Appendix A Review of Fundamental Concepts of Algebra

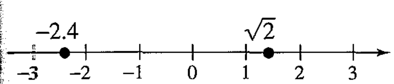
A.1 Real Numbers and Their Properties

▶ What you should learn

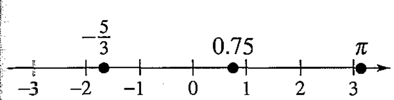
- How to represent and classify real numbers
- How to order real numbers and use inequalities
- How to find the absolute values of real numbers and find the distance between two real numbers
- How to evaluate algebraic expressions
- How to use the basic rules and properties of algebra

▶ Why you should learn it

Real numbers are used to represent many real-life quantities. For example, in Exercise 69 on page A9, you will use real numbers to represent the federal deficit.



Every point on the real number line corresponds to exactly one real number.



Every real number corresponds to exactly one point on the real number line.

FIGURE A.2 *One-to-One Correspondence*

Real Numbers

Real numbers are used in everyday life to describe quantities such as age, miles per gallon, container size, and population. Real numbers are represented by symbols such as

$$-5, 9, 0, \frac{4}{3}, 0.666\ldots, 28.21, \sqrt{2}, \pi, \text{ and } \sqrt[3]{-32}.$$

Here are some important subsets of the real numbers.

$\{1, 2, 3, 4, \ldots\}$	Set of natural numbers
$\{0, 1, 2, 3, 4, \ldots\}$	Set of whole numbers
$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$	Set of integers

A real number is **rational** if it can be written as the ratio p/q of two integers, where $q \neq 0$. For instance, the numbers

$$\frac{1}{3} = 0.3333\ldots = 0.\overline{3}, \frac{1}{8} = 0.125, \text{ and } \frac{125}{111} = 1.126126\ldots = 1.\overline{126}$$

are rational. The decimal representation of a rational number either repeats $\left(\text{as in } \frac{173}{55} = 3.1\overline{45}\right)$ or terminates $\left(\text{as in } \frac{1}{2} = 0.5\right)$. A real number that cannot be written as the ratio of two integers is called **irrational.** Irrational numbers have infinite nonrepeating decimal representations. For instance, the numbers

$$\sqrt{2} \approx 1.4142136 \quad \text{and} \quad \pi \approx 3.1415927$$

are irrational. (The symbol $\approx$ means "is approximately equal to.")

Real numbers are represented graphically by a **real number line.** The point 0 on the real number line is the **origin.** Numbers to the right of 0 are positive, and numbers to the left of 0 are negative, as shown in Figure A.1. The term **nonnegative** describes a number that is either positive or zero.

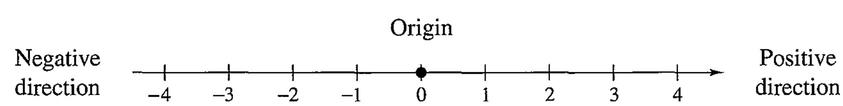


FIGURE A.1 *The Real Number Line*

As illustrated in Figure A.2, there is a *one-to-one correspondence* between real numbers and points on the real number line.

The icon identifies examples and concepts related to features of the Learning Tools CD-ROM and the *Interactive* and *Internet* versions of this text. For more details see the chart on pages *xxi–xxv*.

Ordering Real Numbers

One important property of real numbers is that they are *ordered.*

Definition of Order on the Real Number Line

If a and b are real numbers, a is less than b if $b - a$ is positive. The **order** of a and b is denoted by the **inequality**

$$a < b.$$

This relationship can also be described by saying that b is *greater than a* and writing $b > a$. The inequality $a \le b$ means that a is *less than or equal to b*, and the inequality $b \ge a$ means that b is *greater than or equal to a.* The symbols $<$, $>$, $\le$, and $\ge$ are *inequality symbols.*

Geometrically, this definition implies that $a < b$ if and only if a lies to the *left* of b on the real number line, as shown in Figure A.3.

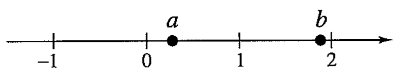


FIGURE A.3 $a < b$ *if and only if a lies to the left of b.*

Example 1 ▶ Interpreting Inequalities

Describe the subset of real numbers represented by each inequality.

a. $x \le 2$ **b.** $-2 \le x < 3$

Solution

a. The inequality $x \le 2$ denotes all real numbers less than or equal to 2, as sh[own] in Figure A.4.

b. The inequality $-2 \le x < 3$ means that $x \ge -2$ and $x < 3$. This "do[uble] inequality" denotes all real numbers between -2 and 3, including -2 bu[t not] including 3, as shown in Figure A.5.

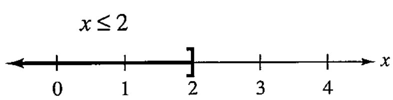


FIGURE A.4

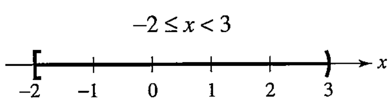
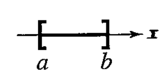
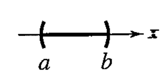
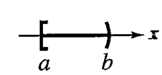
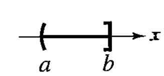


FIGURE A.5

Inequalities can be used to describe subsets of real numbers called **interv[als]**. In the bounded intervals below, the real numbers a and b are the **endpoints** [of] each interval.

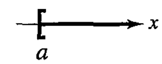
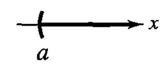
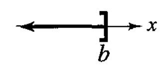
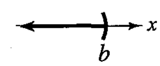
The *Interactive* CD-ROM and *Internet* versions of this text offer a Try It for each example in the text.

Bounded Intervals on the Real Number Line

Notation	*Interval Type*	*Inequality*	*Graph*
$[a, b]$	Closed	$a \le x \le b$	
(a, b)	Open	$a < x < b$	
$[a, b)$		$a \le x < b$	
$(a, b]$		$a < x \le b$	

The symbols ∞, **positive infinity,** and $-\infty$, **negative infinity,** do not represent real numbers. They are simply convenient symbols used to describe the unboundedness of an interval such as $(1, \infty)$ or $(-\infty, 3]$.

Unbounded Intervals on the Real Number Line

Notation	*Interval Type*	*Inequality*	*Graph*
$[a, \infty)$		$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$		$x \leq b$	
$(-\infty, b)$	Open	$x < b$	
$(-\infty, \infty)$	Entire real line	$-\infty < x < \infty$	

Example 2 ▶ Using Inequalities to Represent Intervals

Use inequality notation to describe each of the following.

a. c is at most 2.

b. m is at least -3.

c. All x in the interval $(-3, 5]$

Solution

a. The statement "c is at most 2" can be represented by $c \leq 2$.

b. The statement "m is at least -3" can be represented by $m \geq -3$.

c. "All x in the interval $(-3, 5]$" can be represented by $-3 < x \leq 5$.

Example 3 ▶ Interpreting Intervals

Give a verbal description of each interval.

a. $(-1, 0)$ **b.** $[2, \infty)$ **c.** $(-\infty, 0)$

Solution

a. This interval consists of all real numbers that are greater than -1 and less than 0.

b. This interval consists of all real numbers that are greater than or equal to 2.

c. This interval consists of all real numbers that are less than zero (the negative real numbers).

The **Law of Trichotomy** states that for any two real numbers a and b, *precisely* one of three relationships is possible:

$$a = b, \quad a < b, \quad \text{or} \quad a > b. \qquad \text{Law of Trichotomy}$$

Absolute Value and Distance

The **absolute value** of a real number is its *magnitude*, or the distance between the origin and the point representing the real number on the real number line.

Definition of Absolute Value

If a is a real number, then the absolute value of a is

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0. \end{cases}$$

Notice in this definition that the absolute value of a real number is never negative. For instance, if $a = -5$, then $|-5| = -(-5) = 5$. The absolute value of a real number is either positive or zero. Moreover, 0 is the only real number whose absolute value is 0. So, $|0| = 0$.

Example 4 ▶ Evaluating the Absolute Value of a Number

Evaluate $\dfrac{|x|}{x}$ for (a) $x > 0$ and (b) $x < 0$.

Solution

a. If $x > 0$, then $|x| = x$ and $\dfrac{|x|}{x} = \dfrac{x}{x} = 1$.

b. If $x < 0$, then $|x| = -x$ and $\dfrac{|x|}{x} = \dfrac{-x}{x} = -1$.

Properties of Absolute Values

1. $|a| \geq 0$
2. $|-a| = |a|$
3. $|ab| = |a||b|$
4. $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}, \quad b \neq 0$

Absolute value can be used to define the distance between two points on the real number line. For instance, the distance between -3 and 4 is

$$|-3 - 4| = |-7|$$
$$= 7$$

as shown in Figure A.6.

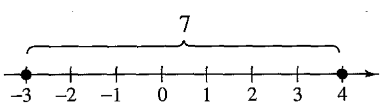


FIGURE A.6 *The distance between −3 and 4 is 7.*

Distance Between Two Points on the Real Line

Let a and b be real numbers. The **distance between a and b** is

$$d(a, b) = |b - a| = |a - b|.$$

Algebraic Expressions

One characteristic of algebra is the use of letters to represent numbers. The letters are **variables,** and combinations of letters and numbers are **algebraic expressions.** Here are a few examples of algebraic expressions.

$$5x, \qquad 2x - 3, \qquad \frac{4}{x^2 + 2}, \qquad 7x + y$$

Definition of an Algebraic Expression

An **algebraic expression** is a collection of letters **(variables)** and real numbers **(constants)** combined using the operations of addition, subtraction, multiplication, division, and exponentiation.

The **terms** of an algebraic expression are those parts that are separated by *addition*. For example,

$$x^2 - 5x + 8 = x^2 + (-5x) + 8$$

has three terms: x^2 and $-5x$ are the **variable terms** and 8 is the **constant term.** The numerical factor of a variable term is the **coefficient** of the variable term. For instance, the coefficient of $-5x$ is -5, and the coefficient of x^2 is 1.

To **evaluate** an algebraic expression, substitute numerical values for each of the variables in the expression. Here are two examples.

The *Interactive* CD-ROM and *Internet* versions of this text offer a Quiz for every section of the text.

Expression	*Value of Variable*	*Substitute*	*Value of Expression*
$-3x + 5$	$x = 3$	$-3(3) + 5$	$-9 + 5 = -4$
$3x^2 + 2x - 1$	$x = -1$	$3(-1)^2 + 2(-1) - 1$	$3 - 2 - 1 = 0$

When an algebraic expression is evaluated, the **Substitution Principle** is used. It states that "If $a = b$, then a can be replaced by b in any expression involving a." In the first evaluation shown above, for instance, 3 is *substituted* for x in the expression $-3x + 5$.

Basic Rules of Algebra

There are four arithmetic operations with real numbers: *addition*, *multiplication*, *subtraction*, and *division*, denoted by the symbols $+$, $\times$ or $\cdot$, $-$, and $\div$. Of these, addition and multiplication are the two primary operations. Subtraction and division are the inverse operations of addition and multiplication, respectively.

Subtraction: Add the opposite. *Division:* Multiply by the reciprocal.

$$a - b = a + (-b) \qquad \text{If } b \neq 0, \text{ then } a/b = a\left(\frac{1}{b}\right) = \frac{a}{b}.$$

STUDY TIP

Try to formulate a verbal description of each property. For instance, the first property states that *the order in which two real numbers are added does not affect their sum.*

In these definitions, $-b$ is the **additive inverse** (or opposite) of b, and $1/b$ is the **multiplicative inverse** (or reciprocal) of b. In the fractional form a/b, a is the **numerator** of the fraction and b is the **denominator.**

Because the properties of real numbers on page A6 are true for variables and algebraic expressions as well as for real numbers, they are often called the **Basic Rules of Algebra.**

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Try to formulate a verbal description of each property. For instance, the first property states that *the order in which two real numbers are added does not affect their sum.*

Basic Rules of Algebra

Let a, b, and c be real numbers, variables, or algebraic expressions.

	Property	*Example*
Commutative Property of Addition:	$a + b = b + a$	$4x + x^2 = x^2 + 4x$
Commutative Property of Multiplication:	$ab = ba$	$(4 - x)x^2 = x^2(4 - x)$
Associative Property of Addition:	$(a + b) + c = a + (b + c)$	$(x + 5) + x^2 = x + (5 + x^2)$
Associative Property of Multiplication:	$(ab)c = a(bc)$	$(2x \cdot 3y)(8) = (2x)(3y \cdot 8)$
Distributive Properties:	$a(b + c) = ab + ac$	$3x(5 + 2x) = 3x \cdot 5 + 3x \cdot 2x$
	$(a + b)c = ac + bc$	$(y + 8)y = y \cdot y + 8 \cdot y$
Additive Identity Property:	$a + 0 = a$	$5y^2 + 0 = 5y^2$
Multiplicative Identity Property:	$a \cdot 1 = a$	$(4x^2)(1) = 4x^2$
Additive Inverse Property:	$a + (-a) = 0$	$5x^3 + (-5x^3) = 0$
Multiplicative Inverse Property:	$a \cdot \dfrac{1}{a} = 1, \quad a \neq 0$	$(x^2 + 4)\left(\dfrac{1}{x^2 + 4}\right) = 1$

Because subtraction is defined as "adding the opposite," the Distributive Properties are also true for subtraction. So, the first Distributive Property can be applied to an expression of the form $a(b - c)$ as follows.

$$a(b - c) = ab - ac$$

STUDY TIP

Be sure you see the difference between the *opposite of a number* and a *negative number.* If a is already negative, then its opposite, $-a$, is positive. For instance, if $a = -5$, then

$$-a = -(-5) = 5.$$

Properties of Negation

Let a and b be real numbers, variables, or algebraic expressions.

Property	*Example*
1. $(-1)a = -a$	$(-1)7 = -7$
2. $-(-a) = a$	$-(-6) = 6$
3. $(-a)b = -(ab) = a(-b)$	$(-5)3 = -(5 \cdot 3) = 5(-3)$
4. $(-a)(-b) = ab$	$(-2)(-x) = 2x$
5. $-(a + b) = (-a) + (-b)$	$-(x + 8) = (-x) + (-8)$
	$= -x - 8$

Properties of Equality

Let a, b, and c be real numbers, variables, or algebraic expressions.

1. If $a = b$, then $a + c = b + c$.	Add c to each side.
2. If $a = b$, then $ac = bc$.	Multiply each side by c.
3. If $a + c = b + c$, then $a = b$.	Subtract c from each side.
4. If $ac = bc$ and $c \neq 0$, then $a = b$.	Divide each side by c.

STUDY TIP

The "or" in the Zero-Factor Property includes the possibility that either or both factors may be zero. This is an **inclusive or,** and it is the way the word "or" is generally used in mathematics.

Properties of Zero

Let a and b be real numbers, variables, or algebraic expressions.

1. $a + 0 = a$ and $a - 0 = a$

2. $a \cdot 0 = 0$

3. $\dfrac{0}{a} = 0, \quad a \neq 0$

4. $\dfrac{a}{0}$ is undefined.

5. Zero-Factor Property: If $ab = 0$, then $a = 0$ or $b = 0$.

STUDY TIP

In Property 1 of fractions, the phrase "if and only if" implies two statements. One statement is: If $a/b = c/d$, then $ad = bc$. The other statement is: If $ad = bc$, where $b \neq 0$ and $d \neq 0$, then $a/b = c/d$.

Properties and Operations of Fractions

Let a, b, c, and d be real numbers, variables, or algebraic expressions such that $b \neq 0$ and $d \neq 0$.

1. Equivalent Fractions: $\dfrac{a}{b} = \dfrac{c}{d}$ if and only if $ad = bc$.

2. Rules of Signs: $-\dfrac{a}{b} = \dfrac{-a}{b} = \dfrac{a}{-b}$ and $\dfrac{-a}{-b} = \dfrac{a}{b}$

3. Generate Equivalent Fractions: $\dfrac{a}{b} = \dfrac{ac}{bc}, \quad c \neq 0$

4. Add or Subtract with Like Denominators: $\dfrac{a}{b} \pm \dfrac{c}{b} = \dfrac{a \pm c}{b}$

5. Add or Subtract with Unlike Denominators: $\dfrac{a}{b} \pm \dfrac{c}{d} = \dfrac{ad \pm bc}{bd}$

6. Multiply Fractions: $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$

7. Divide Fractions: $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c} = \dfrac{ad}{bc}, \quad c \neq 0$

Example 5 ▶ Properties and Operations of Fractions

a. Equivalent fractions: $\dfrac{x}{5} = \dfrac{3 \cdot x}{3 \cdot 5} = \dfrac{3x}{15}$

b. Divide fractions: $\dfrac{7}{x} \div \dfrac{3}{2} = \dfrac{7}{x} \cdot \dfrac{2}{3} = \dfrac{14}{3x}$

c. Add fractions with unlike denominators: $\dfrac{x}{3} + \dfrac{2x}{5} = \dfrac{5 \cdot x + 3 \cdot 2x}{3 \cdot 5} = \dfrac{11x}{15}$

If a, b, and c are integers such that $ab = c$, then a and b are **factors** or **divisors** of c. A **prime number** is an integer that has exactly two positive factors:— itself and 1—such as 2, 3, 5, 7, and 11. The numbers 4, 6, 8, 9, and 10 are **composite** because they can be written as the product of two or more prime numbers. The number 1 is neither prime nor composite. The **Fundamental Theorem of Arithmetic** states that every positive integer greater than 1 can be written as the product of prime numbers in precisely one way (disregarding order). For instance, the *prime factorization* of 24 is $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

A.1 Exercises

The *Interactive* CD-ROM and *Internet* versions of this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

In Exercises 1–6, determine which numbers are (a) natural numbers, (b) integers, (c) rational numbers, and (d) irrational numbers.

1. $-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, 2, -11$

2. $\sqrt{5}, -7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -3, 12, 5$

3. $2.01, 0.666\ldots, -13, 0.010110111\ldots, 1, -6$

4. $2.3030030003\ldots, 0.7575, -4.63, \sqrt{10}, -75, 4$

5. $-\pi, -\frac{1}{3}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -1, 8, -22$

6. $25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 7, -11.1, 13$

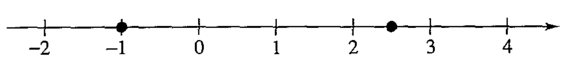
In Exercises 7–10, use a calculator to find the decimal form of the rational number. If it is a nonterminating decimal, write the repeating pattern.

7. $\frac{5}{8}$ **8.** $\frac{1}{3}$

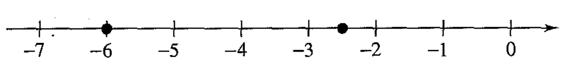
9. $\frac{41}{333}$ **10.** $\frac{6}{11}$

In Exercises 11 and 12, approximate the numbers and place the correct symbol (< or >) between them.

11.



12.



In Exercises 13–18, plot the two real numbers on the real number line. Then place the appropriate inequality symbol (< or >) between them.

13. $-4, -8$ **14.** $-3.5, 1$

15. $\frac{3}{2}, 7$ **16.** $1, \frac{16}{3}$

17. $\frac{5}{6}, \frac{2}{3}$ **18.** $-\frac{8}{7}, -\frac{3}{7}$

In Exercises 19–28, verbally describe the subset of real numbers represented by the inequality. Then sketch the subset on the real number line. State whether the interval is bounded or unbounded.

19. $x \le 5$ **20.** $x \ge -2$

21. $x < 0$ **22.** $x > 3$

23. $x \ge 4$ **24.** $x < 2$

25. $-2 < x < 2$ **26.** $0 \le x \le 5$

27. $-1 \le x < 0$ **28.** $0 < x \le 6$

In Exercises 29–36, use inequality notation to describe the set.

29. All x in the interval $(-2, 4]$

30. All y in the interval $[-6, 0)$

31. y is nonnegative.

32. y is no more than 25.

33. t is at least 10 and at most 22.

34. k is less than 5 but no less than -3.

35. The dog's weight W is more than 65 pounds.

36. The annual rate of inflation r is expected to be at least 2.5% but no more than 5%.

In Exercises 37–40, give a verbal description of the interval.

37. $[0, 8)$ **38.** $[-5, 7]$

39. $(-6, \infty)$ **40.** $(-\infty, 4]$

In Exercises 41–50, evaluate the expression.

41. $|-10|$ **42.** $|0|$

43. $|3-8|$ **44.** $|4-1|$

45. $|-1| - |-2|$ **46.** $-3 - |-3|$

47. $\dfrac{-5}{|-5|}$ **48.** $-3|-3|$

49. $\dfrac{|x+2|}{x+2}, \quad x < -2$ **50.** $\dfrac{|x-1|}{x-1}, \quad x > 1$

In Exercises 51–56, place the correct symbol (<, >, or =) between the pair of real numbers.

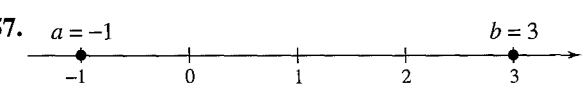
51. $|-3| \quad -|-3|$ **52.** $|-4| \quad |4|$

53. $-5 \quad -|5|$ **54.** $-|-6| \quad |-6|$

55. $-|-2| \quad -|2|$ **56.** $-(-2) \quad -2$

In Exercises 57–64, find the distance between *a* and *b*.

57.



58.

59. $a = 126, b = 75$ **60.** $a = -126, b = -75$

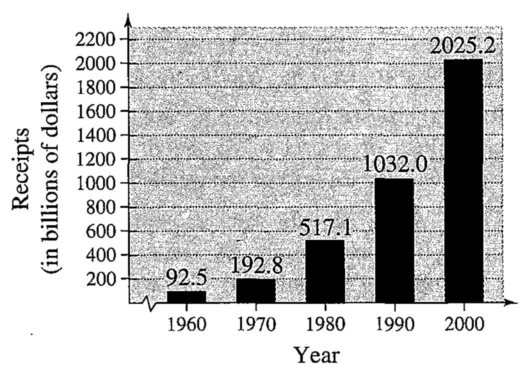
61. $a = -\frac{5}{2}, b = 0$ **62.** $a = \frac{1}{4}, b = \frac{11}{4}$

63. $a = \frac{16}{5}, b = \frac{112}{75}$ **64.** $a = 9.34, b = -5.65$

Budget Variance **In Exercises 65–68, the accounting department of a sports drink bottling company is checking to see whether the actual expenses of a department differ from the budgeted expenses by more than \$500 or by more than 5%. Fill in the missing parts of the table, and determine whether each actual expense passes the "budget variance test."**

	Budgeted Expense, b	*Actual Expense, a*	$\lvert a - b \rvert$	$0.05b$
65. Wages	\$112,700	\$113,356		
66. Utilities	\$9,400	\$9,772		
67. Taxes	\$37,640	\$37,335		
68. Insurance	\$2,575	\$2,613		

69. ***Federal Deficit*** The bar graph shows the federal government receipts (in billions of dollars) for selected years from 1960 through 2000. (Source: U.S. Office of Management and Budget)



(a) Complete the table. (*Hint:* Find |Receipts − Expenditures|.)

Year	Expenditures (in billions)	Surplus or deficit (in billions)
1960	\$92.2	
1970	\$195.6	
1980	\$590.9	
1990	\$1253.2	
2000	\$1788.8	

(b) Use the table in part (a) to construct a bar graph showing the magnitude of the surplus or deficit for each year.

70. ***Veterans*** The table shows the number of surviving spouses of deceased veterans of United States wars (as of May 2001). Construct a circle graph showing the percent of surviving spouses for each war as a fraction of the total number of surviving spouses of deceased war veterans. (Source: Department of Veteran Affairs)

War	Number of surviving spouses
Civil War	1
Indian Wars	0
Spanish-American War	386
Mexican Border War	181
World War I	25,573
World War II	272,793
Korean War	63,579
Vietnam War	114,514
Gulf War	6,261

In Exercises 71–78, use absolute value notation to describe the situation.

71. While traveling on the Pennsylvania Turnpike, you pass milepost 57 near Pittsburgh, then milepost 236 near Gettysburg. How far do you travel during that time period?

72. While traveling on the Pennsylvania Turnpike, you pass milepost 326 near Valley Forge, then milepost 351 near Philadelphia. How far do you travel during that time period?

73. The temperature in Bismarck, North Dakota, was 60° at noon, then 23° at midnight. What was the change in temperature over the 12-hour period?

74. The temperature in Chicago, Illinois was 48° last night at midnight, then 82° at noon today. What was the change in temperature over the 12-hour period?

75. The distance between x and 5 is no more than 3.

76. The distance between x and -10 is at least 6.

77. y is at least six units from 0.

78. y is at most two units from a.

In Exercises 79–84, identify the terms. Then identify the coefficients of the variable terms of the expression.

79. $7x + 4$

80. $6x^3 - 5x$

81. $\sqrt{3}x^2 - 8x - 11$

82. $3\sqrt{3}x^2 + 1$

83. $4x^3 + \dfrac{x}{2} - 5$ **84.** $3x^4 - \dfrac{x^2}{4}$

In Exercises 85–90, evaluate the expression for each value of *x*. (If not possible, state the reason.)

Expression	*Values*	
85. $4x - 6$	(a) $x = -1$	(b) $x = 0$
86. $9 - 7x$	(a) $x = -3$	(b) $x = 3$
87. $x^2 - 3x + 4$	(a) $x = -2$	(b) $x = 2$
88. $-x^2 + 5x - 4$	(a) $x = -1$	(b) $x = 1$
89. $\dfrac{x+1}{x-1}$	(a) $x = 1$	(b) $x = -1$
90. $\dfrac{x}{x+2}$	(a) $x = 2$	(b) $x = -2$

In Exercises 91–100, identify the rule(s) of algebra illustrated by the statement.

91. $x + 9 = 9 + x$ **92.** $2\left(\frac{1}{2}\right) = 1$

93. $\dfrac{1}{h+6}(h + 6) = 1, \quad h \neq -6$

94. $(x + 3) - (x + 3) = 0$

95. $2(x + 3) = 2x + 6$ **96.** $(z - 2) + 0 = z - 2$

97. $1 \cdot (1 + x) = 1 + x$

98. $x + (y + 10) = (x + y) + 10$

99. $x(3y) = (x \cdot 3)y = (3x)y$

100. $\frac{1}{7}(7 \cdot 12) = \left(\frac{1}{7} \cdot 7\right)12 = 1 \cdot 12 = 12$

In Exercises 101–108, perform the operation(s). (Write fractional answers in simplest form.)

101. $\frac{3}{16} + \frac{5}{16}$ **102.** $\frac{6}{7} - \frac{4}{7}$

103. $\frac{5}{8} - \frac{5}{12} + \frac{1}{6}$ **104.** $\frac{10}{11} + \frac{6}{33} - \frac{13}{66}$

105. $12 \div \frac{1}{4}$ **106.** $-\left(6 \cdot \frac{4}{8}\right)$

107. $\dfrac{2x}{3} - \dfrac{x}{4}$ **108.** $\dfrac{5x}{6} \cdot \dfrac{2}{9}$

109. (a) Use a calculator to complete the table.

n	1	0.5	0.01	0.0001	0.000001
$5/n$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n approaches 0.

110. (a) Use a calculator to complete the table.

n	1	10	100	10,000	100,000
$5/n$					

(b) Use the result from part (a) to make a conjecture about the value of $5/n$ as n increases without bound.

Synthesis

True or False? **In Exercises 111 and 112, determine whether the statement is true or false. Justify your answer.**

111. If $a < b$, then $\dfrac{1}{a} < \dfrac{1}{b}$, where $a \neq b \neq 0$.

112. Because $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$, then $\dfrac{c}{a+b} = \dfrac{c}{a} + \dfrac{c}{b}$.

113. ***Exploration*** Consider $|u + v|$ and $|u| + |v|$.

(a) Are the values of the expressions always equal? If not, under what conditions are they unequal?

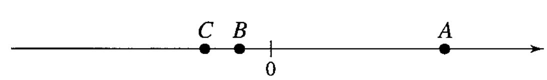
(b) If the two expressions are not equal for certain values of u and v, is one of the expressions always greater than the other? Explain.

114. ***Think About It*** Is there a difference between saying that a real number is positive and saying that a real number is nonnegative? Explain.

115. ***Think About It*** Because every even number is divisible by 2, is it possible that there exist any even prime numbers? Explain.

116. ***Writing*** Describe the differences among the sets of natural numbers, integers, rational numbers, and irrational numbers.

In Exercises 117 and 118, use the real numbers *A*, *B*, and *C* shown on the number line. Determine the sign of each expression.



117. (a) $-A$ **118.** (a) $-C$

(b) $B - A$ (b) $A - C$

119. ***Writing*** You may hear it said that to take the absolute value of a real number you simply remove any negative sign and make the number positive. Can it ever be true that $|a| = -a$ for a real number a? Explain.