A.2 Exponents and Radicals

▶ What you should learn

- How to use properties of exponents
- How to use scientific notation to represent real numbers
- How to use properties of radicals
- How to simplify and combine radicals
- How to rationalize denominators and numerators
- How to use properties of rational exponents

▶ Why you should learn it

Real numbers and algebraic expressions are often written with exponents and radicals. For instance, in Exercise 113 on page A22, you will use an expression involving rational exponents to find the time required for a funnel to empty for different water heights.

Integer Exponents

Repeated *multiplication* can be written in **exponential form.**

Repeated Multiplication	*Exponential Form*
$a \cdot a \cdot a \cdot a \cdot a$	a^5
$(-4)(-4)(-4)$	$(-4)^3$
$(2x)(2x)(2x)(2x)$	$(2x)^4$

In general, if a is a real number and n is a positive integer, then

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

where n is the **exponent** and a is the **base.** The expression a^n is read "a to the nth **power.**" In Property 3 below, be sure you see how to use a negative exponent.

Properties of Exponents

Let a and b be real numbers, variables, or algebraic expressions, and let m and n be integers. (All denominators and bases are nonzero.)

Property	*Example*
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^4 = 3^{2+4} = 3^6 = 729$
2. $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{x^7}{x^4} = x^{7-4} = x^3$
3. $a^{-n} = \dfrac{1}{a^n} = \left(\dfrac{1}{a}\right)^n$	$y^{-4} = \dfrac{1}{y^4} = \left(\dfrac{1}{y}\right)^4$
4. $a^0 = 1, \quad a \neq 0$	$(x^2 + 1)^0 = 1$
5. $(ab)^m = a^m b^m$	$(5x)^3 = 5^3x^3 = 125x^3$
6. $(a^m)^n = a^{mn}$	$(y^3)^{-4} = y^{3(-4)} = y^{-12} = \dfrac{1}{y^{12}}$
7. $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	$\left(\dfrac{2}{x}\right)^3 = \dfrac{2^3}{x^3} = \dfrac{8}{x^3}$
8. $\lvert a^2\rvert = \lvert a\rvert^2 = a^2$	$\lvert(-2)^2\rvert = \lvert -2\rvert^2 = (2)^2 = 4$

It is important to recognize the difference between expressions such as $(-2)^4$ and -2^4. In $(-2)^4$, the parentheses indicate that the exponent applies to the negative sign as well as to the 2, but in $-2^4 = -(2^4)$, the exponent applies only to the 2. So,

$$(-2)^4 = 16 \quad \text{and} \quad -2^4 = -16.$$

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STUDY TIP

Rarely in algebra is there only one way to solve a problem. Don't be concerned if the steps you use to solve a problem are not exactly the same as the steps presented in this text. The important thing is to use steps that you understand and, of course, steps that are justified by the rules of algebra. For instance, you might prefer the following steps for Example 2(d).

$$\left(\frac{3x^2}{y}\right)^{-2} = \left(\frac{y}{3x^2}\right)^2 = \frac{y^2}{9x^4}$$

Note how Property 3 is used in the first step of this solution. The fractional form of this property is

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m.$$

The properties of exponents listed on the preceding page apply to *all* integers m and n, not just to positive integers. For instance, by Property 2, you can write

$$\frac{3^4}{3^{-5}} = 3^{4-(-5)} = 3^{4+5} = 3^9.$$

Example 1 ▶ Using Properties of Exponents

Use the properties of exponents to simplify each expression.

a. $(-3ab^4)(4ab^{-3})$ **b.** $(2xy^2)^3$ **c.** $3a(-4a^2)^0$ **d.** $\left(\frac{5x^3}{y}\right)^2$

Solution

a. $(-3ab^4)(4ab^{-3}) = (-3)(4)(a)(a)(b^4)(b^{-3}) = -12a^2b$

b. $(2xy^2)^3 = 2^3(x)^3(y^2)^3 = 8x^3y^6$

c. $3a(-4a^2)^0 = 3a(1) = 3a, \quad a \neq 0$

d. $\left(\frac{5x^3}{y}\right)^2 = \frac{5^2(x^3)^2}{y^2} = \frac{25x^6}{y^2}$

Example 2 ▶ Rewriting the Positive Exponents

Rewrite each expression with positive exponents.

a. x^{-1} **b.** $\frac{1}{3x^{-2}}$ **c.** $\frac{12a^3b^{-4}}{4a^{-2}b}$ **d.** $\left(\frac{3x^2}{y}\right)^{-2}$

Solution

a. $x^{-1} = \frac{1}{x}$ Property 3

b. $\frac{1}{3x^{-2}} = \frac{1(x^2)}{3} = \frac{x^2}{3}$ The exponent -2 does not apply to 3.

c. $\frac{12a^3b^{-4}}{4a^{-2}b} = \frac{12a^3 \cdot a^2}{4b \cdot b^4}$ Property 3

$= \frac{3a^5}{b^5}$ Property 1

d. $\left(\frac{3x^2}{y}\right)^{-2} = \frac{3^{-2}(x^2)^{-2}}{y^{-2}}$ Properties 5 and 7

$= \frac{3^{-2}x^{-4}}{y^{-2}}$ Property 6

$= \frac{y^2}{3^2x^4}$ Property 3

$= \frac{y^2}{9x^4}$ Simplify.

Technology

You can use a calculator to evaluate expressions with exponents. For instance, evaluate -3^{-2} as follows.

Scientific:

3 [+/−] [y^x] 2 [+/−] [=]

Graphing:

[(−)] 3 [^] [(−)] 2

The display will be as follows.

−.1111111111

Scientific Notation

Exponents provide an efficient way of writing and computing with very large (or very small) numbers. For instance, there are about 359 billion billion gallons of water on Earth—that is, 359 followed by 18 zeros.

$$359{,}000{,}000{,}000{,}000{,}000{,}000$$

It is convenient to write such numbers in **scientific notation.** This notation has the form $c \times 10^n$, where $1 \le c < 10$ and n is an integer. So, the number of gallons of water on Earth can be written in scientific notation as

$$3.59 \times 100{,}000{,}000{,}000{,}000{,}000{,}000 = 3.59 \times 10^{20}.$$

The *positive* exponent 20 indicates that the number is *large* (10 or more) and that the decimal point has been moved 20 places. A *negative* exponent indicates that the number is *small* (less than 1). For instance, the mass (in grams) of one electron is approximately

$$9.0 \times 10^{-28} = 0.\underbrace{0000000000000000000000000009}_{\text{28 decimal places}}.$$

Example 3 ▶ Scientific Notation

Write the number in scientific notation.

a. 0.0000782 **b.** 836,100,000

Solution

a. $0.0000782 = 7.82 \times 10^{-5}$

b. $836{,}100{,}000 = 8.361 \times 10^{8}$

Example 4 ▶ Decimal Notation

Write the number in decimal notation.

a. 9.36×10^{-6} **b.** 1.345×10^{2}

Solution

a. $9.36 \times 10^{-6} = 0.00000936$ **b.** $1.345 \times 10^{2} = 134.5$

Technology

Most calculators automatically switch to scientific notation when they are showing large (or small) numbers that exceed the display range.

To *enter* numbers in scientific notation, your calculator should have an exponential entry key labeled

EE or EXP.

Consult the user's guide for your calculator for instructions on keystrokes and how numbers in scientific notation are displayed.

Simplifying Radicals

An expression involving radicals is in **simplest form** when the following conditions are satisfied.

1. All possible factors have been removed from the radical.
2. All fractions have radical-free denominators (accomplished by a process called *rationalizing the denominator*).
3. The index of the radical is reduced.

To simplify a radical, factor the radicand into factors whose exponents are multiples of the index. The roots of these factors are written outside the radical and the "leftover" factors make up the new radicand.

STUDY TIP

When you simplify a radical, it is important that both expressions are defined for the same values of the variable. For instance, in Example 7(b), $\sqrt{75x^3}$ and $5x\sqrt{3x}$ are both defined only for nonnegative values of x. Similarly, in Example 7(c), $\sqrt[4]{(5x)^4}$ and $5|x|$ are both defined for all real values of x.

Example 7 ▶ Simplifying Even Roots

a. $\sqrt[4]{48} = \sqrt[4]{\overset{\text{Perfect 4th power}}{\overset{\downarrow}{16}} \cdot \overset{\text{Leftover factor}}{\overset{\downarrow}{3}}} = \sqrt[4]{2^4 \cdot 3} = 2\sqrt[4]{3}$

b. $\sqrt{75x^3} = \sqrt{\overset{\text{Perfect square}}{\overset{\downarrow}{25x^2}} \cdot \overset{\text{Leftover factor}}{\overset{\downarrow}{3x}}}$ Find largest square factor.

$= \sqrt{(5x)^2 \cdot 3x}$

$= 5x\sqrt{3x}$ Find root of perfect square.

c. $\sqrt[4]{(5x)^4} = |5x| = 5|x|$

In Example 7(b), the expression $\sqrt{75x^3}$ makes sense only for nonnegative values of x.

Example 8 ▶ Simplifying Odd Roots

a. $\sqrt[3]{24} = \sqrt[3]{\overset{\text{Perfect cube}}{\overset{\downarrow}{8}} \cdot \overset{\text{Leftover factor}}{\overset{\downarrow}{3}}} = \sqrt[3]{2^3 \cdot 3} = 2\sqrt[3]{3}$

b. $\sqrt[3]{24a^4} = \sqrt[3]{\overset{\text{Perfect cube}}{\overset{\downarrow}{8a^3}} \cdot \overset{\text{Leftover factor}}{\overset{\downarrow}{3a}}}$ Find largest cube factor.

$= \sqrt[3]{(2a)^3 \cdot 3a}$

$= 2a\sqrt[3]{3a}$ Find root of perfect cube.

c. $\sqrt[3]{-40x^6} = \sqrt[3]{(-8x^6) \cdot 5}$ Find largest cube factor.

$= \sqrt[3]{(-2x^2)^3 \cdot 5}$

$= -2x^2\sqrt[3]{5}$ Find root of perfect cube.

Radical expressions can be combined (added or subtracted) if they are **like radicals**—that is, if they have the same index and radicand. For instance, $\sqrt{2}$, $3\sqrt{2}$, and $\frac{1}{2}\sqrt{2}$ are like radicals, but $\sqrt{3}$ and $\sqrt{2}$ are unlike radicals. To determine whether two radicals can be combined, you should first simplify each radical.

Example 9 ▶ **Combining Radicals**

a.

$$\begin{aligned} 2\sqrt{48} - 3\sqrt{27} &= 2\sqrt{16 \cdot 3} - 3\sqrt{9 \cdot 3} && \text{Find square factors.} \\ &= 8\sqrt{3} - 9\sqrt{3} && \text{Find square roots.} \\ &= (8 - 9)\sqrt{3} && \text{Combine like terms.} \\ &= -\sqrt{3} && \text{Simplify.} \end{aligned}$$

b.

$$\begin{aligned} \sqrt[3]{16x} - \sqrt[3]{54x^4} &= \sqrt[3]{8 \cdot 2x} - \sqrt[3]{27 \cdot x^3 \cdot 2x} && \text{Find cube factors.} \\ &= 2\sqrt[3]{2x} - 3x\sqrt[3]{2x} && \text{Find cube roots.} \\ &= (2 - 3x)\sqrt[3]{2x} && \text{Combine like terms.} \end{aligned}$$

Rationalizing Denominators and Numerators

To rationalize a denominator or numerator of the form $a - b\sqrt{m}$ or $a + b\sqrt{m}$, multiply both numerator and denominator by a **conjugate:** $a + b\sqrt{m}$ and $a - b\sqrt{m}$ are conjugates of each other. If $a = 0$, then the rationalizing factor for $\sqrt{m}$ is itself, $\sqrt{m}$. For cube roots, choose a rationalizing factor that generates a perfect cube.

Example 10 ▶ **Rationalizing Single-Term Denominators**

Rationalize the denominator of each expression.

a. $\dfrac{5}{2\sqrt{3}}$ **b.** $\dfrac{2}{\sqrt[3]{5}}$

Solution

a.

$$\begin{aligned} \frac{5}{2\sqrt{3}} &= \frac{5}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} && \sqrt{3} \text{ is rationalizing factor.} \\ &= \frac{5\sqrt{3}}{2(3)} \\ &= \frac{5\sqrt{3}}{6} \end{aligned}$$

b.

$$\begin{aligned} \frac{2}{\sqrt[3]{5}} &= \frac{2}{\sqrt[3]{5}} \cdot \frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}} && \sqrt[3]{5^2} \text{ is rationalizing factor.} \\ &= \frac{2\sqrt[3]{5^2}}{\sqrt[3]{5^3}} \\ &= \frac{2\sqrt[3]{25}}{5} \end{aligned}$$

Example 11 ► Rationalizing a Denominator with Two Terms

$$\begin{aligned}
\frac{2}{3+\sqrt{7}} &= \frac{2}{3+\sqrt{7}} \cdot \frac{3-\sqrt{7}}{3-\sqrt{7}} && \text{Multiply numerator and denominator by conjugate of denominator.}\\
&= \frac{2\left(3-\sqrt{7}\right)}{3(3)+3\left(-\sqrt{7}\right)+\sqrt{7}(3)-\left(\sqrt{7}\right)\left(\sqrt{7}\right)} && \text{Use Distributive Property.}\\
&= \frac{2\left(3-\sqrt{7}\right)}{(3)^2-\left(\sqrt{7}\right)^2} && \text{Simplify.}\\
&= \frac{2\left(3-\sqrt{7}\right)}{9-7} && \text{Square terms of denominator.}\\
&= \frac{2\left(3-\sqrt{7}\right)}{2} = 3-\sqrt{7} && \text{Simplify.}
\end{aligned}$$

Sometimes it is necessary to rationalize the numerator of an expression. For instance, in Section A.4 you will use the technique shown in the next example to rationalize the numerator of an expression from calculus.

Example 12 ► Rationalizing a Numerator

$$\begin{aligned}
\frac{\sqrt{5}-\sqrt{7}}{2} &= \frac{\sqrt{5}-\sqrt{7}}{2} \cdot \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} && \text{Multiply numerator and denominator by conjugate of numerator.}\\
&= \frac{\left(\sqrt{5}\right)^2-\left(\sqrt{7}\right)^2}{2\left(\sqrt{5}+\sqrt{7}\right)} && \text{Simplify.}\\
&= \frac{5-7}{2\left(\sqrt{5}+\sqrt{7}\right)} && \text{Square terms of numerator.}\\
&= \frac{-2}{2\left(\sqrt{5}+\sqrt{7}\right)} = \frac{-1}{\sqrt{5}+\sqrt{7}} && \text{Simplify.}
\end{aligned}$$

STUDY TIP

Do not confuse the expression $\sqrt{5}+\sqrt{7}$ with the expression $\sqrt{5+7}$. In general, $\sqrt{x+y}$ does not equal $\sqrt{x}+\sqrt{y}$. Similarly, $\sqrt{x^2+y^2}$ does not equal $x+y$.

Rational Exponents

Definition of Rational Exponents

If a is a real number and n is a positive integer such that the principal nth root of a exists, then $a^{1/n}$ is defined as

$a^{1/n} = \sqrt[n]{a}$, where $1/n$ is the **rational exponent** of a.

Moreover, if m is a positive integer that has no common factor with n, then

$$a^{m/n} = \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m \quad \text{and} \quad a^{m/n} = \left(a^m\right)^{1/n} = \sqrt[n]{a^m}.$$

The symbol ∫ indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

STUDY TIP

Rational exponents can be tricky, and you must remember that the expression $b^{m/n}$ is not defined unless $\sqrt[n]{b}$ is a real number. This restriction produces some unusual-looking results. For instance, the number $(-8)^{1/3}$ is defined because $\sqrt[3]{-8} = -2$, but the number $(-8)^{2/6}$ is undefined because $\sqrt[6]{-8}$ is not a real number.

The numerator of a rational exponent denotes the *power* to which the base is raised, and the denominator denotes the *index* or the *root* to be taken.

Power

Index

$$b^{m/n} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

When you are working with rational exponents, the properties of integer exponents still apply. For instance,

$$2^{1/2}2^{1/3} = 2^{(1/2)+(1/3)} = 2^{5/6}.$$

Example 13 ▶ Changing from Radical to Exponential Form

a. $\sqrt{3} = 3^{1/2}$

b. $\sqrt{(3xy)^5} = \sqrt[2]{(3xy)^5} = (3xy)^{(5/2)}$

c. $2x\sqrt[4]{x^3} = (2x)(x^{3/4}) = 2x^{1+(3/4)} = 2x^{7/4}$

Example 14 ▶ Changing from Exponential to Radical Form

a. $(x^2 + y^2)^{3/2} = \left(\sqrt{x^2 + y^2}\right)^3 = \sqrt{(x^2 + y^2)^3}$

b. $2y^{3/4}z^{1/4} = 2(y^3z)^{1/4} = 2\sqrt[4]{y^3z}$

c. $a^{-3/2} = \dfrac{1}{a^{3/2}} = \dfrac{1}{\sqrt{a^3}}$ **d.** $x^{0.2} = x^{1/5} = \sqrt[5]{x}$

Rational exponents are useful for evaluating roots of numbers on a calculator, for reducing the index of a radical, and for simplifying expressions in calculus.

Technology

There are four methods of evaluating radicals on most graphing calculators. For square roots, you can use the square root key $\boxed{\sqrt{\ }}$. For cube roots, you can use the cube root key $\boxed{\sqrt[3]{\ }}$. For other roots, you can first convert the radical to exponential form and then use the exponential key $\boxed{\wedge}$, or you can use the *n*th root key $\boxed{\sqrt[x]{\ }}$.

Example 15 ▶ Simplifying with Rational Exponents

a. $(-32)^{-4/5} = \left(\sqrt[5]{-32}\right)^{-4} = (-2)^{-4} = \dfrac{1}{(-2)^4} = \dfrac{1}{16}$

b. $(-5x^{5/3})(3x^{-3/4}) = -15x^{(5/3)-(3/4)} = -15x^{11/12}, \quad x \neq 0$

c. $\sqrt[9]{a^3} = a^{3/9} = a^{1/3} = \sqrt[3]{a}$ Reduce index.

d. $\sqrt[3]{\sqrt{125}} = \sqrt[6]{125} = \sqrt[6]{(5)^3} = 5^{3/6} = 5^{1/2} = \sqrt{5}$

e.
$$\begin{aligned}(2x-1)^{4/3}(2x-1)^{-1/3} &= (2x-1)^{(4/3)-(1/3)}\\ &= 2x - 1, \quad x \neq \frac{1}{2}\end{aligned}$$

f.
$$\begin{aligned}\frac{x-1}{(x-1)^{-1/2}} &= \frac{x-1}{(x-1)^{-1/2}} \cdot \frac{(x-1)^{1/2}}{(x-1)^{1/2}}\\ &= \frac{(x-1)^{3/2}}{(x-1)^0}\\ &= (x-1)^{3/2}, \quad x \neq 1\end{aligned}$$

A.2 Exercises

In Exercises 1–4, write the expression as a repeated multiplication problem.

1. 8^5 **2.** $(-2)^7$
3. -0.4^6 **4.** 11.3^4

In Exercises 5–8, write the expression using exponential notation.

5. $(4.9)(4.9)(4.9)(4.9)(4.9)(4.9)$
6. $(2\sqrt{5})(2\sqrt{5})(2\sqrt{5})(2\sqrt{5})$
7. $(-10)(-10)(-10)(-10)(-10)$
8. $-\left(\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}\right)$

In Exercises 9–16, evaluate each expression.

9. (a) $3^2 \cdot 3$ (b) $3 \cdot 3^3$
10. (a) $\dfrac{5^5}{5^2}$ (b) $\dfrac{3^2}{3^4}$
11. (a) $(3^3)^2$ (b) -3^2
12. (a) $(2^3 \cdot 3^2)^2$ (b) $\left(-\frac{3}{5}\right)^3\left(\frac{5}{3}\right)^2$
13. (a) $\dfrac{3 \cdot 4^{-4}}{3^{-4} \cdot 4^{-1}}$ (b) $32(-2)^{-5}$
14. (a) $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}}$ (b) $(-2)^0$
15. (a) $2^{-1} + 3^{-1}$ (b) $(2^{-1})^{-2}$
16. (a) $3^{-1} + 2^{-2}$ (b) $(3^{-2})^2$

In Exercises 17–20, use a calculator to evaluate the expression. (If necessary, round your answer to three decimal places.)

17. $(-4)^3(5^2)$ **18.** $(8^{-4})(10^3)$
19. $\dfrac{3^6}{7^3}$ **20.** $\dfrac{4^3}{3^{-4}}$

In Exercises 21–28, evaluate the expression for the given value of *x*.

	Expression	*Value*
21.	$-3x^3$	2
22.	$7x^{-2}$	4
23.	$6x^0$	10
24.	$5(-x)^3$	3
25.	$2x^3$	-3
26.	$-3x^4$	-2
27.	$4x^2$	$-\frac{1}{2}$
28.	$5(-x)^3$	$\frac{1}{3}$

In Exercises 29–34, simplify each expression.

29. (a) $(-5z)^3$ (b) $5x^4(x^2)$
30. (a) $(3x)^2$ (b) $(4x^3)^2$
31. (a) $6y^2(2y^4)^2$ (b) $\dfrac{3x^5}{x^3}$
32. (a) $(-z)^3(3z^4)$ (b) $\dfrac{25y^8}{10y^4}$
33. (a) $\dfrac{7x^2}{x^3}$ (b) $\dfrac{12(x+y)^3}{9(x+y)}$
34. (a) $\dfrac{r^4}{r^6}$ (b) $\left(\dfrac{4}{y}\right)^3\left(\dfrac{3}{y}\right)^4$

In Exercises 35–40, rewrite each expression with positive exponents and simplify.

35. (a) $(x+5)^0, \quad x \neq -5$ (b) $(2x^2)^{-2}$
36. (a) $(2x^5)^0, \quad x \neq 0$ (b) $(z+2)^{-3}(z+2)^{-1}$
37. (a) $(-2x^2)^3(4x^3)^{-1}$ (b) $\left(\dfrac{x}{10}\right)^{-1}$
38. (a) $(4y^{-2})(8y^4)$ (b) $\left(\dfrac{x^{-3}y^4}{5}\right)^{-3}$
39. (a) $3^n \cdot 3^{2n}$ (b) $\left(\dfrac{a^{-2}}{b^{-2}}\right)\left(\dfrac{b}{a}\right)^3$
40. (a) $\dfrac{x^2 \cdot x^n}{x^3 \cdot x^n}$ (b) $\left(\dfrac{a^{-3}}{b^{-3}}\right)\left(\dfrac{a}{b}\right)^3$

In Exercises 41–44, write the number in scientific notation.

41. Land area of Earth: 57,300,000 square miles
42. Light year: 9,460,000,000,000 kilometers
43. Relative density of hydrogen: 0.0000899 gram per cubic centimeter
44. One micron (millionth of a meter): 0.00003937 inch

In Exercises 45–48, write the number in decimal notation.

45. Worldwide daily consumption of Coca-Cola: 4.568×10^9 servings (Source: The Coca-Cola Company)

46. Interior temperature of the sun: 1.5×10^7 degrees Celsius

47. Charge of an electron: 1.602×10^{-19} coulomb

48. Width of a human hair: 9.0×10^{-5} meter

In Exercises 49 and 50, evaluate each expression without using a calculator.

49. (a) $\sqrt{25 \times 10^8}$ (b) $\sqrt[3]{8 \times 10^{15}}$

50. (a) $(1.2 \times 10^7)(5 \times 10^{-3})$ (b) $\dfrac{(6.0 \times 10^8)}{(3.0 \times 10^{-3})}$

In Exercises 51–54, use a calculator to evaluate each expression. (Round your answer to three decimal places.)

51. (a) $750\left(1 + \dfrac{0.11}{365}\right)^{800}$

(b) $\dfrac{67{,}000{,}000 + 93{,}000{,}000}{0.0052}$

52. (a) $(9.3 \times 10^6)^3(6.1 \times 10^{-4})$

(b) $\dfrac{(2.414 \times 10^4)^6}{(1.68 \times 10^5)^5}$

53. (a) $\sqrt{4.5 \times 10^9}$ (b) $\sqrt[3]{6.3 \times 10^4}$

54. (a) $(2.65 \times 10^{-4})^{1/3}$ (b) $\sqrt{9 \times 10^{-4}}$

In Exercises 55–66, fill in the missing form of the expression.

	Radical Form	*Rational Exponent Form*
55.	$\sqrt{9}$	
56.	$\sqrt[3]{64}$	
57.		$32^{1/5}$
58.		$-(144^{1/2})$
59.		$196^{1/2}$
60.	$\sqrt[3]{614.125}$	
61.	$\sqrt[3]{-216}$	
62.		$(-243)^{1/5}$
63.		$27^{2/3}$
64.	$(\sqrt[4]{81})^3$	
65.	$\sqrt[4]{81^3}$	
66.		$16^{5/4}$

In Exercises 67–74, evaluate each expression without using a calculator.

67. (a) $\sqrt{9}$ (b) $\sqrt[3]{8}$

68. (a) $\sqrt{49}$ (b) $\sqrt[3]{\frac{27}{8}}$

69. (a) $(\sqrt[3]{-125})^3$ (b) $27^{1/3}$

70. (a) $\sqrt[4]{562^4}$ (b) $36^{3/2}$

71. (a) $32^{-3/5}$ (b) $\left(\frac{16}{81}\right)^{-3/4}$

72. (a) $100^{-3/2}$ (b) $\left(\frac{9}{4}\right)^{-1/2}$

73. (a) $\left(-\dfrac{1}{64}\right)^{-1/3}$ (b) $\left(\dfrac{1}{\sqrt{32}}\right)^{-2/5}$

74. (a) $\left(-\dfrac{125}{27}\right)^{-1/3}$ (b) $-\left(\dfrac{1}{125}\right)^{-4/3}$

In Exercises 75–78, use a calculator to approximate the number. (Round your answer to three decimal places.)

75. (a) $\sqrt{57}$ (b) $\sqrt[5]{-27^3}$

76. (a) $\sqrt[3]{45^2}$ (b) $\sqrt[6]{125}$

77. (a) $(-12.4)^{-1.8}$ (b) $(5\sqrt{3})^{-2.5}$

78. (a) $\dfrac{7 - (4.1)^{-3.2}}{2}$ (b) $\left(\dfrac{13}{3}\right)^{-3/2} - \left(-\dfrac{3}{2}\right)^{13/3}$

In Exercises 79–84, simplify by removing all possible factors from each radical.

79. (a) $\sqrt{8}$ (b) $\sqrt[3]{24}$

80. (a) $\sqrt[3]{\frac{16}{27}}$ (b) $\sqrt{\frac{75}{4}}$

81. (a) $\sqrt{72x^3}$ (b) $\sqrt{\dfrac{18^2}{z^3}}$

82. (a) $\sqrt{54xy^4}$ (b) $\sqrt{\dfrac{32a^4}{b^2}}$

83. (a) $\sqrt[3]{16x^5}$ (b) $\sqrt{75x^2y^{-4}}$

84. (a) $\sqrt[4]{(3x^2)^4}$ (b) $\sqrt[5]{96x^5}$

In Exercises 85–88, perform the operations and simplify.

85. $\dfrac{(2x^2)^{3/2}}{2^{1/2}x^4}$ **86.** $\dfrac{x^{4/3}y^{2/3}}{(xy)^{1/3}}$

87. $\dfrac{x^{-3} \cdot x^{1/2}}{x^{3/2} \cdot x^{-1}}$ **88.** $\dfrac{5^{-1/2} \cdot 5x^{5/2}}{(5x)^{3/2}}$

In Exercises 89–92, rationalize the denominator of the expression. Then simplify your answer.

89. $\dfrac{1}{\sqrt{3}}$ **90.** $\dfrac{5}{\sqrt{10}}$

91. $\dfrac{2}{5 - \sqrt{3}}$ **92.** $\dfrac{3}{\sqrt{5} + \sqrt{6}}$

∫ **In Exercises 93–96, rationalize the numerator of the expression. Then simplify your answer.**

93. $\dfrac{\sqrt{8}}{2}$

94. $\dfrac{\sqrt{2}}{3}$

95. $\dfrac{\sqrt{5}+\sqrt{3}}{3}$

96. $\dfrac{\sqrt{7}-3}{4}$

In Exercises 97 and 98, reduce the index of each radical.

97. (a) $\sqrt[4]{3^2}$ (b) $\sqrt[6]{(x+1)^4}$

98. (a) $\sqrt[6]{x^3}$ (b) $\sqrt[4]{(3x^2)^4}$

In Exercises 99 and 100, write each expression as a single radical. Then simplify your answer.

99. (a) $\sqrt{\sqrt{32}}$ (b) $\sqrt{\sqrt[4]{2x}}$

100. (a) $\sqrt{\sqrt{243(x+1)}}$ (b) $\sqrt{\sqrt[3]{10a^7b}}$

In Exercises 101–106, simplify each expression.

101. (a) $2\sqrt{50}+12\sqrt{8}$ (b) $10\sqrt{32}-6\sqrt{18}$

102. (a) $4\sqrt{27}-\sqrt{75}$ (b) $\sqrt[3]{16}+3\sqrt[3]{54}$

103. (a) $5\sqrt{x}-3\sqrt{x}$ (b) $-2\sqrt{9y}+10\sqrt{y}$

104. (a) $8\sqrt{49x}-14\sqrt{100x}$
(b) $-3\sqrt{48x^2}+7\sqrt{75x^2}$

105. (a) $3\sqrt{x+1}+10\sqrt{x+1}$
(b) $7\sqrt{80x}-2\sqrt{125x}$

106. (a) $-\sqrt{x^3-7}+5\sqrt{x^3-7}$
(b) $11\sqrt{245x^3}-9\sqrt{45x^3}$

In Exercises 107–110, complete the statement with <, =, or >.

107. $\sqrt{5}+\sqrt{3}$ $\quad$ $\sqrt{5+3}$

108. $\sqrt{\dfrac{3}{11}}$ $\quad$ $\dfrac{\sqrt{3}}{\sqrt{11}}$

109. 5 $\quad$ $\sqrt{3^2+2^2}$

110. 5 $\quad$ $\sqrt{3^2+4^2}$

111. ***Period of a Pendulum*** The period T (in seconds) of a pendulum is

$$T = 2\pi\sqrt{\frac{L}{32}}$$

where L is the length of the pendulum (in feet). Find the period of a pendulum whose length is 2 feet.

112. ***Erosion*** A stream of water moving at the rate of v feet per second can carry particles of size $0.03\sqrt{v}$ inches. Find the size of the largest particle that can be carried by a stream flowing at the rate of $\frac{3}{4}$ foot per second.

113. ***Mathematical Modeling*** A funnel is filled with water to a height of h centimeters. The time t (in seconds) for the funnel to empty is

$$t = 0.03[12^{5/2} - (12-h)^{5/2}], \quad 0 \le h \le 12.$$

(a) Use the *table* feature of a graphing utility to find the times required for the funnel to empty for water heights of $h = 0, h = 1, h = 2, \ldots$ $h = 12$ centimeters.

(b) Is there a limiting value of time required for the water to empty as the height of the water becomes closer to 12 centimeters? Explain.

114. ***Speed of Light*** The speed of light is approximately 11,180,000 miles per minute. The distance from the sun to Earth is approximately 93,000,000 miles. Find the time for light to travel from the sun to Earth.

Synthesis

True or False? **In Exercises 115 and 116, determine whether the statement is true or false. Justify your answer.**

115. $\dfrac{x^{k+1}}{x} = x^k$

116. $(a^n)^k = a^{n^k}$

117. Verify that $a^0 = 1, a \ne 0$. (*Hint:* Use the property of exponents $a^m/a^n = a^{m-n}$.)

118. Explain why each of the following pairs is not equal.

(a) $(3x)^{-1} \ne \dfrac{3}{x}$ (b) $y^3 \cdot y^2 \ne y^6$

(c) $(a^2b^3)^4 \ne a^6b^7$ (d) $(a+b)^2 \ne a^2+b^2$

(e) $\sqrt{4x^2} \ne 2x$ (f) $\sqrt{2}+\sqrt{3} \ne \sqrt{5}$

119. ***Exploration*** List all possible digits that occur in the units place of the square of a positive integer. Use that list to determine whether $\sqrt{5233}$ is an integer.

120. ***Think About It*** Square the real number $2/\sqrt{5}$ and note that the radical is eliminated from the denominator. Is this equivalent to rationalizing the denominator? Why or why not?

The symbol ∫ indicates an example or exercise that highlights algebraic techniques specifically used in calculus.

The symbol indicates an exercise or parts of an exercise in which you are instructed to use a graphing utility.