Using Fundamental Identities

In this lesson we will use the fundamental trig identities to do the following:

1. Evaluate trig functions
2. Simplify trig expressions
3. Develop additional trig identities
4. Solve trig equations

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

*Cofuntion Identities**

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

* 90° can be substituted for $\frac{\pi}{2}$ in the cofunction identities.

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \csc(-u) = -\csc u$$

$$\cos(-u) = \cos u \qquad \sec(-u) = \sec u$$

$$\tan(-u) = -\tan u \qquad \cot(-u) = -\cot u$$

Note: Sometimes variations of these are used, such as $\cos^2 u = 1 - \sin^2 u$.

Using Fundamental Identities

Example: If csc u = -5/3, and cos u > 0, find the values of the other five trig functions.

(By allsintancos we know we are in Quadrant IV.)

$$\sin u = \frac{-3}{5}$$

(csc and sin are reciprocals)

$$\cos^2 u = 1 - \sin^2 u = 1 - \left(\frac{-3}{5}\right)^2 = \frac{16}{25}$$

$$\cos u = \frac{4}{5}$$

$$\sec u = \frac{5}{4}$$

(sec and cos are reciprocals)

$$\tan u = \frac{\sin u}{\cos u} = \frac{\frac{-3}{5}}{\frac{4}{5}} = \frac{-3}{5} \cdot \frac{5}{4} = \frac{-3}{4}$$

$$\cot u = \frac{-4}{3}$$

(cot and tan are reciprocals)

Example: Simplify $\csc^2 x \cot x - \cot x$.

$$\begin{aligned}\csc^2 x \cot x - \cot x &= \cot x(\csc^2 x - 1)\\ &= \cot x(\cot^2 x)\\ &= \cot^3 x\end{aligned}$$

Example: Simplify $\tan x \sin x + \cos x$.

$$\begin{aligned}\tan x \sin x + \cos x &= \frac{\sin x}{\cos x} \cdot \sin x + \cos x\\ &= \frac{\sin^2 x}{\cos x} + \cos x\\ &= \frac{\sin^2 x}{\cos x} + \cos x\left(\frac{\cos x}{\cos x}\right)\\ &= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}\\ &= \frac{\sin^2 x + \cos^2 x}{\cos x}\\ &= \frac{1}{\cos x}\\ &= \sec x\end{aligned}$$

Note: Sometimes it is helpful to write everything in terms of sine and cosine. Other times you will see forms of the trig identities, as in the following example.

Example: Simplify $\frac{\sec t}{\tan t} - \frac{\tan t}{1+\sec t}$.

$$\frac{\sec t}{\tan t} - \frac{\tan t}{1+\sec t} = \frac{\sec t}{\tan t} \cdot \frac{(1+\sec t)}{(1+\sec t)} - \frac{\tan t}{1+\sec t} \cdot \frac{(\tan t)}{(\tan t)}$$

$$= \frac{\sec t + \sec^2 t}{\tan t(1+\sec t)} - \frac{\tan^2 t}{\tan t(1+\sec t)}$$

$$= \frac{\sec t + \sec^2 t - \tan^2 t}{\tan t(1+\sec t)}$$

$$= \frac{\sec t + 1}{\tan t(1+\sec t)}$$

$$= \frac{(1+\sec t)}{\tan t(1+\sec t)}$$

$$= \frac{1}{\tan t}$$

$$= \cot t$$

Example: Factor $\cos^2 x - 1$.

$$\cos^2 x - 1$$
$$(\cos x + 1)(\cos x - 1)$$

This is the difference of squares.

Example: Factor $\sin^2 u - 3\sin u - 10$.

$$\sin^2 u - 3\sin u - 10$$
$$(\sin u + 2)(\sin u - 5)$$

This is backwards FOIL.

Example: Factor $\sec^2 t - \tan t - 3$.

Write first in terms of one trig function.

$$\begin{aligned} \sec^2 t - \tan t - 3 &= (\tan^2 t + 1) - \tan t - 3 \\ &= \tan^2 t - \tan t - 2 \\ &= (\tan t + 1)(\tan t - 2) \end{aligned}$$

Example: Rewrite $\dfrac{1}{\sec x - 1}$ so that it is not a fraction.

Multiply by 1 using the conjugate of the denominator.

$$\frac{1}{\sec x - 1} = \frac{1}{\sec x - 1} \cdot \frac{(\sec x + 1)}{(\sec x + 1)}$$

$$= \frac{\sec x + 1}{\sec^2 x - 1}$$

$$= \frac{\sec x + 1}{\tan^2 x}$$

$$= \frac{\sec x}{\tan^2 x} + \frac{1}{\tan^2 x}$$

$$= \sec x\left(\frac{1}{\tan^2 x}\right) + \frac{1}{\tan^2 x}$$

$$= \sec x \cot^2 x + \cot^2 x$$

$$= \left(\frac{1}{\cos x}\right)\left(\frac{\cos^2 x}{\sin^2 x}\right) + \cot^2 x$$

$$= \left(\frac{\cos x}{\sin^2 x}\right) + \cot^2 x$$

$$= \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) + \cot^2 x$$

$$= \csc x \cot x + \cot$$

Example: Perform the addition and simplify.

$$\frac{\sin x}{1+\cos x}+\frac{\cos x}{\sin x}$$

$$=\frac{\sin x}{(1+\cos x)}\cdot\frac{(\sin x)}{(\sin x)}+\frac{\cos x}{\sin x}\cdot\frac{(1+\cos x)}{(1+\cos x)}$$

$$=\frac{\sin^2 x}{(1+\cos x)(\sin x)}+\frac{\cos x+\cos^2 x}{(1+\cos x)(\sin x)}$$

$$=\frac{\sin^2 x+\cos x+\cos^2 x}{(1+\cos x)(\sin x)}$$

$$=\frac{[\sin^2 x+\cos^2 x]+\cos x}{(1+\cos x)(\sin x)}$$

$$=\frac{1+\cos x}{(1+\cos x)(\sin x)}$$

$$=\frac{(1+\cos x)}{(1+\cos x)(\sin x)}$$

$$=\frac{1}{\sin x}$$

$$=\csc x$$