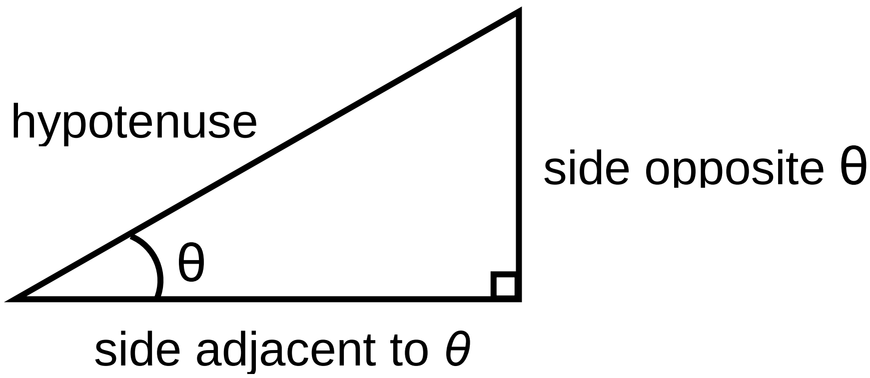
Right Angle Trigonometry

Look at the right triangle:



The six trigonometric functions, **sine, cosine, tangent, cotangent, secant, and cosecant,** can be defined as follows.

Let θ be an acute angle of a right triangle. Then

$$\text{sine}\,\theta = \sin\theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

$$\text{cosine}\,\theta = \cos\theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

$$\text{tangent}\,\theta = \tan\theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta} = \frac{\text{opp}}{\text{adj}}$$

$$\text{cosecant}\,\theta = \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\text{secant}\,\theta = \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\text{cotangent}\,\theta = \cot\theta = \frac{\text{adj}}{\text{opp}}$$

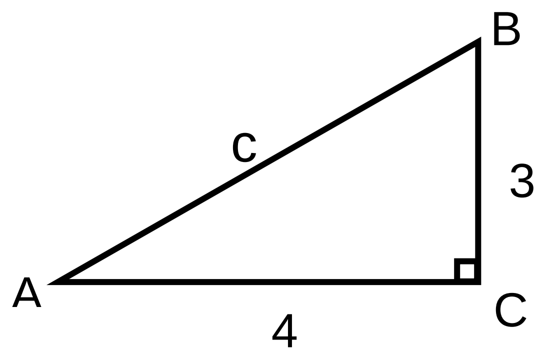
In general, we have:

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta = \frac{\text{hyp}}{\text{opp}}$
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}}$
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$

**Notice that <u>sine</u> and <u>cosecant</u> are reciprocals,
<u>cosine</u> and <u>secant</u> are reciprocals,
and <u>tangent</u> and <u>cotangent</u> are reciprocals.

SOHCAHTOA!!!

Example: Find the hypotenuse and then find all 6 trigonometric values for both angles.



1. Find the hypotenuse by the Pythagorean Theorem.

$$c^2 = 3^2 + 4^2$$

$$c^2 = 9 + 16$$

$$c^2 = 25$$

$$c = 5$$

2.

$$\sin A = \frac{opp}{hyp} = \frac{3}{5} \qquad \sin B = \frac{opp}{hyp} = \frac{4}{5}$$

$$\cos A = \frac{adj}{hyp} = \frac{4}{5} \qquad \cos B = \frac{adj}{hyp} = \frac{3}{5}$$

$$\tan A = \frac{opp}{adj} = \frac{3}{4} \qquad \tan B = \frac{opp}{adj} = \frac{4}{3}$$

$$\csc A = \frac{hyp}{opp} = \frac{5}{3} \qquad \csc B = \frac{hyp}{opp} = \frac{5}{4}$$

$$\sec A = \frac{hyp}{adj} = \frac{5}{4} \qquad \sec B = \frac{hyp}{adj} = \frac{5}{3}$$

$$\cot A = \frac{adj}{opp} = \frac{4}{3} \qquad \cot B = \frac{adj}{opp} = \frac{3}{4}$$

<u>Notice that</u>:

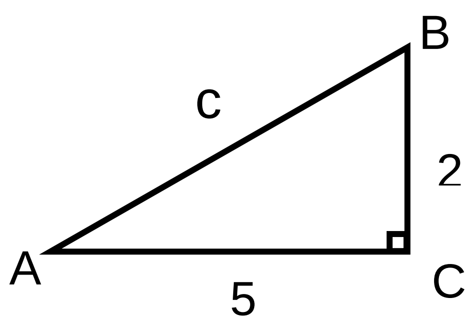
sin A = cos B
cos A = sin B

and

sec A = csc B
csc A = sec B

<u>Note</u>: Sine and Cosine of complementary angles are =.
Secant and Cosecant of complementary angles are =.
Tangent and Cotangent of complementary angles are =.
Example: sin 60° = cos 30°

Example: Find the hypotenuse and then find the sine, cosine and tangent for both angles.



1. Find the hypotenuse by the Pythagorean Theorem.

$$c^2 = 2^2 + 5^2$$
$$c^2 = 4 + 25$$
$$c^2 = 29$$
$$c = \sqrt{29}$$

2.

$$\sin A = \frac{opp}{hyp} = \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$

$$\cos A = \frac{adj}{hyp} = \frac{5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$$

$$\tan A = \frac{opp}{adj} = \frac{2}{5}$$

$$\sin B = \frac{opp}{hyp} = \frac{5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$$

$$\cos B = \frac{adj}{hyp} = \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$

$$\tan B = \frac{opp}{adj} = \frac{5}{2}$$

Calculator Conversion

Fractional parts of degrees can be denoted as decimal degrees or as degrees, minutes and seconds.

$$1° = 60' \text{ (minutes)}$$
$$1' = 60'' \text{ (seconds)}$$

This also means that, $1' = \frac{1}{60}°$

$1'' = \frac{1}{60}'$ or $\frac{1}{3600}°$

example: 34° 15′ 30″

*To convert to decimal degrees:

$$34 + \frac{15}{60} + \frac{30}{3600} = 34.2583°$$

On your graphing calculator:

Enter 34° 15′ 30″. Use the [ANGLE] menu for ° and ′ and [ALPHA] [+] for the ″. Press [MATH] [►Dec] [ENTER] to convert to decimal degrees.

Example: Convert 68° 22′ 46″ to decimal degrees.

answer: 68.3794°

Example: Convert 43° 21′ 13″ to decimal degrees.

answer: 43.3536°

<u>To convert decimal degrees to degrees, minutes, and seconds (DMS):</u>

Enter the decimal degree. Press [ANLGE] [►DMS] [ENTER] to convert. Round the seconds to the nearest second.

Example: Convert 8.875° to degrees, minutes and seconds.

answer: 8° 52′ 30″

Example: Convert 56.674° to degrees, minutes and seconds.

answer: 56.674° - 56° = .674°
.674° · 60 = 40.44’
.44’ · 60 =26”

answer: 56° 40′ 26″

Evaluating Trig Functions with a Calculator

*Before using your calculator to evaluate trig functions, make sure the MODE is set to match the angle unit you are using (DEGREE or RADIAN).

DEGREE Mode

Example: Set in Degree Mode and find:

(a) cos 15.3°

[COS] 15.3 [ENTER] ≈ 0.9646

(b) sin 24° 32′

[SIN] (24° 32′) [ENTER] ≈ .4152

or [SIN] $(24 + \frac{32}{60})$ [ENTER] ≈ .4152

More examples: Find the following.

a) tan 45.67° answer: 1.0237
b) sin 65° answer: .9063
c) cos 40° 32′ 23″ answer: .7599
d) sin 14° 20’ answer: .2476

<u>Finding the angle when given the Trig value</u>

If sin θ = .5, we know that θ must be 30°.

On the calculator: [2^{nd}] [SIN] (.5)[ENTER] = 30°.

Use [SIN] if you know the angle and want the sine.
Use [SIN^{-1}] if you know the sine and want the angle.

Example: Solve for θ in the following:

a) sin θ = 0.8145 answer: 54.5°
b) cos θ = 0.9848 answer: 10.0°
c) tan θ = 0.3125 answer: 17.4°

If we know the sine of an angle, but not the angle, we use the inverse function called <u>arcsine</u>, symbolized by arcsin x or $\sin^{-1} x$.

Both mean "What angle has the sine of x?"

arcsin(sin x) = x and sin(arcsinx) = x

Solving Right Triangles

When solving a right triangle, you are usually given one side of a right triangle and one of the acute angles and are asked to find one of the other sides, or you are given two sides and are asked to find one of the acute angles.

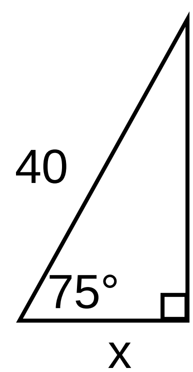
Example: Solve for x in the following right triangle.

Look at what values you are given and how they are related.

$$\cos x = \frac{adj}{hyp}$$

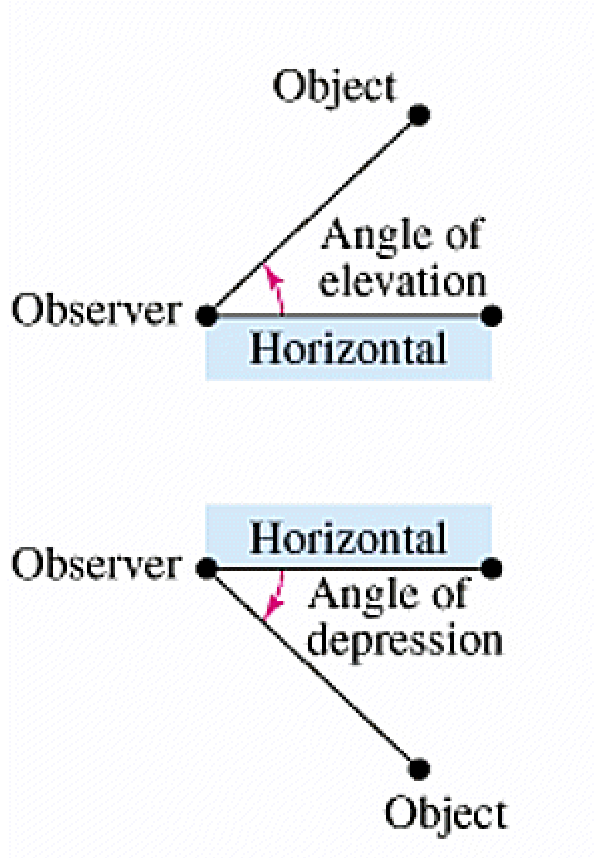
$$\cos 75° = \frac{x}{40}$$

$$x = 40\cos 75° \approx 10.35$$

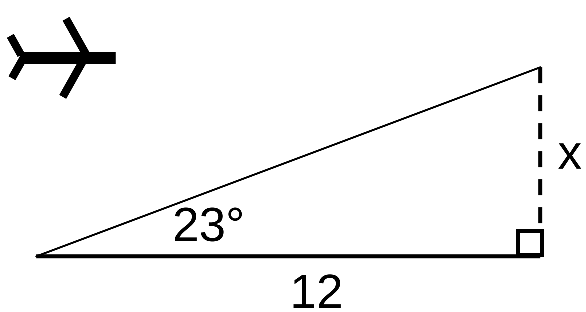


- The term **angle of elevation** means the angle from the horizontal upward to an object.

- The term **angle of depression** means for objects that lie below the horizontal, the angle from the horizontal downward to an object.



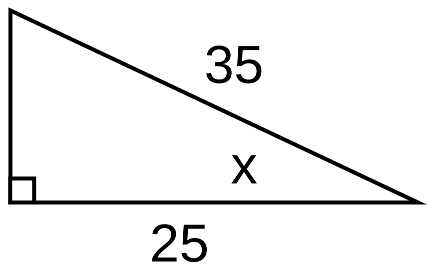
Example: A plane flying over level ground will pass directly over a radar antenna. It is 12 miles on the ground from the antenna to the point directly under the plane and the angle of elevation from that point on the ground to the top of the antenna is 23°. Find the altitude at which the plane is flying.



$$\tan 23° = \frac{x}{12}$$
$$x = 12\tan 23° \approx 5.09$$

The plane is flying at 5.09 miles.

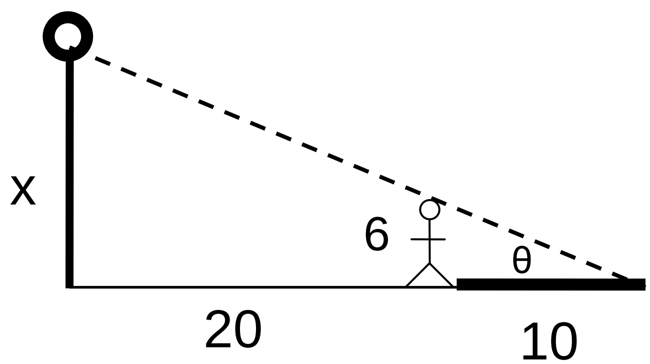
Example: If a rope tied to the top of a flagpole is 35 feet long, then what angle is formed by the rope and the ground when the rope is pulled to the ground, 25 feet from the base of the pole?



$$\cos x = \frac{25}{35} = 0.7143$$

$$x = \cos^{-1}(0.7143) \approx 44.4°$$

Example: A six-foot man standing 20 feet from a street light casts a 10-foot shadow. How tall is the street light?



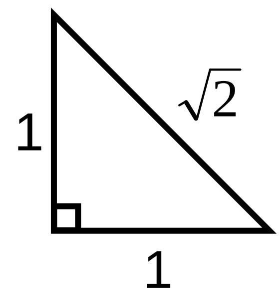
We know from the smaller triangle that tan θ = $\frac{6}{10}$ = 0.6.

Using the larger triangle we get:

$$\tan\theta = \frac{x}{30} \;\Rightarrow\; 0.6 = \frac{x}{30} \;\Rightarrow\; x = 30(0.6) = 18$$

Special Right Triangles

45°- 45° Triangle:



$$c^2 = 1^2 + 1^2$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

(If the 2 angles are equal, then the 2 legs must be equal. You can use whatever you want for the legs as long as they are equal.)

$$\sin 45° = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

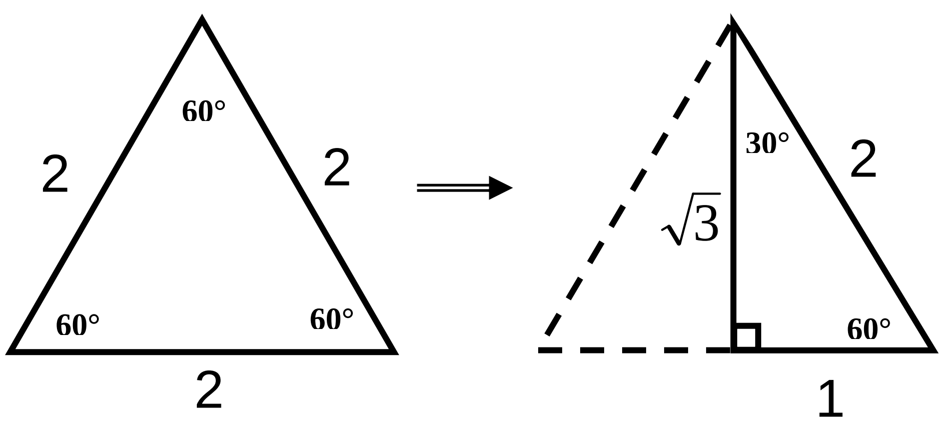
$$\cos 45° = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{opp}{adj} = \frac{1}{1} = 1$$

Memorize these!!!!

30°- 60° Triangle:

Start with an equilateral triangle. It does not matter what the lengths of the sides are.



$$2^2 = 1^2 + b^2$$
$$4 = 1 + b^2$$
$$b^2 = 3$$
$$b = \sqrt{3}$$

$$\sin 30° = \frac{opp}{hyp} = \frac{1}{2} \qquad \sin 60° = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{adj}{hyp} = \frac{1}{2}$$

$$\tan 30° = \frac{opp}{adj} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan 60° = \frac{opp}{adj} = \frac{\sqrt{3}}{1}$$

Memorize these!!!!