Definitions:

section 1.1

set – a collection of objects

elements – objects in the set

2 ways to describe elements:

List method: C = {spatula, scraper, whisk, spoon}

Set-builder notation: C = {x| x is a cooking utensil}

Set Notation

Let B = {2, 4, 6, 8}

We can say that $4 \in \{2, 4, 6, 8\}$ or $4 \in B$

We can also say that $7 \notin B$

$\in$ means "is an element of"
$\notin$ means "is not an element of"

<u>Subsets</u>

Let $A = \{1, 2, 3, 4\}$ $P = \{5\}$
$N = \{2, 3\}$ $Q = \{3, 5\}$

$N \subseteq A \Rightarrow$ "N is a subset of A" since all elements in set N are in set A.

$P \not\subset A \Rightarrow$ "P is not a subset of A" since the element(s) in P are not in set A.

$P \subseteq P \Rightarrow$ Any set is a subset of itself.

$\{\ \} \subseteq Q \Rightarrow$ The empty set (null set) is a subset of every set. (also written $\varnothing$)

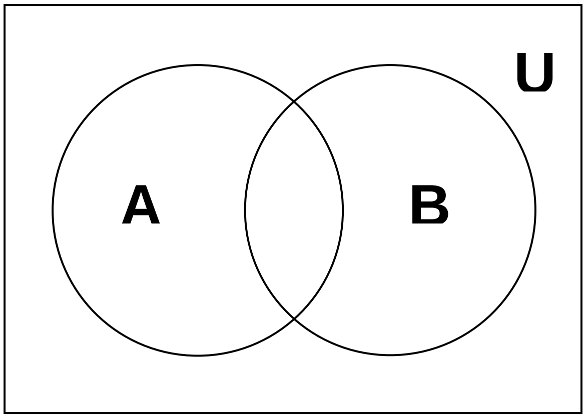
$N \subset A \Rightarrow$ "N is a proper subset of A" since the set is a subset of A but not the same as A.

Definitions:

<u>equal</u> - 2 sets are <u>equal</u> if they are the same set (ie. same elements)

<u>equivalent</u> – 2 sets are <u>equivalent</u> if they have the same number of elements

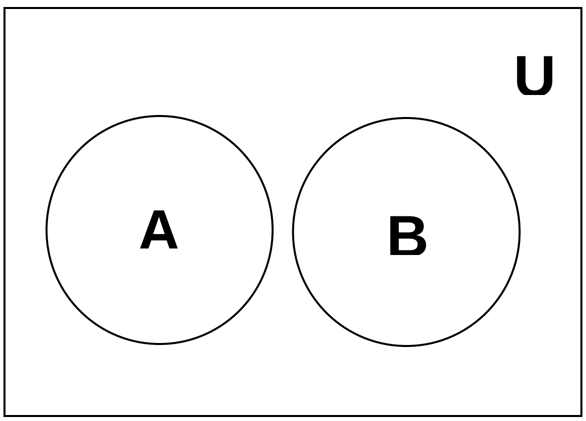
<u>universal set</u> – the <u>universal set</u> is denoted by U and it contains all the elements being considered for a particular problem.



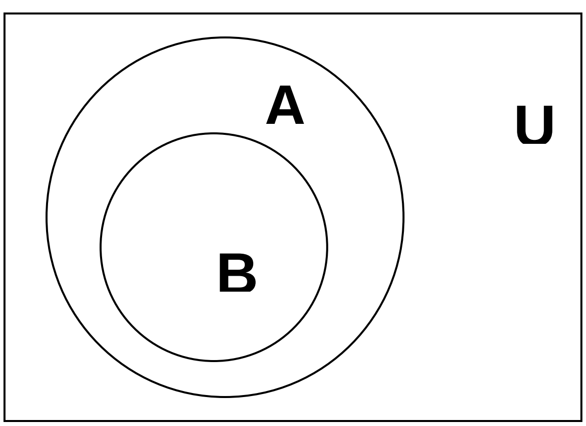
U = {Students in CHAT}

A = {CHAT students in Speech}

B = {CHAT students in Geometry}



There are no students in both Speech and Geometry



All students in Geometry are also in Speech.

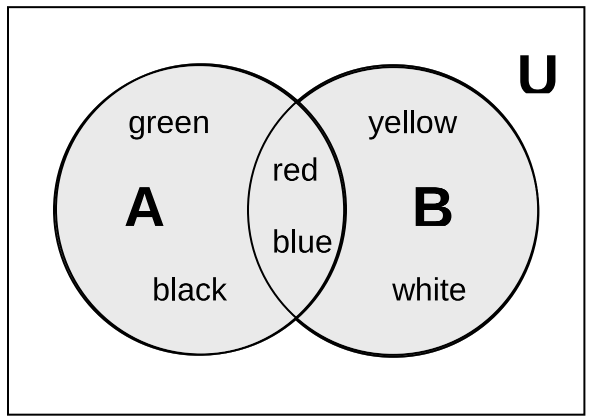
Definitions:

<u>union</u> – The <u>union</u> of 2 sets is the set containing all of the elements of both sets.

A U B = {x| x∈A or x∈B}

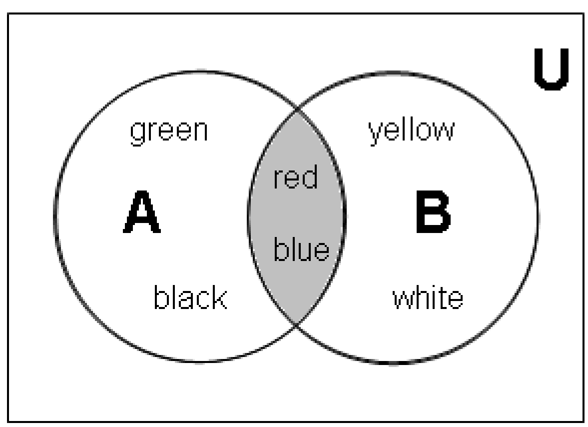
Let A = {red, blue, green, black}
Let B = {yellow, white, red, blue}

A U B = {red, blue, green, black, yellow, white}



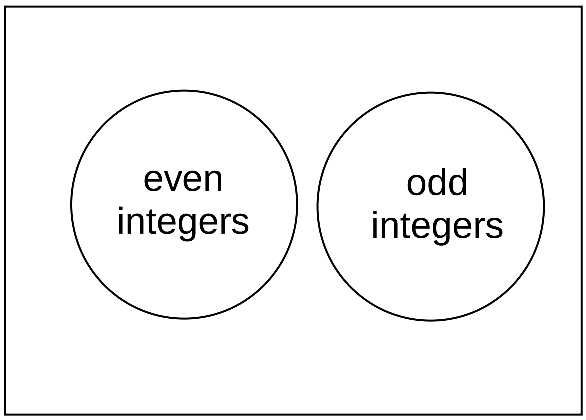
<u>intersection</u> – the <u>intersection</u> of 2 sets is the set that contains elements that are in both sets. A ∩ B = {x| x∈A and x∈B}

A ∩ B = {red, blue}



Definitions:

<u>disjoint sets</u> – sets that have no elements in common



$\{\text{even integers}\} \cap \{\text{odd integers}\} = \{\ \}$ or $\varnothing$

<u>complement</u> – the <u>complement</u> of a set is the set of all elements in the universal set that are not in the original set.

$A' = \{x \mid x \in U \text{ and } x \notin A\}$

<u>Note</u>: $A \cup A' = U$

$A \cap A' = \varnothing$

A and A′ are always disjoint!

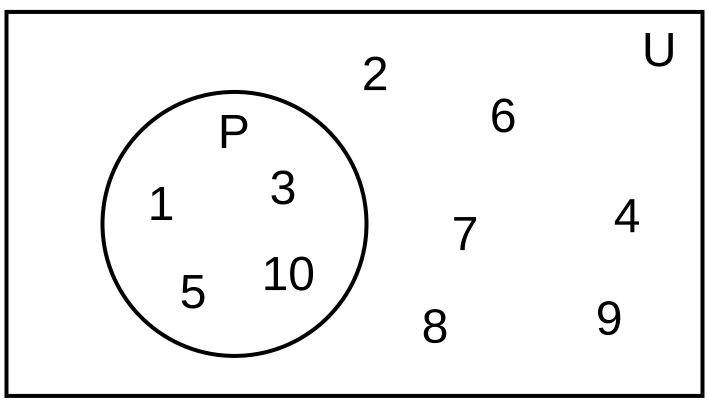
Sample Problem:

Let U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
P = {1, 3, 5, 10}

1. Find P′

P′ = {2, 4, 6, 7, 8, 9}

2. Write the Venn diagram representation.



Sample Problem:

Let A = {1, 2, 3, 4} B = {3, 4, 5} U = {1, 2, 3, 4, 5, 6}

1. Find A ∩ B′

answer: {1, 2}

2. Find (A′ ∪ B) ′

answer: {1, 2}

*For intersections using Venn diagrams, shade both sets and the look for the double-shaded part.

Definitions:

binary operations – done on 2 sets (eg. intersection and union)

unary operations – done on 1 set (eg. complements)

Definitions must be:

section 1.3

1. **Clear**. The definition must communicate the point and state the term being defined. Avoid vague or ambiguous language.

2. **Useful**. The definition must use only words that have been previously defined or are commonly accepted as undefined.

3. **Precise**. The definition must be accurate and reversible. Identify the class to which the object belongs and its distinguishing characteristics.

4. **Concise**. The definition must be a good sentence and use good grammar. Stick to the point and avoid unnecessary words.

5. **Objective**. The definition must be neutral. Avoid emotional words, figures of speech, and limitations of time or place.

Practice:

Tell the criteria that are missing in these poor definitions:

1. A baboon is a monkey.

    Imprecise since it is not reversible.

2. A sprint is an ambulatory motion at an individual's maximal rate for a limited duration.

    Useless – circular, definition involves more difficult vocabulary than the term itself.

3. A trowel is a small hand-held shovel for digging in the garden with.

    Not concise due to redundant words (small, hand-held) and poor grammar (ends with preposition).

4. Reporters are deceivers who censor truth and sensationalize stories to promote the liberal agenda.

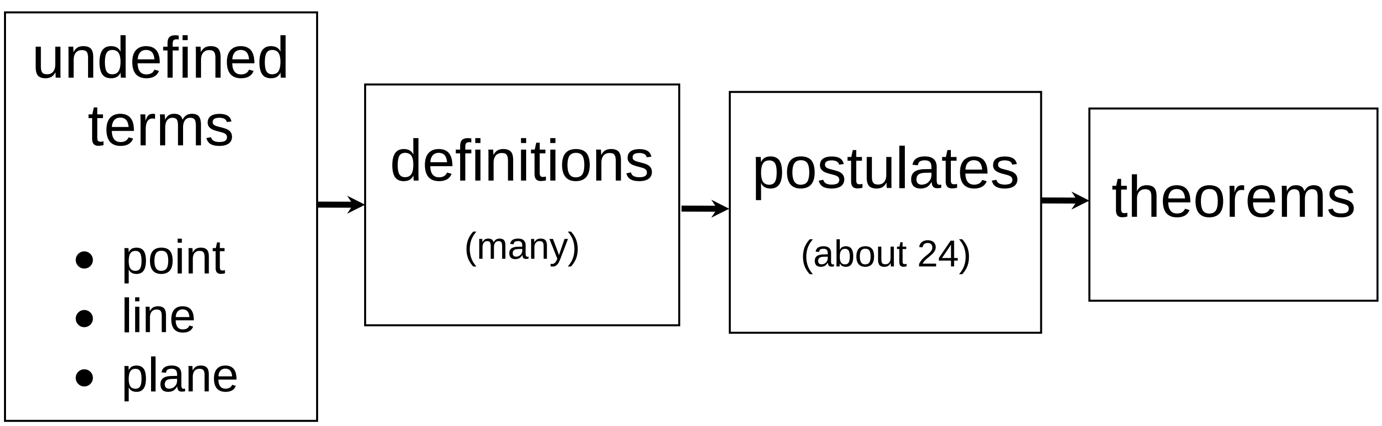
    Not objective – many emotion-laden words deceivers, censor, sensationalize, liberal)

5. A schism divides people.

Unclear – does it saw them in half, or confine them in separate cells, or make them uncooperative?

section 1.4

Geometry has 3 undefined words: point, line, plane.



Definitions:

collinear points – points that lie on the same line

noncollinear points – points that do not lie on the same line

concurrent lines – lines that intersect at a single point

coplanar points – points that lie in the same plane

noncoplanar points – points that do not lie in the same plane

coplanar lines – lines that lie in the same plane

parallel lines – coplanar lines that do not intersect

skew lines – lines that are not coplanar

parallel planes – planes in space that do not intersect

postulates – statements which are assumed to be true without proof (using both defined and undefined terms) They show relationships between defined and undefined terms.

theorems – statements that can be shown true by a logical progression of previous terms and statements.

undefined terms → building blocks for definitions

postulates → building blocks for theorems

Note: We also use definitions and other theorems to prove theorems.

Postulates cannot be proved.

Theorems must be proved.

Practice problem:

Undefined terms: puppy, dog, mammal

Postulates: 1. Dogs are mammals.
2. Dogs bear live young called puppies.
3. All mammals nurse their young.

What conclusions (theorems) can be proved logically (deduced) from these?

Theorem 1: Dogs nurse their puppies

Theorem 2: Some mammals bear live young.

Definition: Pulis are long-haired dogs from Hungary.

What new theorems are there?

Theorem 3: Pulis are mammals.

Theorem 4: Pulis bear live young called puppies.

Theorem 5: Pulis nurse their puppies.

Note: Theorem 5 follows from the definition and Theorem 1.