Systems of Inequalities

Consider the inequality $x - y > 2$.

The solution of the inequality is the set of all points (x, y) that make the inequality true.

To Solve an Inequality by Graphing:

1. Replace the inequality sign by an equal sign. The resulting equation is the border equation. Graph it.

   - $<$ or $>$ means the border is *dotted* (not included in the solution)
   - $\leq$ or $\geq$ means the border is *solid* (part of the solution)

2. Test one point that is clearly not on the border in the original inequality.

   - If the point makes the inequality true, then shade that side.
   - If the point makes the inequality false, then shade the other side.

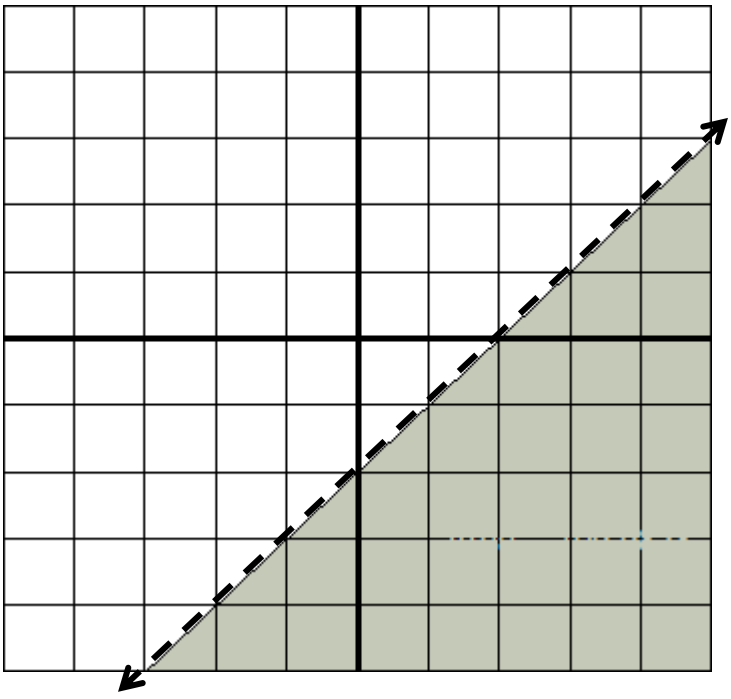
Example: Graph the inequality $x - y > 2$.

1. The border is $x - y = 2$ (i.e. $y = x - 2$) and is dotted.

2. The easiest point to test is (0, 0).

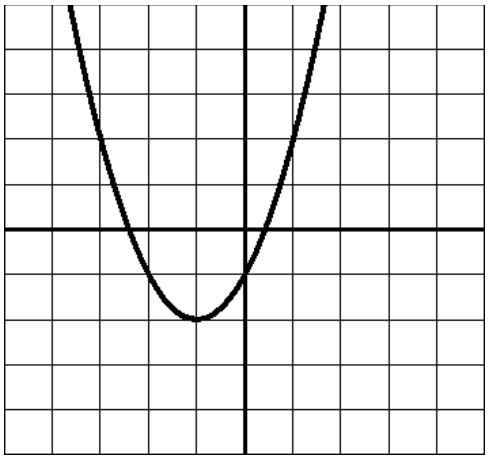
$$x - y > 2$$
$$0 - 0 > 2$$
$$0 > 2$$

This is *false*, so shade on the *opposite* side of the border equation.



Example: Graph $y \geq (x+1)^2 - 2$.

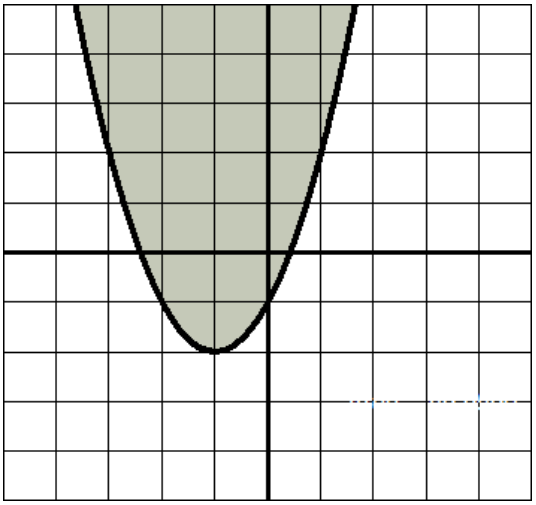
1. <u>Border</u>: $y \geq (x+1)^2 - 2$ (solid)



2. Test the point (0, 0)

$y \geq (x+1)^2 - 2$
$0 \geq (0+1)^2 - 2$
$0 \geq 1 - 2$
$0 \geq -1$

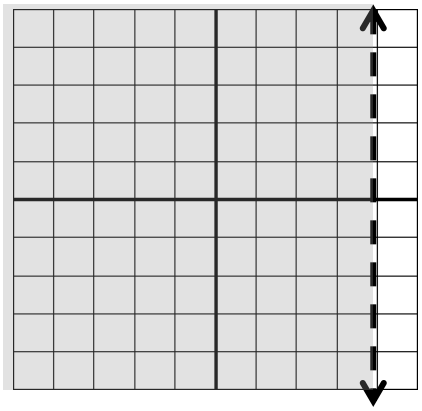
This is true, so shade the region containing the point (0, 0).



Example: Graph $x < 4$.

1. Border: $x = 4$ (dotted)

2. Test (0, 0): $x < 4$
   $0 < 4$
   True, so shade (0, 0) side.



Inequalities on a Graphing Calculator

To show an inequality on a graphing calculator, you must first solve the inequality for y. Then, if the inequality is > or $\geq$, then shade above. If it is < or $\leq$ then shade below the curve.

1. Enter the border equation for $y \geq (x+1)^2 - 2$.
2. Move the cursor to the left as far as it will go.
3. Press [ENTER] until you see the upper shaded triangle. This means it will shade above the equation.
4. Press [GRAPH].

Note: It will not graph dotted or solid. You will have to keep track of that mentally.

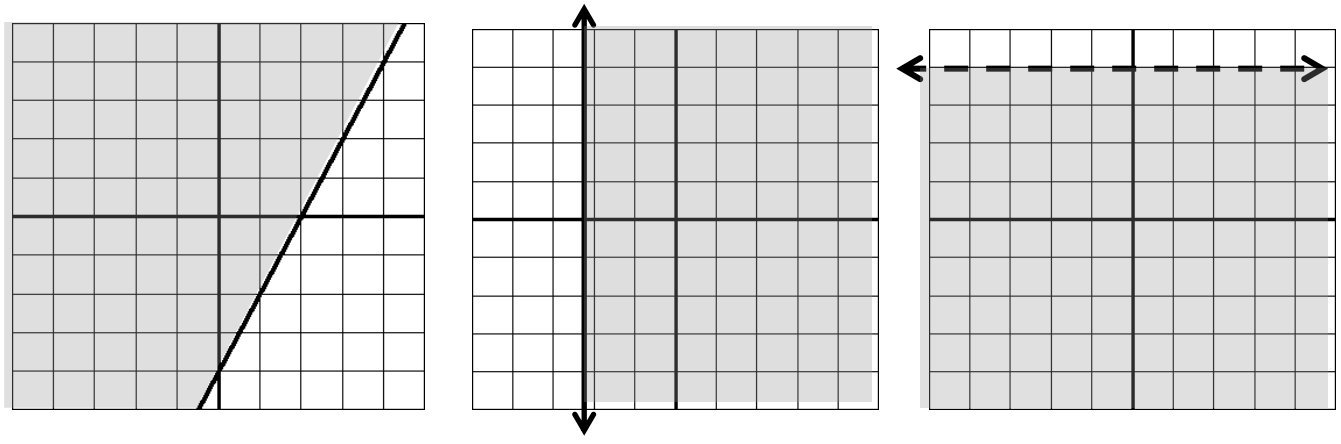
Systems of Inequalities

Look at the following system:

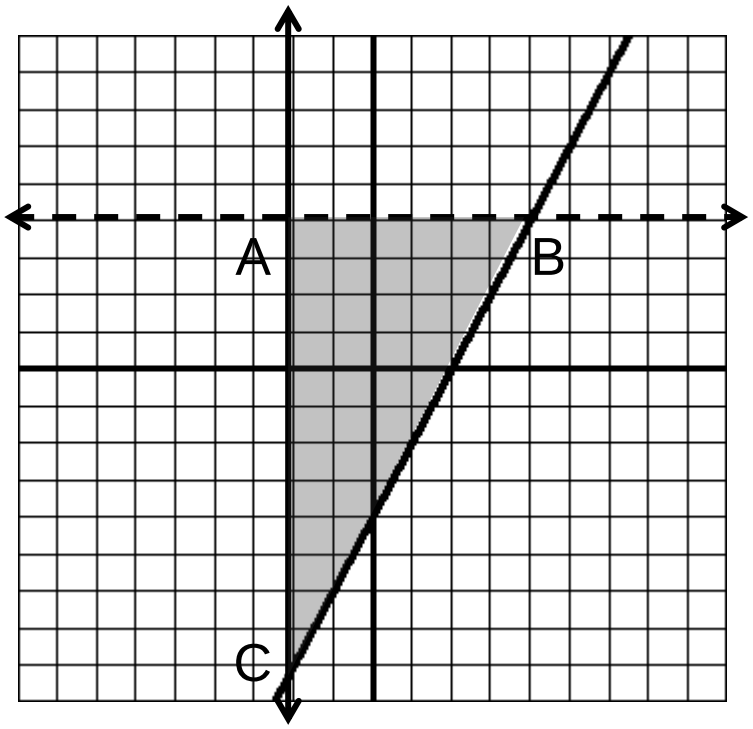
$$\begin{cases} 2x - y \le 4 \\ x \ge -2 \\ 2y < 8 \end{cases}$$

This is a system of inequalities. The solution to this system is the set of ordered pairs (x, y) such that each ordered pair satisfies each inequality in the system.

To solve systems of inequalities, first sketch the graph of each individual inequality (on the same coordinate system) and then find the region that is common to every graph in the system (i.e. where the inequalities overlap).



If all 3 are graphed on the same coordinate system, the region that would overlap would be:



<u>To Find the Vertices of the Region:</u>

Solve the 3 systems of corresponding equations obtained by taking pairs of equation representing the boundaries of the individual regions.

Vertex A (-2, 4)

$$\begin{cases} y = 4 \\ x = -2 \end{cases}$$

Vertex B (4, 4)

$$\begin{cases} y = 4 \\ 2x - y = 4 \end{cases}$$

Vertex C (-2, -8)

$$\begin{cases} x = -2 \\ 2x - y = 4 \end{cases}$$

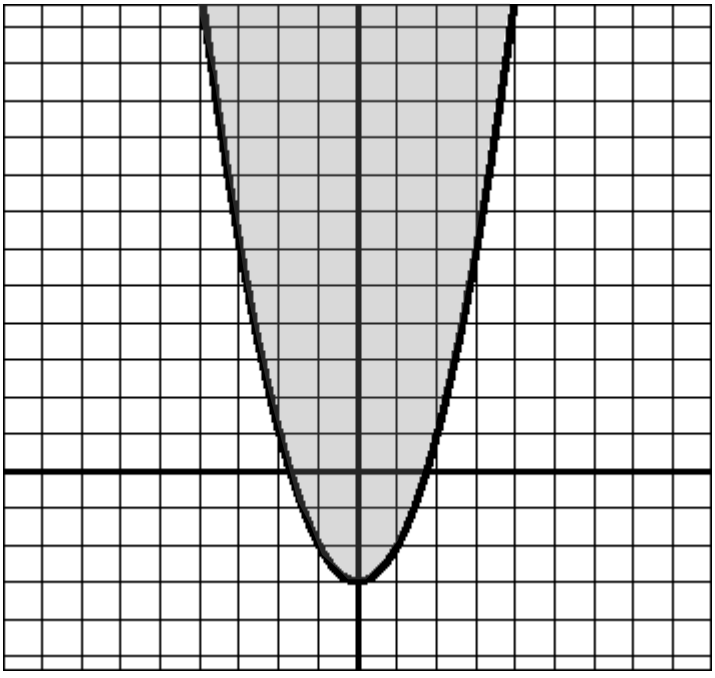
Example: Solve the system.

$$\begin{cases} y \geq x^2 - 3 \\ y < 2x + 3 \end{cases}$$

1. Graph $y \geq x^2 - 3$

Border: $y \geq x^2 - 3 \rightarrow y = x^2 - 3$ (solid)

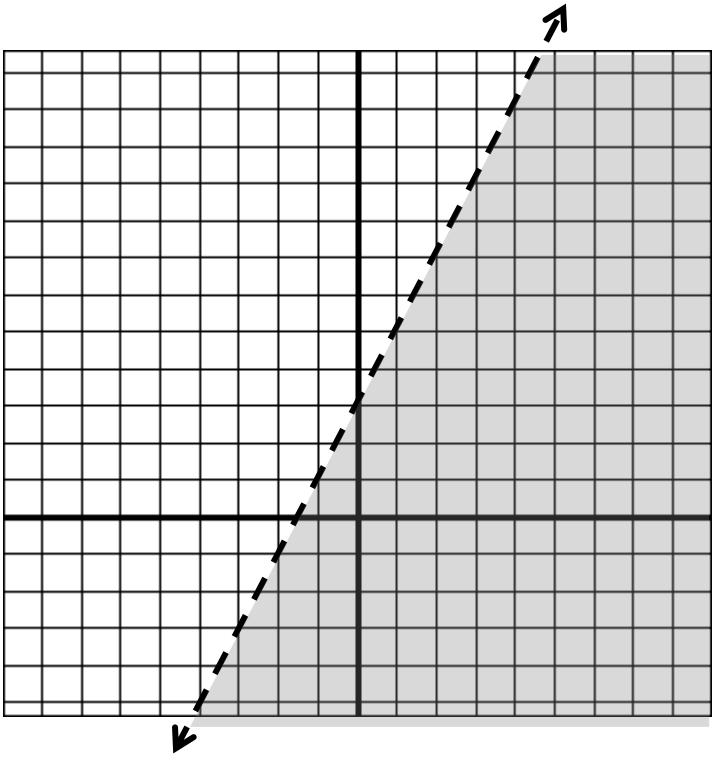
Test (0, 0) $\rightarrow$ True, so shade inside the parabola.



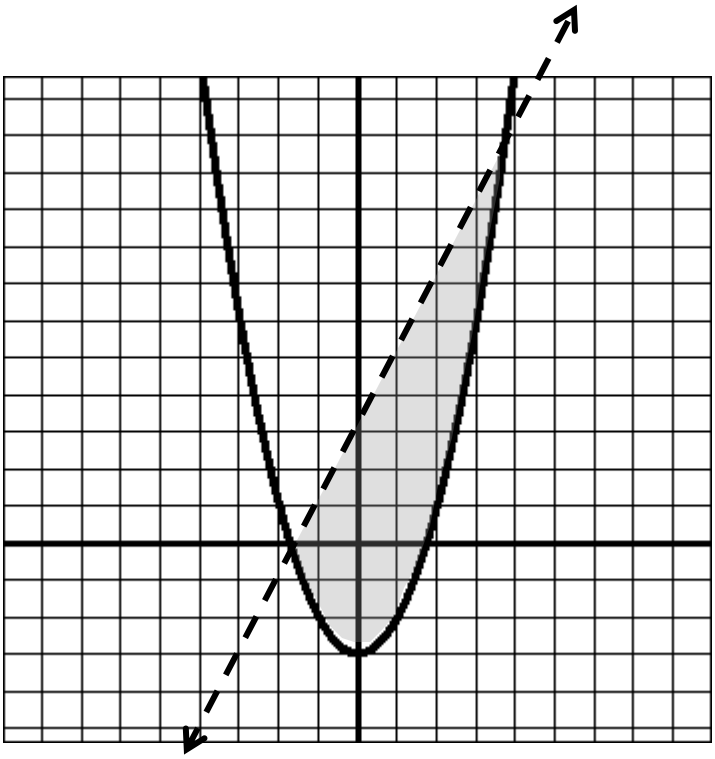
2. Graph $y < 2x + 3$

Border: $y < 2x + 3 \rightarrow y = 2x + 3$ (dotted)

Test (0, 0) → True, so shade below the line.

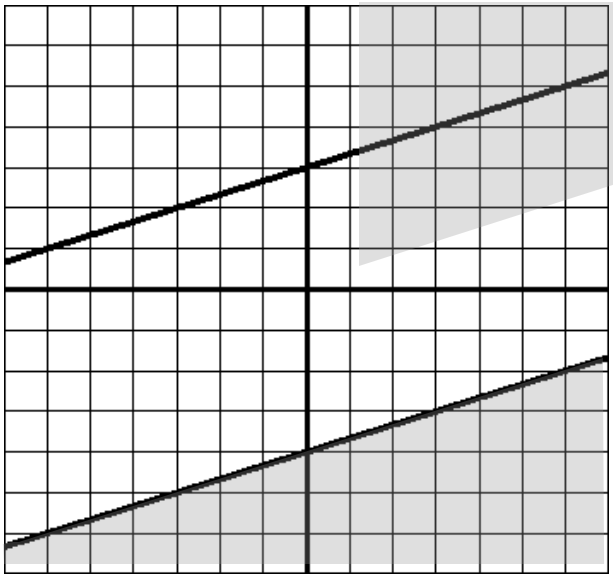


Together, they give us the following graph:



Example: Solve the system.

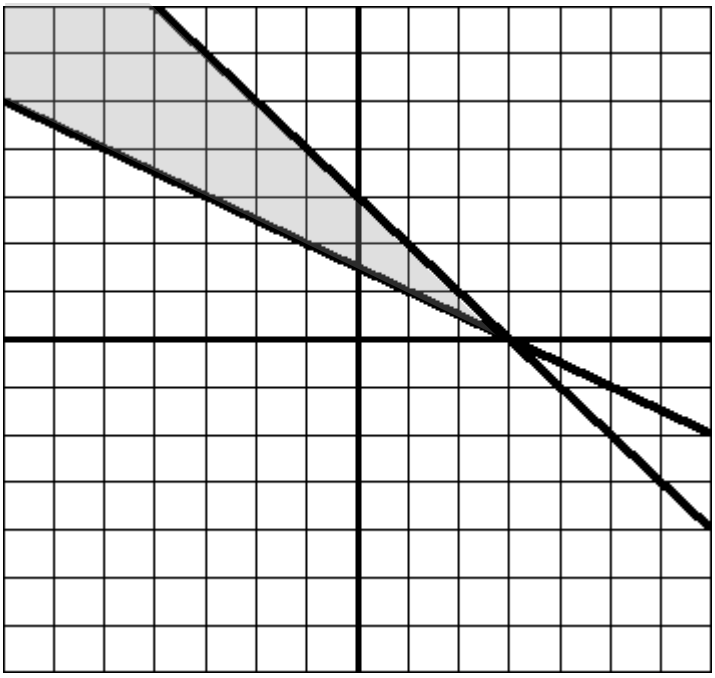
$$\begin{cases} y > \frac{1}{3}x + 3 \\ y < \frac{1}{3}x - 4 \end{cases}$$



This system has no solution because there is no overlap.

Example: Solve the system.

$$\begin{cases} x + y \le 3 \\ x + 2y \ge 3 \end{cases}$$

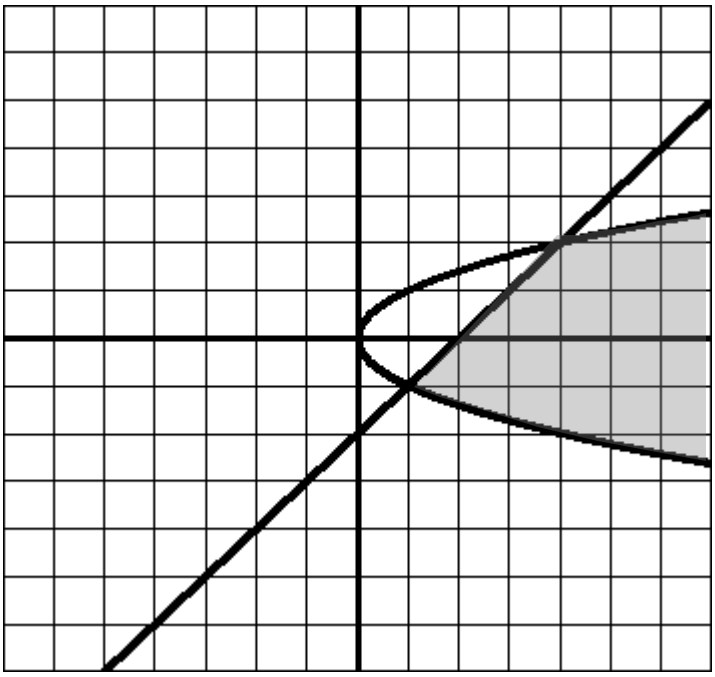


The intersection of these two half-planes is called an infinite wedge, because it is unbounded.

Additional Examples:

Solve the system and label the vertices, if they exist.

$$\begin{cases} x - y^2 \geq 0 \\ x - y \geq 2 \end{cases}$$



Vertices:

$$\begin{cases} x - y^2 = 0 \\ x - y = 2 \end{cases} \rightarrow \begin{cases} x - y^2 = 0 \\ x = 2 + y \end{cases} \rightarrow \text{use substitution}$$

$$(2 + y) - y^2 = 0$$
$$y^2 - y - 2 = 0$$
$$(y - 2)(y + 1) = 0$$
$$y = 2, y = -1$$

$$\underline{y = 2} \qquad\qquad \underline{y = -1}$$
$$x = 2 + 2 \qquad\qquad x = 2 - 1$$
$$x = 4 \qquad\qquad x = 1$$

The vertices are (4, 2) and (1, -1).

Solve the system.

$$\begin{cases} x^2 + y^2 \le 16 \\ 2x + 3y \ge 6 \end{cases}$$