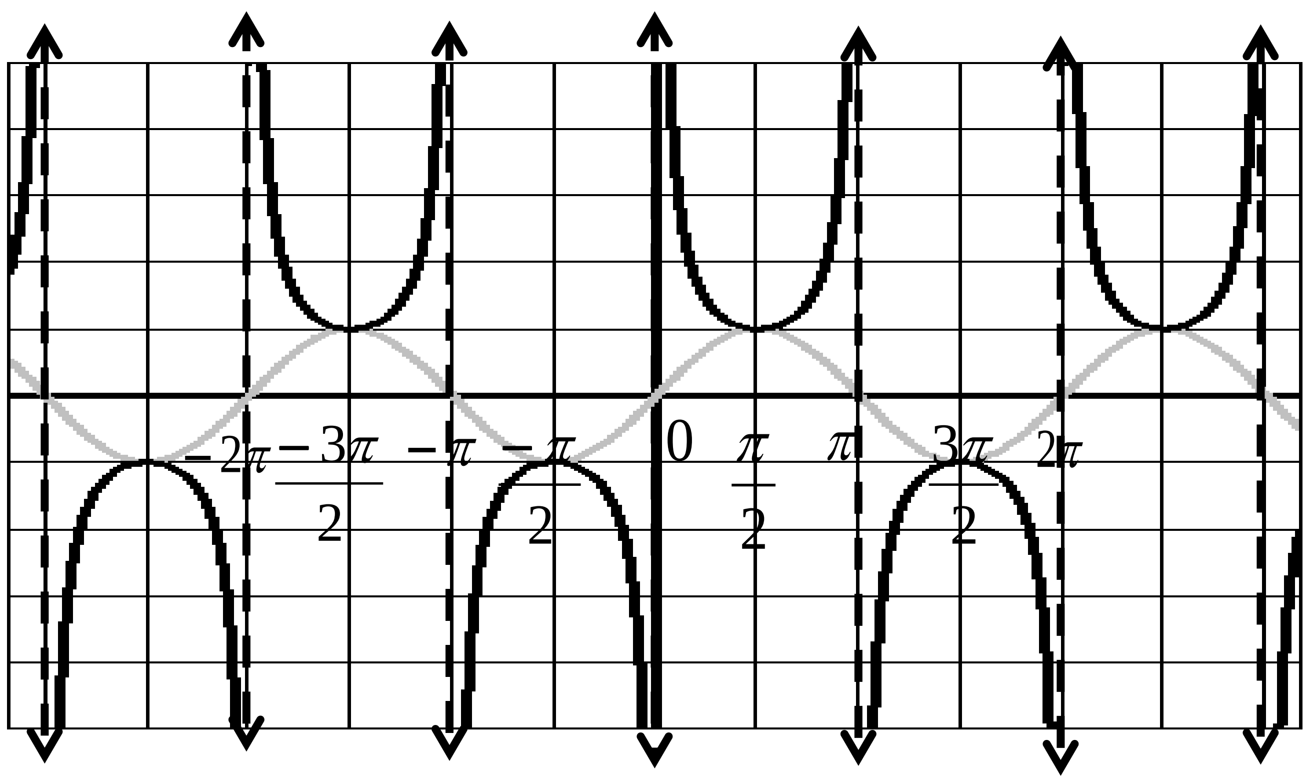
This means:

- Where one function is zero, its reciprocal function has a vertical asymptote.
- Where one function has a relative maximum, its reciprocal function has a relative minimum.
- Every y-coordinate for one function will become its reciprocal for the reciprocal function.

**When graphing the secant or cosecant function, we first sketch its reciprocal function. Then take the reciprocals of the y-coordinates to obtain points on the graph.

Graph $y = \csc x$

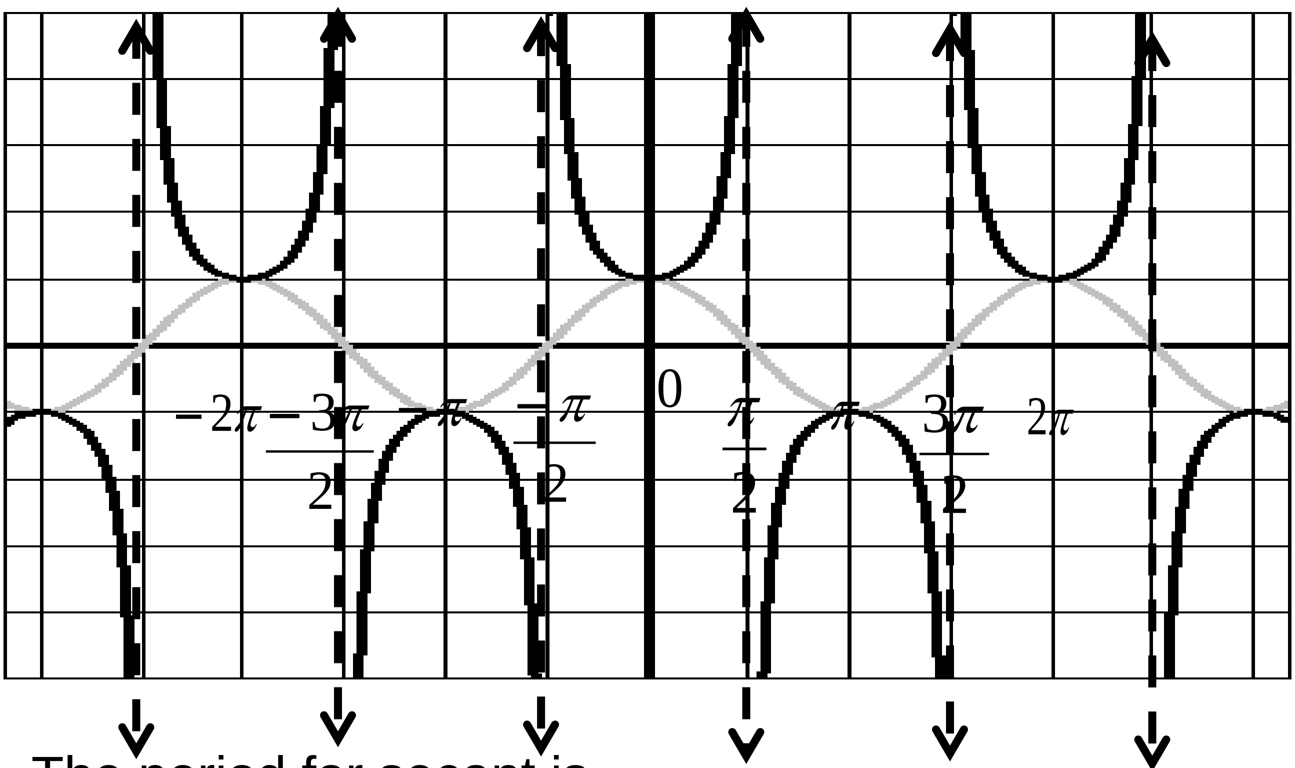
- First graph $y = \sin x$.
- Draw in asymptotes where the sin $x = 0$.
- Then take the reciprocals of the y-coordinates to obtain some points.



- The period for cosecant is π.
- One cycle is $0 < x < 2\pi$.
- There is no amplitude.
- One cycle goes from 0 to 2π.
- There is a vertical asymptote at $x = n\pi$
  (at every x-value for which the sine is zero)
- The Domain is all $x \neq n\pi$
- The Range is all real numbers.

Graph $y = \sec x$

- First graph $y = \cos x$.
- Draw in asypmtotes where the cos x = 0.
- Then take the reciprocals of the y-coordinates to obtain some points.



- The period for secant is π.
- One cycle is $0 < x < 2\pi$.
- There is no amplitude.
- One cycle goes from 0 to 2π.
- There is a vertical asymptote at $x = \frac{\pi}{2} + n\pi$
  (at every x-value for which the cosine is zero)
- The Domain is all $x \neq \frac{\pi}{2} + n\pi$
- The Range is all real numbers.

Example : Graph $y = -2\cot 2x$

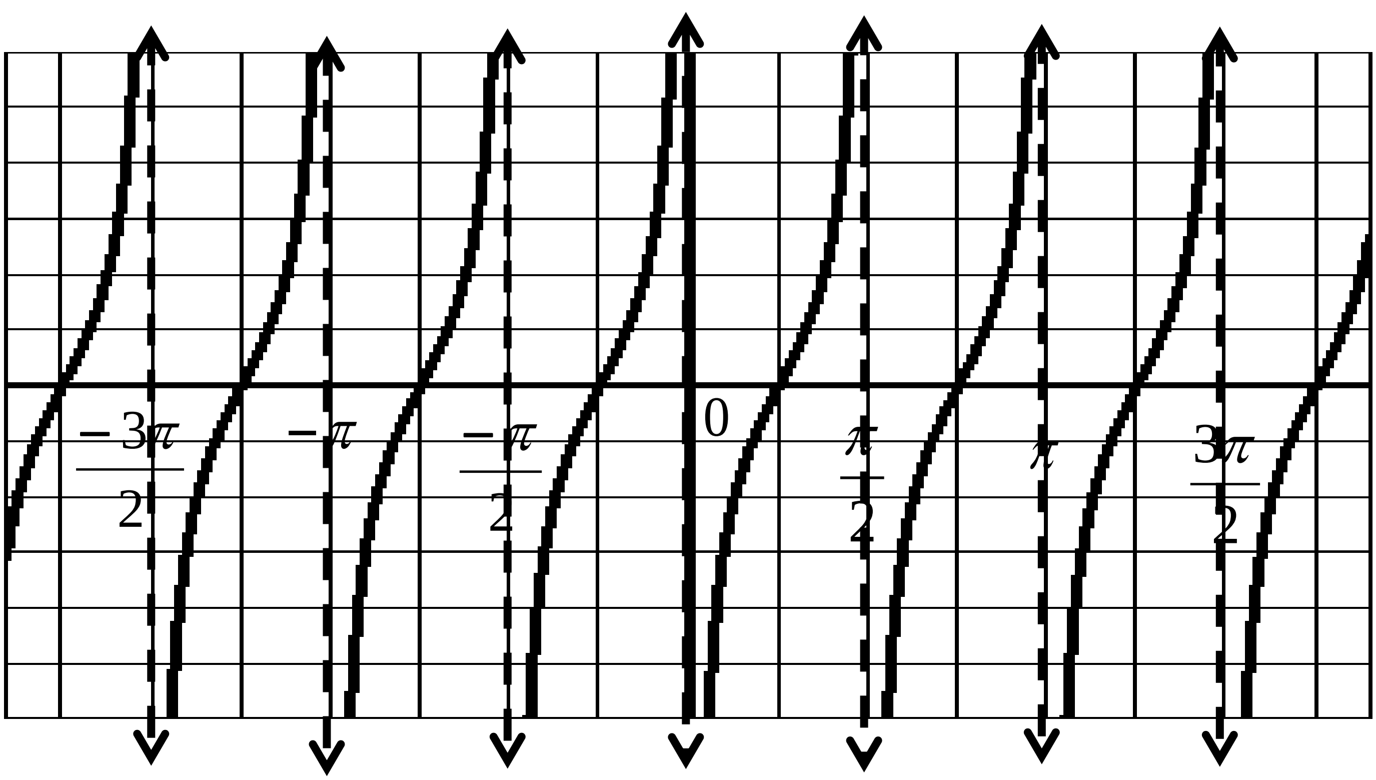
One cycle for cotangent is $0 < x < \pi$, so

$$0 < 2x < \pi$$

$$0 < x < \frac{\pi}{2}$$ This is one cycle of our graph.

There is an asymptote at every $\frac{\pi}{2}$ units starting at 0.
The graph is "up-side-down" from the normal cotangent graph.

x	$\frac{\pi}{4}$	$\frac{\pi}{8}$	$\frac{3\pi}{8}$
y	0	-2	2

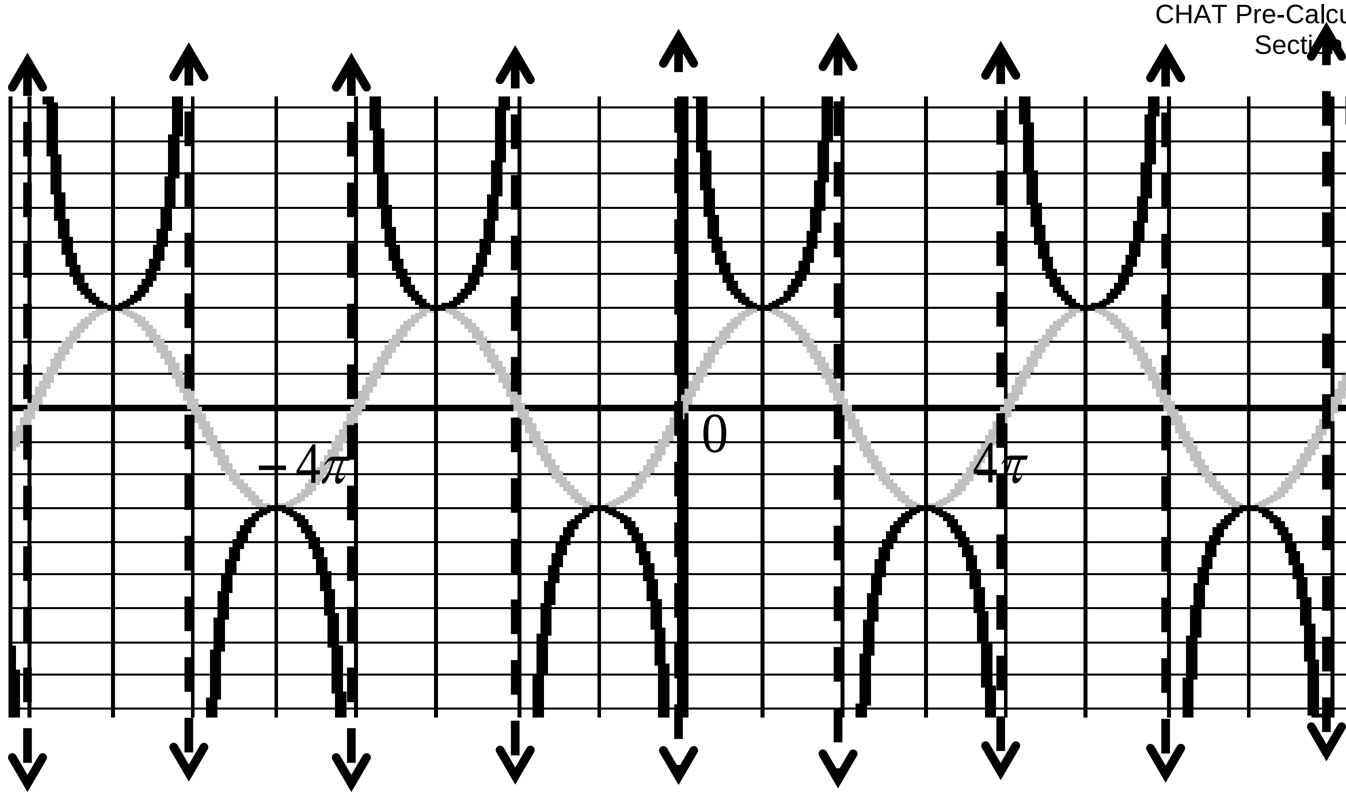


Example : Graph $y = 3\csc\frac{x}{2}$

- $y = 3\left(\csc\frac{x}{2}\right)$ is the same as $y = 3\left(\frac{1}{\sin\frac{x}{2}}\right)$, so graph $y = 3\sin\frac{x}{2}$ first.
- amplitude = 3
- period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$
- One cycle for sine is $0 \le x \le 2\pi$, so we have
- $0 \le \frac{x}{2} \le 2\pi$

  $0 \le x \le 4\pi$ This is one cycle of our graph.
- Where the sine curve equals zero (crosses the x-axis), our cosecant curve has asymptotes.
- Put a couple of values into the table.

x	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$3\sin\frac{x}{2}$	$3\left(\frac{\sqrt{2}}{2}\right) \approx 2.1$	3(1)=3	$3\left(\frac{\sqrt{2}}{2}\right) \approx 2.1$
$3\csc\frac{x}{2}$	$3\left(\frac{2}{\sqrt{2}}\right) \approx .4.2$	3(1)=3	$3\left(\frac{2}{\sqrt{2}}\right) \approx .4.2$



Example : Graph $y = -2\sec(4x + \pi) + 2$

- $y = -2\sec(4x + \pi) + 2$ is the same as $y = -2\left(\frac{1}{\cos(4x + \pi)}\right) + 2.$

Graph $y = -2\cos(4x + \pi) + 2$ first.

- amplitude = 2
- Graph reflected over x-axis (ie. up-side-down)
- period = $\frac{2\pi}{4} = \frac{\pi}{2}$
- One cycle for cosine is $0 \leq x \leq 2\pi$, so we have

$$0 \leq 4x + \pi \leq 2\pi$$

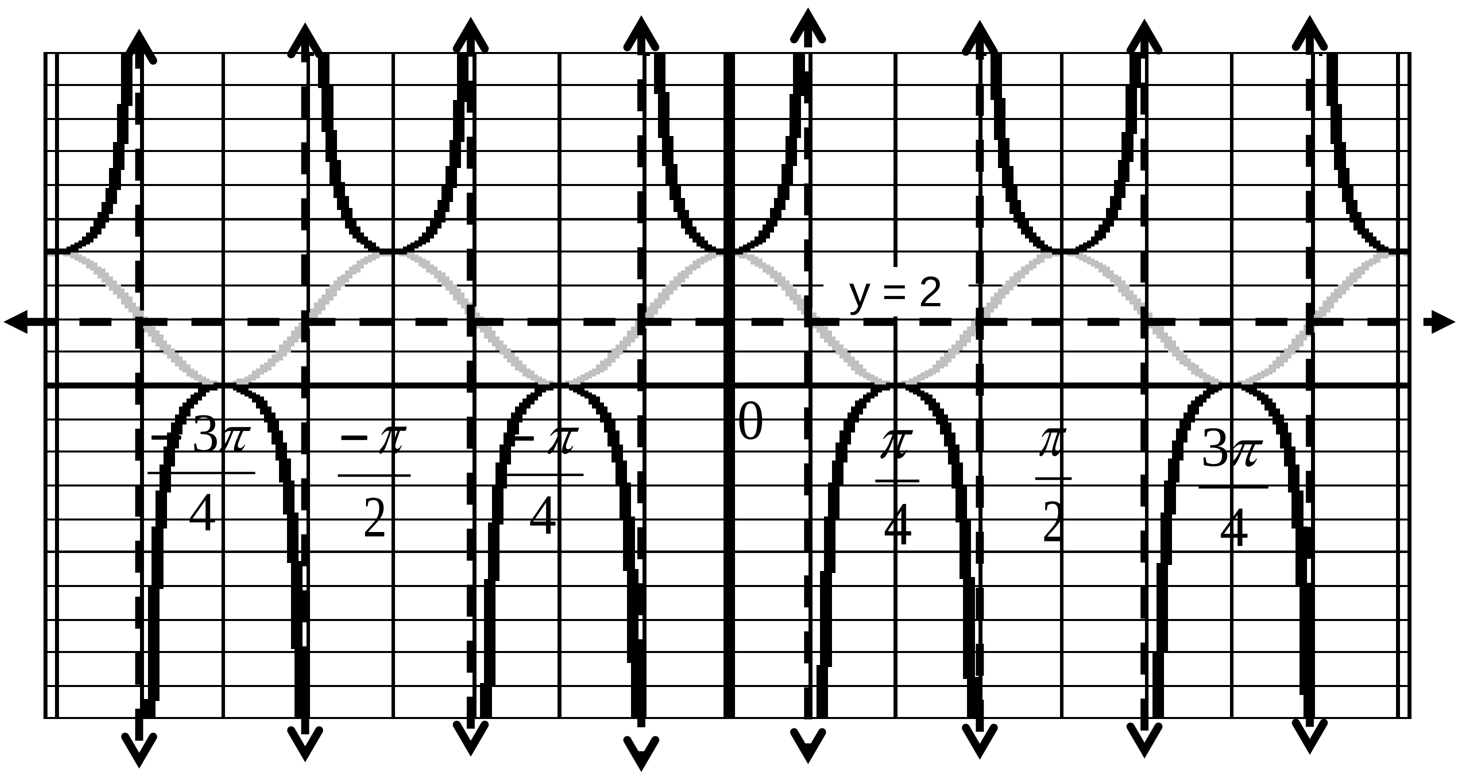
$$-\pi \leq 4x \leq \pi$$

$$\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$$

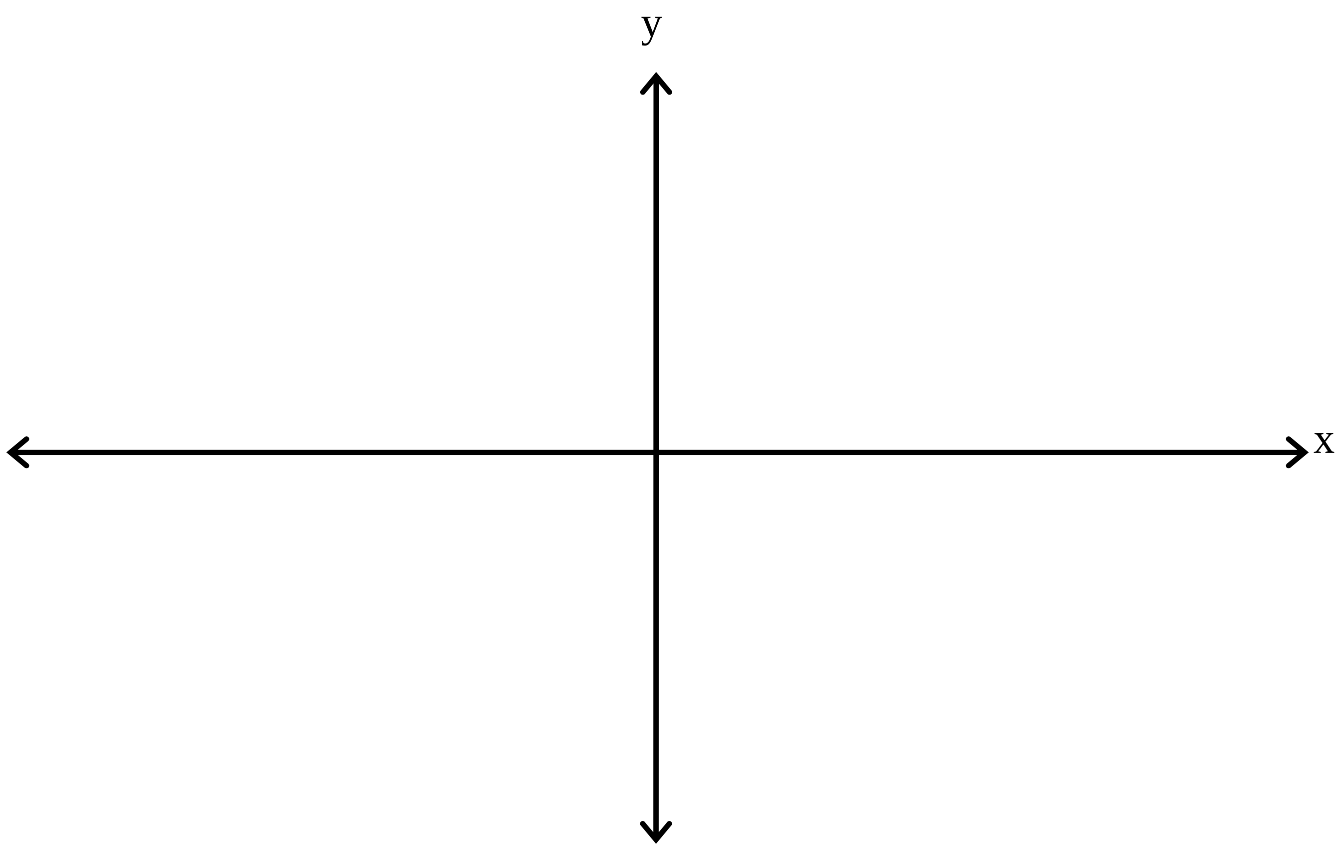
This is one cycle of our graph.

- Our graph is shifted 2 units up, so draw a dotted line at y = 2 to represent the line that our graph oscillates around.

- Where the cosine curve crosses the line y = 2, our secant curve has asymptotes.
- Sketch the graph.



***Look at the summary of the 6 basic trigonometric functions on page 317.


y
x