

Definition:

A proof is a system of reasoning or argument to convince a person of the truth of a statement.

*Do not accept everything that you hear or read as truth without some solid evidence that it is true! (Just because you read it on the internet does not make it true!)

2 Main Types of Reasoning:

Inductive Reasoning is an argument to establish that a statement is probably true.

Deductive Reasoning is an argument to establish that a statement is absolutely certain.

Example Problems: Classify the arguments as inductive or deductive.

1. All squares are rectangles and all rectangles are polygons. Therefore all squares are polygons.

Answer: The conclusion is definitely true if the premises are. The argument is deductive.

2. A meteorologist studied weather patterns for the last ten years in his city. Based on the trends, he predicts a high of 25° F in January.

Answer: The conclusion may be probable but it is not certain. The argument is inductive.

******Inductive reasoning** usually involves instances and examples that show a pattern or lend credibility to what you are trying to prove.

Example: Scientists who conduct lab experiments gather data and results that lead to a reasonable conclusion. The more evidence, the more likely the result will be valid.

*Mathematicians must prove conclusions deductively.
He uses a sequence of logical steps.

Terminology:

In a reasoned argument, the supporting statements are called premises.

The statement that they support is called the conclusion.

Definitions:

An argument is valid if the reasoning proceeds logically from the premises to the conclusion.

An argument is sound if it is valid and the premises are true.

Example:

- 1. All dogs are mammals.**
- 2. Fido is a dog.**
- 3. Therefore, Fido is a mammal.**

- The argument is deductive.
- The first 2 statements are the premises.
- Statement 3 is the conclusion.
- It is valid because the logic is good (proceeds logically).
- It is sound because the premises are true.

Using Symbolic Logic

- Example:**
1. If a number is an integer, then it is a rational number.
 2. If a number is a rational number, then it is a real number.
 3. Therefore, if a number is an integer, then it is a real number.

Let p : A number is an integer.

Let q : A number is a rational number.

Let r : A number is a real number.

Our premises are $p \rightarrow q$ and $q \rightarrow r$.

Then our conclusion of $p \rightarrow r$ is valid logically.

Sample Problems: Classify each argument as valid or invalid, and then decide if it is sound or unsound.

1. All dogs are horses. Fido is a dog. Therefore, Fido is a horse.

This is a valid argument, but because the first premise is false, the argument is unsound, and the conclusion is false.

2. All dogs are mammals. Fido is a mammal.
Therefore Fido is a dog.

This is invalid (Fido could be a cat.) Since it is invalid, it is automatically unsound even though the premises are both true.

Let p: It is a dog.
Let q: It is a mammal.
Let r: It's name is Fido.

The above argument says $p \rightarrow q$ and $r \rightarrow q$, but that does not mean that $r \rightarrow p$.

3. All horses are mammals. Fido is a horse.
Therefore Fido is a mammal.

This argument is valid and sound.

Counterexamples

**You can show that a conclusion is false by giving just one counterexample.

Definition:

A counterexample is a specific instance that fits the hypothesis of a conditional statement but not the conclusion.

Example: There are no even prime numbers.

Counterexample: The number 2 is prime and is even.

******Just because you can't find a counterexample, it doesn't mean the statement is automatically true.

Inductive Types of Proofs:

1. Appeals to tendencies because of statistical premises.
2. Lack of counterexample
3. Appeals to authority
4. Appeals to experience
5. Appeals to analogy.

****Note:** Because we will be focusing on deductive proofs, only a couple of the types of inductive proofs have been discussed (numbers 1 & 2).

In the study of reasoning, you must be aware that validity and truth are two different things. An argument that is valid can result in a false conclusion, and an argument that is invalid can result in a true conclusion. You must be able to distinguish valid arguments from true conclusions.

Look at:

1. Pigs are dirty animals.
2. Cows provide milk.
3. Therefore rural areas have a low population density.

This conclusion is true and the premises are true, but the argument is invalid because there is no logical connection between premises and the conclusion. Since it is not valid, it is automatically unsound.

*****When we use poor reasoning in an argument, we commit a fallacy.**

- Every invalid argument contains a fallacy.
- Every unsound argument contains either a logical fallacy or a false premise.

Fallacies Common to Mathematical Proofs:

1. **Hasty generalizations** – jumping to conclusions based on insufficient evidence
2. **Circular Argument** – when you assume what you are trying to prove
3. **Accident** – misapplying a general principle for a situation for which it was not intended.
(e.g. applying measurement to a sketch. Just because it measures to be 90° do not assume an angle = 90° unless that fact is given.)

Summary:

V = valid, S = sound, I = invalid, U = unsound

Premises	Reasoning	Argument
T	V	S
T	I	U
F	V	U
F	I	U

Types of Deductive Reasoning Proofs:

1. Proof by direct argument
2. Proof by contradiction

**There are many kinds of direct proofs, but we will only study 4 types.

Types of Deductive Arguments that we will study:

1. Law of Deduction
2. Modus ponens
3. Modus tollens
4. Transitivity

Definitions:

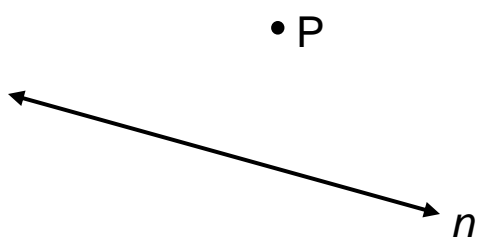
The Law of Deduction is a method of deductive proof with the following symbolic form:

\mathbf{p}	(assumed premise)
$\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \dots, \mathbf{q}_n$	(statements known to be true)
\mathbf{r}	(deduced from the statements above)
$\mathbf{p} \rightarrow \mathbf{r}$	(conclusion)

***This is what most of our proofs will look like.

Example:

Theorem 1.2: If n is a line and P is a point not on n , then n and P are contained in exactly one plane.



Statement	Reason
1. n is a line, P is a point, and P is not on n .	1. Assumption
2. Line n contains two points R and Q .	2. Expansion Postulate
3. P, Q, R are noncollinear	3. Steps 1 & 2 with def. of noncollinear
4. P, Q, R are contained in exactly one plane	4. Plane Postulate
5. If n is a line, and P a point not on n , then there is exactly one plane containing P and n .	5. Law of Deduction

Definition:

Modus ponens is a method of deductive proof with the following symbolic form:

premise 1: $p \rightarrow q$
premise 2: p
conclusion: q

Example:

premise 1: If there is a blizzard today, CHAT will be cancelled.
premise 2: There is a blizzard today.
conclusion: CHAT will be cancelled.

**In Law of Deduction proofs, we start out with p and try to add statements that lead us to the conclusion of q . In Modus ponens, we start out with $p \rightarrow q$, then show p is true in order to get our conclusion of q .

Definition:

Modus tollens is a method of deductive proof with the following symbolic form:

premise 1: $p \rightarrow q$

premise 2: $\sim q$

conclusion: $\sim p$

(Remember that if $p \rightarrow q$ is true, then the contrapositive $\sim q \rightarrow \sim p$ is also true.)

Definition:

Transitivity is a method of deductive proof with the following symbolic form:

premise 1: $p \rightarrow q$

premise 2: $q \rightarrow r$

conclusion: $p \rightarrow r$

***2 Common Fallacies come from misusing modus ponens and modus tollens. They are assuming the converse or inverse is true.

Example: All dogs are mammals.
Fido is a mammal.
Therefore Fido is a dog.

In symbols: $d \rightarrow m$
 m
 $\therefore d$

*This fallacy is assuming the converse.

Example: All dogs are mammals.
Fido is not a dog.
Therefore Fido is not a mammal.

In symbols: $d \rightarrow m$
 $\sim d$
 $\therefore \sim m$

*This fallacy is assuming the inverse.

Sample Problems: State the form of deductive reasoning in the following:

1. If it rains today, the ballgame will be cancelled. It rained. Therefore the ballgame was cancelled.

answer: modus ponens

2. If we eat carrots, then our eyesight will be good.
Our eyesight is not good. Therefore, we did not eat our carrots.

answer: modus tollens

3. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

answer: transitivity

4. Ocean water is grade A milk. Milk contains essential vitamins and minerals. Any beverage containing essential vitamins and minerals is nourishing. Ocean water is a nourishing beverage. Therefore, if ocean water is grade A milk, then ocean water is a nourishing beverage.

answer: Law of Deduction

(Note: This is valid but unsound because the first premise is not true.)

Constructions

Construction 6: Construct a perpendicular line through a point.

Given: \overleftrightarrow{AB} and a point P

1. Place the point of the compass at point P and make arcs that intersect \overleftrightarrow{AB} in two places. Call these points C and D .
2. Move the point of the compass first to C and then to D , making intersecting arcs in the half-plane that does not contain P (or either half plane if P is on the line). Call this point of intersection Q .
3. Draw PQ to form a line perpendicular to \overleftrightarrow{AB} through P .

Construction 7: Construct a 45° angle.

1. Draw \overleftrightarrow{AB} and construct a perpendicular line through B, thus forming four right angles.
2. Bisect one of the right angles, thus forming two 45° angles; $m\angle ABC = 45^\circ$

Construction 8: Construct an equilateral triangle (and thus a 60° angle).

1. Mark off any segment, \overline{AB} . Place the point of the compass at each endpoint and, using length AB, mark intersecting arcs above \overline{AB} . Call the point where the arcs intersect point C.
2. Draw \overline{AC} and \overline{BC} .

Note: $\angle A$, $\angle B$, and $\angle C$ are all 60° angles.