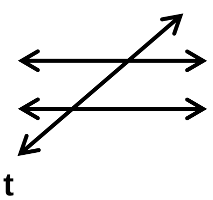
section 6.4

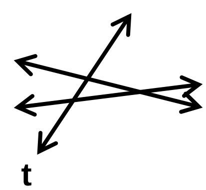
Definitions:

Two coplanar lines that do not intersect are parallel.

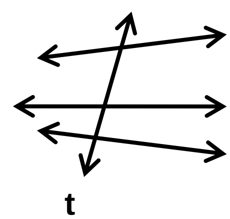
A transversal is a line that intersects two or more distinct coplanar lines in two or more distinct points.



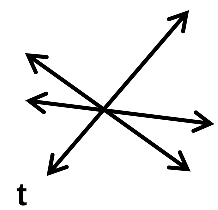
t is a transversal



t is a transversal

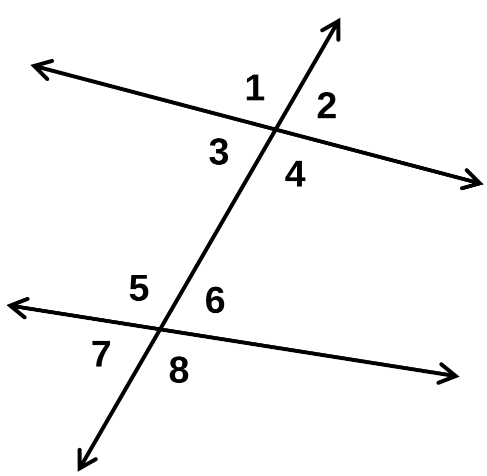


t is a transversal



t is not a transversal

Terms for angles formed when a transversal intersects 2 lines:



$\angle 3$ and $\angle 6$ are alternate interior angles

$\angle 1$ and $\angle 8$ are alternate exterior angles

$\angle 2$ and $\angle 6$ are corresponding angles

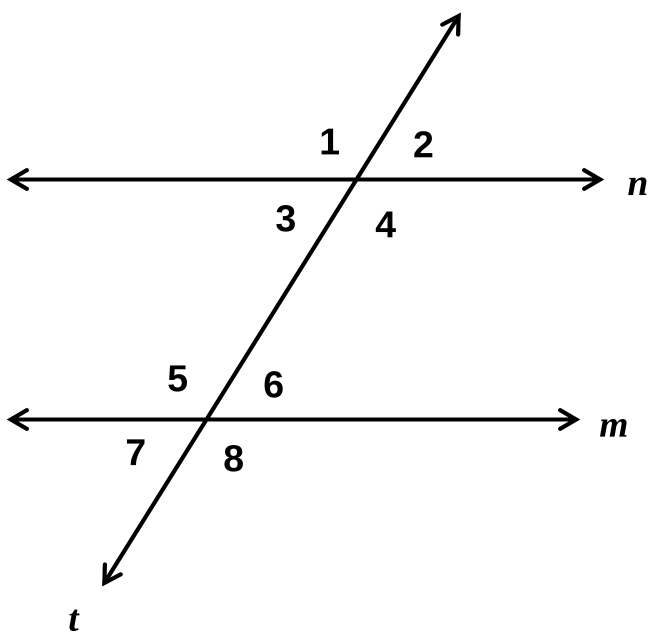
Definitions:

Alternate interior angles are angles which are on opposite sides of the transversal and between the other two lines.

Alternate exterior angles are angels on opposite sides of the transversal and outside the other two lines.

Corresponding angles are angles on the same side of the transversal and on the same side of their respective lines.

Sample Problem:



t is a transversal
n and *m* are parallel lines

1. Name the pairs of alternate interior angles.

   $\angle 3$ *and* $\angle 6$
   $\angle 4$ *and* $\angle 5$

2. Name all pairs of alternate exterior angles.

   $\angle 1$ *and* $\angle 8$
   $\angle 2$ *and* $\angle 7$

3. Name all pairs of corresponding angles.

   $\angle 1$ *and* $\angle 5$
   $\angle 2$ *and* $\angle 6$
   $\angle 3$ *and* $\angle 7$
   $\angle 4$ *and* $\angle 8$

4. Name all pairs of vertical angles.

   $\angle 1$ *and* $\angle 4$
   $\angle 2$ *and* $\angle 3$
   $\angle 5$ *and* $\angle 8$
   $\angle 6$ *and* $\angle 7$

5. What relationship seems to be true regarding alternate interior angles?

   They are congruent.

Parallel Postulate: Two lines intersected by a transversal are parallel if and only if the alternate interior angles are congruent.

Historic Parallel Postulate: Given a line and a point not on the line, there is exactly one line passing through the point that is parallel to the given line.

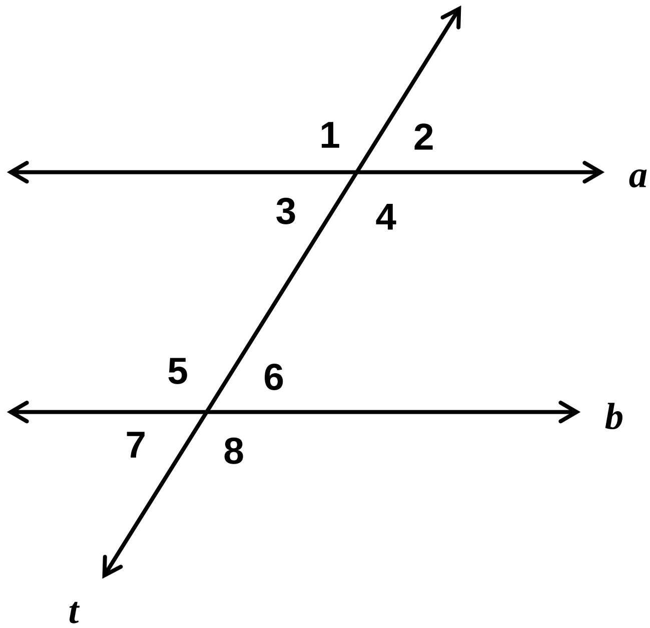
Theorem 6.12: Alternate Exterior Angle Theorem: Two lines intersected by a transversal are parallel if and only if the alternate exterior angles are congruent.

Theorem 6.13: Corresponding Angle Theorem: Two lines intersected by a transversal are parallel if and only if the corresponding angles are congruent.

***<u>Remember</u>: Biconditionals require 2 proofs – one for each conditional.

Example 1: Alternate Exterior Angle Theorem – part 1

If two parallel lines are intersected by a transversal, then the alternate exterior angles are congruent.

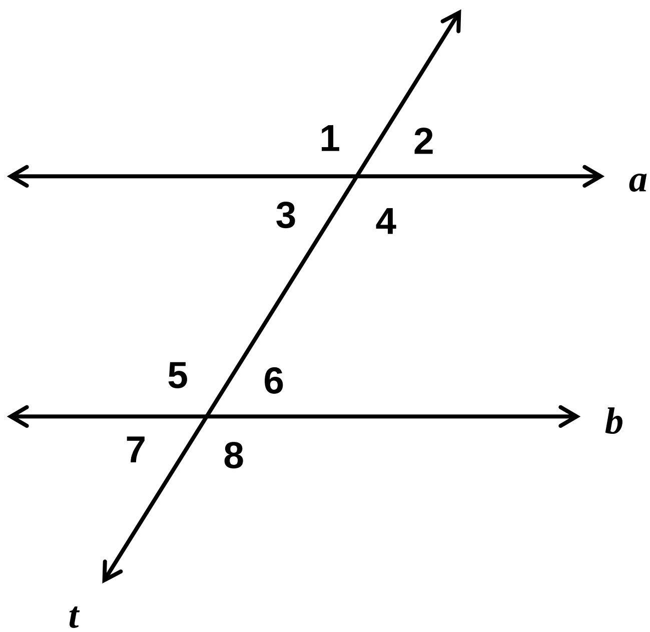


Given: $a \parallel b$ and t is a transversal that forms the eight angles shown.

Prove: $\angle 1 \cong \angle 8$

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Solution:



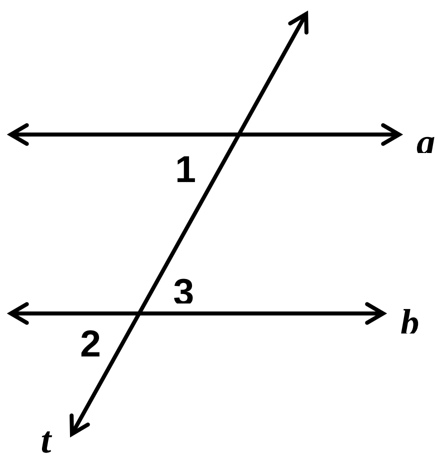
Given: $a \parallel b$ and t is a transversal that forms the eight angles shown.

Prove: $\angle 1 \cong \angle 8$

Statement	Reason
1. $a \parallel b$ and t is a transversal	1. Given
2. $\angle 4 \cong \angle 5$	2. Parallel Postulate
3. $\angle 1 \cong \angle 4$ and $\angle 8 \cong \angle 5$	3. Vertical Angle Theorem
4. $m\angle 4 = m\angle 5$; $m\angle 1 = m\angle 4$; and $m\angle 8 = m\angle 5$	4. Def. of congruent angles
5. $m\angle 1 = m\angle 8$	5. Substitution or transitive (step 4)
6. $\angle 1 \cong \angle 8$	6. Def. of congruent angles
7. If two parallel lines are intersected by a transversal, then the alternate exterior angles are congruent.	7. Law of Deduction

Example 2: Corresponding Angle Theorem – part 1

If a transversal intersects two lines such that the corresponding angles are congruent, then the two lines are parallel.



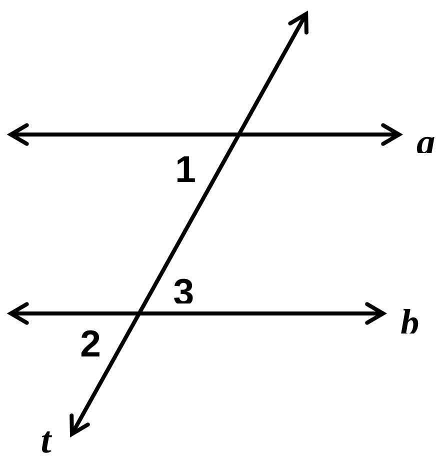
Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

Example 2: Corresponding Angle Theorem – part 1

If a transversal intersects two lines such that the corresponding angles are congruent, then the two lines are parallel.



Given: $\angle 1 \cong \angle 2$

Prove: $a \parallel b$

Statement	Reason
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 2 \cong \angle 3$	2. Vertical angles are equal
3. $\angle 1 \cong \angle 3$	3. Transitive
4. $a \parallel b$	4. Parallel Postulate
5. If a transversal intersects two lines such that the corresponding angles are congruent, then the two lines are parallel.	5. Law of Deduction

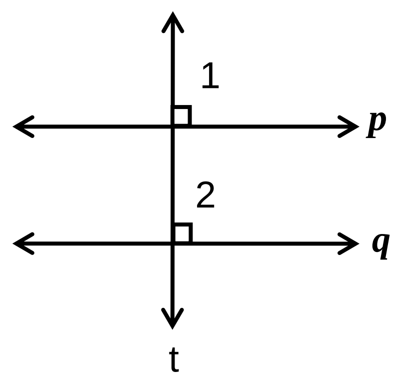
Theorem 6.14: If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other also.

Theorem 6.15: If two lines are perpendicular to the same line, then they are parallel to each other.

Proof of Theorem 6.15:

Given: $p \perp t$ and $q \perp t$

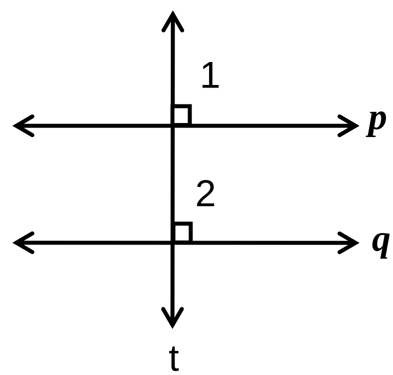
Prove: $p \parallel q$



Statement	Reason
1. $p \perp t$ and $q \perp t$	1. Given
2. $\angle 1$ and $\angle 2$ are right angles	2. Perp. lines intersect to form rt. angles
3. $\angle 1 \cong \angle 2$	3. All right angles are congruent (Thm 4.1)
4. $p \parallel q$	4. Corresponding Angle Theorem
5. If $p \perp t$ and $q \perp t$, then $p \parallel q$	5. Law of Deduction

Theorem 6.15: If two lines are perpendicular to the same line, then they are parallel to each other.

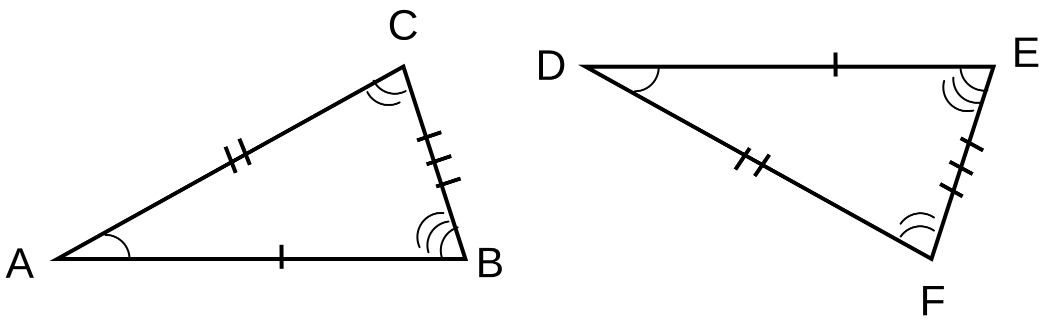
Proof of Theorem 6.15:



Given: $p \perp t$ and $q \perp t$

Prove: $p \parallel q$

Statement	Reason
1. $p \perp t$ and $q \perp t$	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.



These triangles are congruent because their corresponding angles and sides are congruent.

Terms:

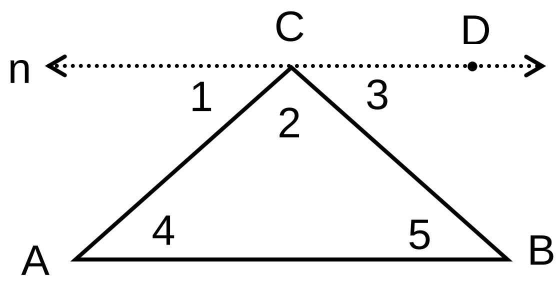
<u>included angle</u>: $\angle A$ is the included angle between $\overline{AC}$ and $\overline{AB}$.

<u>included side</u>: $\overline{DE}$ is the included side between $\angle D$ and $\angle E$.

Theorem 6.16: The sum of the measures of any triangle is 180°.

Proof: *Given*: ΔABC

Draw ΔABC and then draw a line that passes through C and is parallel to AB. Make the new line dotted to show that it is not a part of the given information

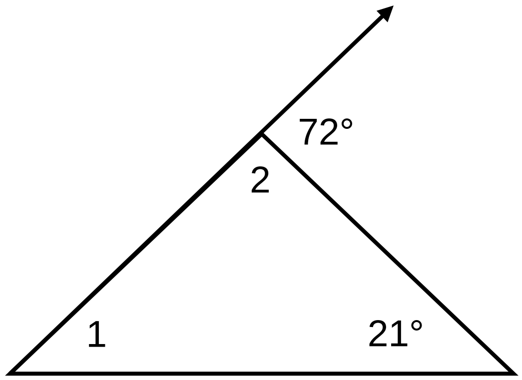


When we add to a drawing it is called an <u>auxiliary figure</u>. These are valid as long as the figure drawn can really exist.

Prove: m∠2+m∠4+m∠5=180°.

Statement	Reason
1. ΔABC; n ∥ $\overleftrightarrow{AB}$ through C	1. Given; auxiliary line
2. m∠2+m∠3 = m∠ACD	2. Angle Addition Postulate
3. ∠1 and ∠ACD are supplementary	3. Linear pairs are supplementary
4. m∠1 + m∠ACD = 180°	4. Def. of supplementary
5. m∠1+m∠2 +m∠3 = 180°	5. Substitution (step 2 into 4)
6. ∠1 ≅ ∠4 ∠3 ≅ ∠5	6. Parallel Postulate
7. m∠1 = m∠4 m∠3 = m∠5	7. Def. of congruent angles
8. m∠2+m∠4+m∠5=180°	8. Substitution (step 7 into 5)

Sample Problem:



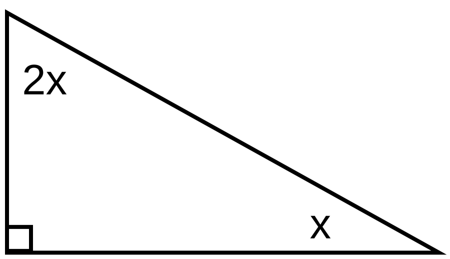
Find m∠1 and m∠2

Answers:

m∠2 = 180°- 72° = 108° since ∠2 and 72° are supplements

m∠1 = 180°- 21° - 108° = 51°

Sample Problem:

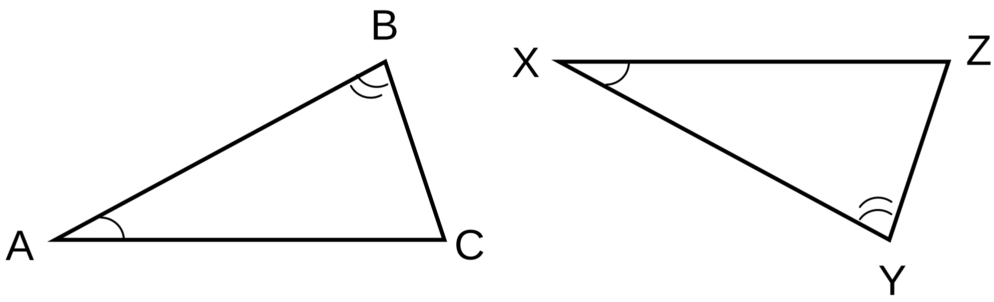


Find the value of x

Answer:

x + 2x + 90 = 180
3x + 90 = 180
3x = 90
x = 30

Theorem 6.17: If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

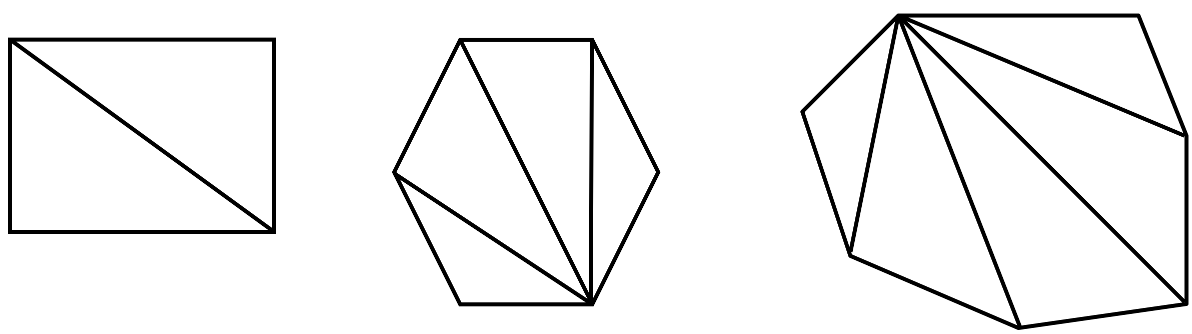


Given: $\angle A \cong \angle X$ and $\angle B \cong \angle Y$
Prove: $\angle C \cong \angle Z$

Statement	Reason
1. $\angle A \cong \angle X$ and $\angle B \cong \angle Y$	1. Given
2. $m\angle A+m\angle B+m\angle C=180$ $m\angle X+m\angle Y+m\angle Z=180$	2. Sum of meas. of angles in Δ =180°
3. $m\angle A+m\angle B+m\angle C=$ $m\angle X+m\angle Y+m\angle Z$	3. Transitive
4. $m\angle A=m\angle X$ $m\angle B=m\angle Y$	4. Def. of congruent angles
5. $m\angle A+m\angle B+m\angle C=$ $m\angle A+m\angle B+m\angle Z$	5. Substitution (step 4 into 3)
6. $m\angle C = m\angle Z$	6. Addition Property of Equality
7. $\angle C \cong \angle Z$	7. Def. of congruent angles
8. If $\angle A \cong \angle X$ and $\angle B \cong \angle Y$, then $\angle C \cong \angle Z$	8. Law of Deduction

Theorem 6.18: The acute angles of a right triangle are complementary

Divide the polygon into triangles. Since each triangle equals 180°, multiply the number of triangles times 180° for the total measure of all angles.



4 sides = 2 Δ's   6 sides = 4 Δ's   7 sides = 5 Δ's

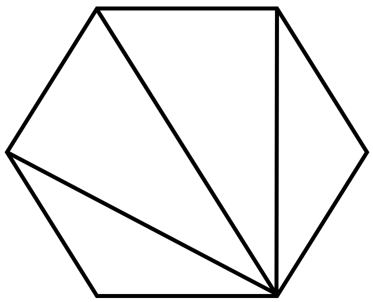
***In general, there are *n*-2 triangles formed if a figure has *n* sides.

***The total angle measure of a polygon of *n* sides is 180(*n*-2).

Interior Angle Measure:

If the polygon is regular, then to find the measure of each angle, take the total angle measure and divide it by the number of angles: $\frac{180(n-2)}{n}$

Example:



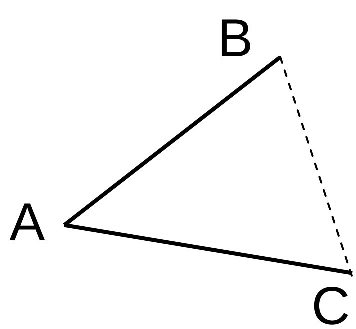
6 sides = 4 triangles
4 · 180 = 720

720° ÷ 6 angles = 120°

section 6.5

Up until now we have had to show all corresponding angles and sides are congruent to show that 2 triangles are congruent. We are now going to learn faster methods.

Look at:

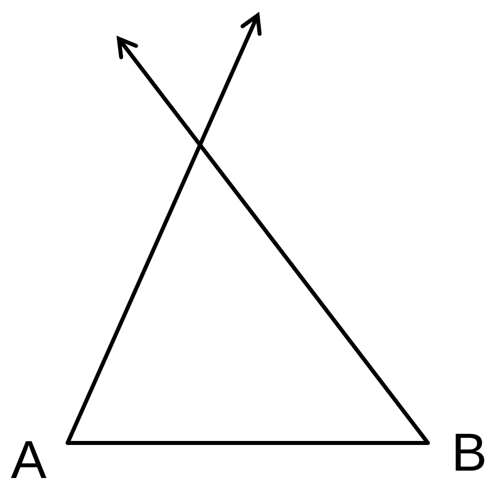


The size and shape of $\angle A$ and $\overline{AB}$ and $\overline{AC}$ determine the size and shape of ΔABC.

*If we draw another angle the same size as $\angle A$, with segments the same length as $\overline{AB}$ and $\overline{AC}$, the triangles will be congruent.

Postulate 6.2: (SAS Congruence Postulate) If 2 sides and an included angle of one triangle are congruent to the corresponding 2 sides and included angle of another triangle, then the two triangles are congruent.

Look at:



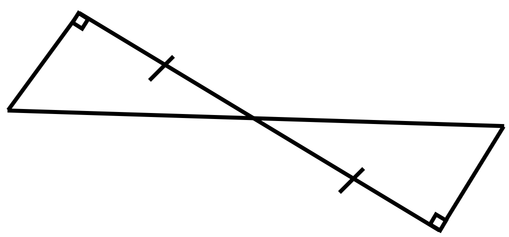
If $\overline{AB}$ is a set length, and $\angle A$ and $\angle B$ are set measures, then there is only one possible triangle.

*If we draw another segment the same length as $\overline{AB}$, and the angles at the endpoints of the segment the same size as $\angle A$ and $\angle B$, the triangles will be congruent.

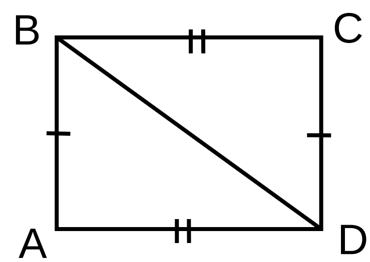
Postulate 6.3: (ASA Congruence Postulate) If 2 angles and an included side of one triangle are congruent to the corresponding 2 angles and the included side of another triangle, then the 2 triangles are congruent.

Sample Problems:

Which triangles can be shown congruent by SAS, ASA, or neither?

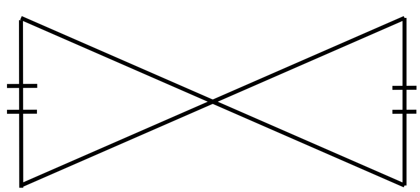


ASA, since the vertical angles are equal

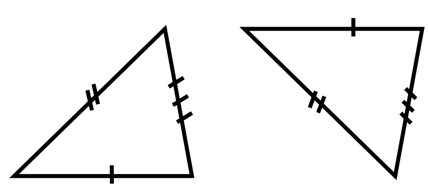


Rectangle ABCD

SAS

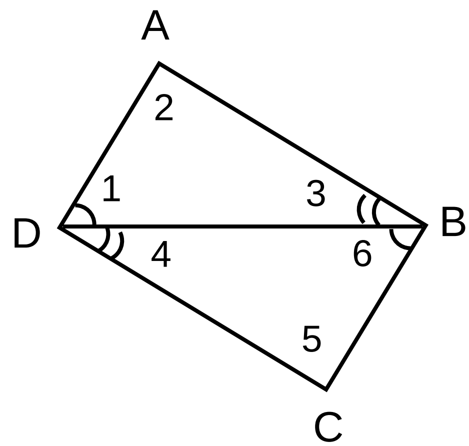


neither



neither

Given: $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$

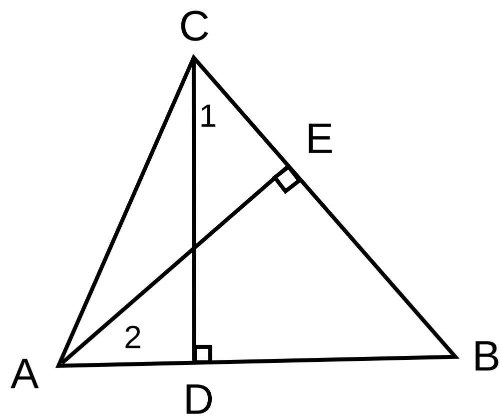


Prove: $\overline{AD} \cong \overline{BC}$

Statement	Reason
1. $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$	1. Given
2. $\overline{DB} = \overline{DB}$	2. Reflexive
3. $\Delta ABD \cong \Delta CDB$	3. ASA
4. $\overline{AD} = \overline{BC}$	4. Corresponding parts of congruent triangles are congruent

Given: $\triangle ABC$ is isosceles with base $\overline{AC}$
$\overline{BC} \cong \overline{AB}$, $\overline{AE} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$

Prove: $\triangle CDB \cong \triangle AEB$



Statement	**Reason**
1. $\triangle ABC$ is isosceles with base $\overline{AC}$, $\overline{BC} \cong \overline{AB}$, $\overline{AE} \perp \overline{BC}$, $\overline{CD} \perp \overline{AB}$	1. Given
2. $\angle B \cong \angle B$	2. Reflexive
3. $\angle AEB$ and $\angle CDB$ are right angles	3. Def. of perpendicular
4. $\angle AEB \cong \angle CDB$	4. Thm. 4.1 (rt. angles are congruent)
5. $\angle 1 \cong \angle 2$	5. Thm. 6.17 (3rd angles are congruent)
6. $\triangle CDB \cong \triangle AEB$	6. ASA