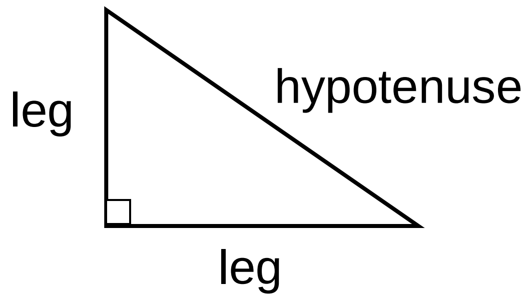
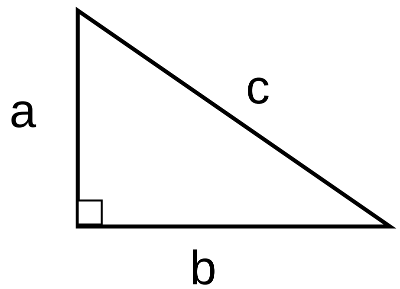
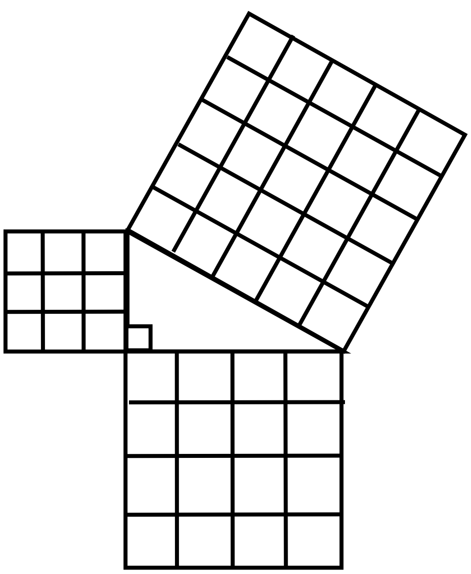
section 8.3





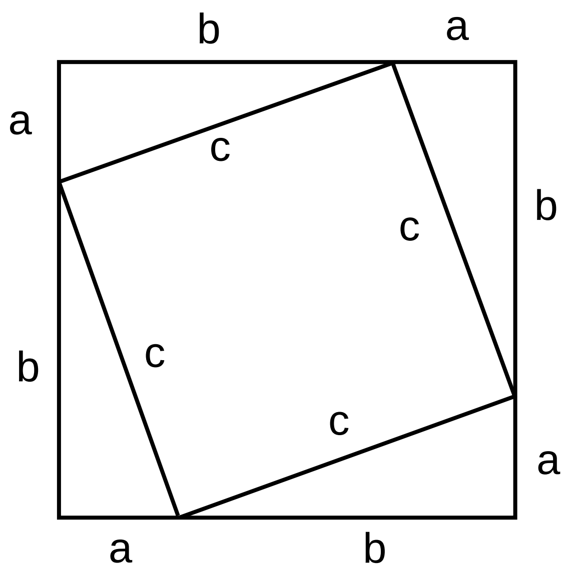
Pythagorean Theorem:

$$a^2 + b^2 = c^2$$



Area:

$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$



Area of large square: $(a+b)^2 = a^2 + 2ab + b^2$
Area of small square: c^2
Area of triangle: ½ ab

Area lg. square = Area sm. square + Area 4 triangles
$a^2 + 2ab + b^2 = c^2 + 4(\frac{1}{2} ab)$
$a^2 + 2ab + b^2 = c^2 + 2ab$
(Subtract 2ab from both sides)
$a^2 + b^2 = c^2$

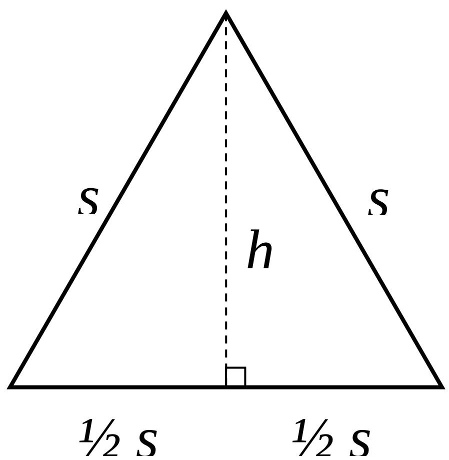
<u>Pythagorean Theorem:</u> In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse: $a^2 + b^2 = c^2$

*<u>**Note**</u>**:** The converse of the Pythagorean Theorem is also true.

Theorem 8.8: The area of an equilateral triangle is $\sqrt{3}/4$ times the square of the length of one side:

$$A = s^2\frac{\sqrt{3}}{4}$$

Consider the right triangles formed and use the Pythagorean Theorem:



$$s^2 = \left(\frac{1}{2}s\right)^2 + h^2$$

$$s^2 = \frac{1}{4}s^2 + h^2$$

$$\frac{3}{4}s^2 = h^2$$

$$\sqrt{\frac{3}{4}s^2} = \sqrt{h^2}$$

$$h = \frac{\sqrt{3}}{2}s$$

Use the formula for the area of a triangle:

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}s\left(\frac{\sqrt{3}}{2}s\right) = \frac{\sqrt{3}}{4}s$$

Sample Problems:

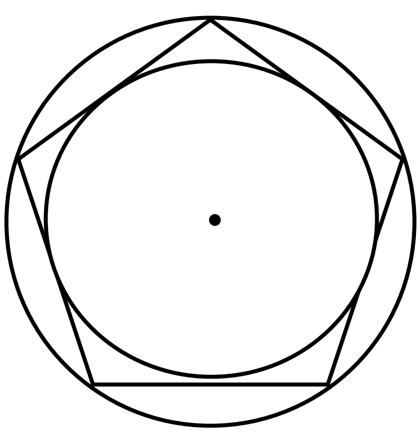
1. Find the altitude of the equilateral triangle with a side of 6.

$$h = 3\sqrt{3}$$

2. The bases of an isosceles trapezoid are 20 and 30 inches. Find the area if the congruent sides are 13 inches.

Area = 300 sq. inches

section 8.4



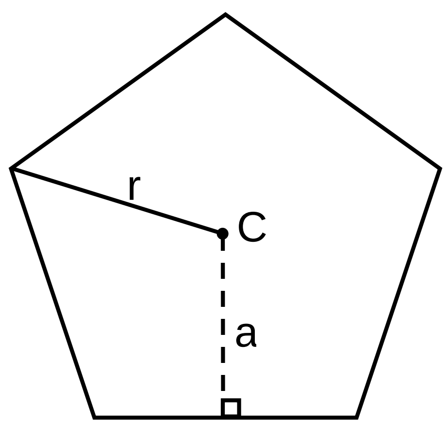
All regular polygons can have a circle inscribed and circumscribed circle. The polygon and circles have a common center.

Definitions:

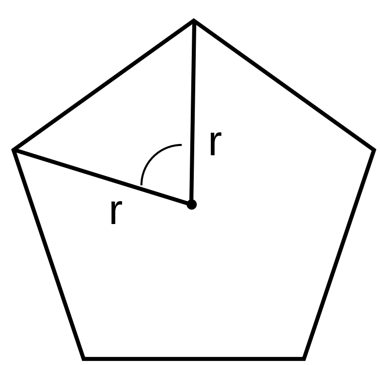
The <u>center of a regular polygon</u> is the common center of the inscribed and circumscribed circles of the regular polygon.

The <u>radius of a regular polygon</u> is a segment that joins the center of a regular polygon with one of its vertices.

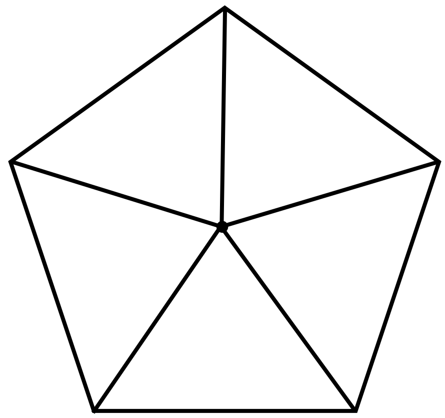
The <u>apothem of a regular polygon</u> is the perpendicular segment that joins that center with a side of a polygon.



A <u>central angle of a regular polygon</u> is the angle formed at the center of the polygon by 2 radii drawn to consecutive vertices.



Theorem 8.9: The central angles of a regular n-gon are congruent and measure $360 \div n$.



5 sides means 5 central angles
$360° \div 5 = 72°$ for each angle

Question: How big are the other 2 angles in the triangle?

Answer: The Δ is isosceles because all radii are =

$$x + x + 72 = 180$$
$$2x = 108$$
$$x = 54°$$

Question: What is the size of each angle of the pentagon?

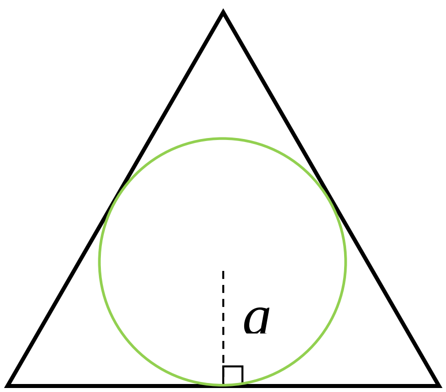
Answer: Each angle is the sum of the base angles of 2 isosceles triangles, which each are 54° each. Therefore the pentagon angles are each 108°.

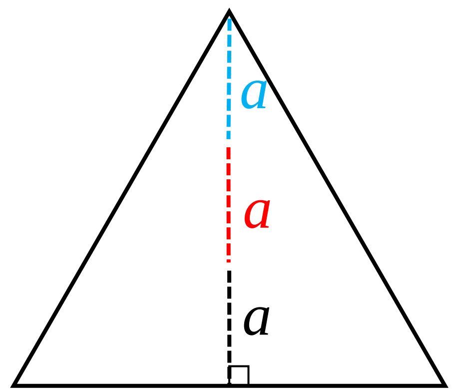
<u>Theorem 8.10</u>: The area of a regular polygon is one-half the product of its apothem and its perimeter:

$$A = \frac{1}{2} ap$$

<u>Theorem 8.11</u>: The apothem of an equilateral triangle is one-third the length of the altitude:

$$a = \frac{1}{3}h$$





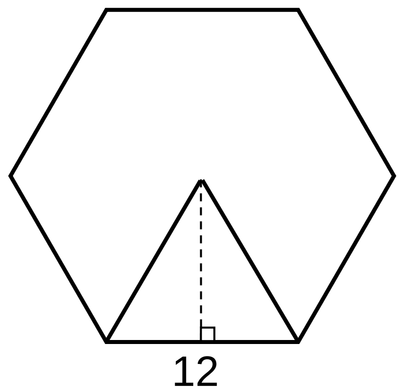
***<u>Note</u>:** This also means that the height is 3 times the apothem: $h = 3a$

<u>Theorem 8.12</u>: The apothem of an equilateral triangle is $\sqrt{3}$ times 1/6 the length of the side:

$$a = \frac{\sqrt{3}}{6}$$

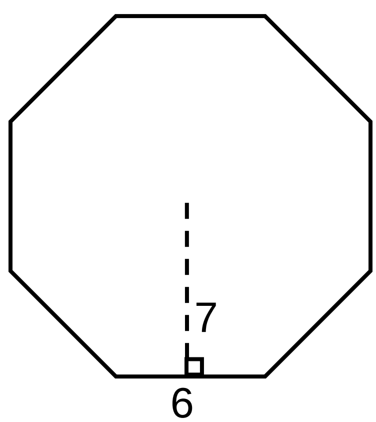
Sample Problems:

1. Find the area of the regular hexagon.



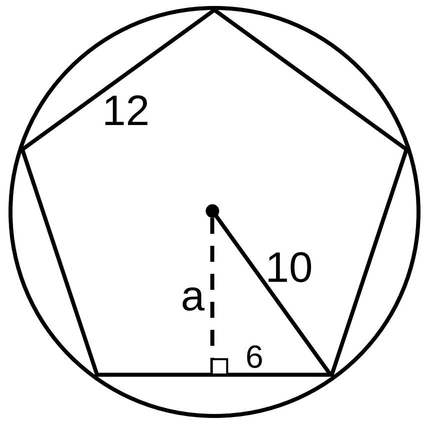
Answer: area = ½ ap = ½ $(6\sqrt{3})(6)(12) = 216\sqrt{3}$

2. Find the area of the regular octagon.



Answer: area = ½ ap = ½ (7)(8 · 6) = 168

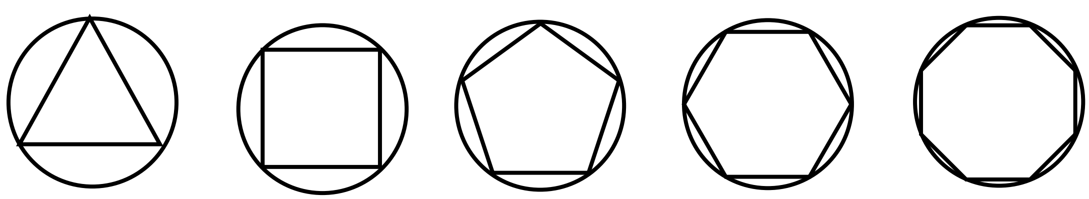
3. Find the apothem and area of the regular pentagon inscribed in a circle of radius 10 and side of 12.



Answer: area = ½ ap = ½ (8)(5 ·12) = 240

section 8.5

Look at:



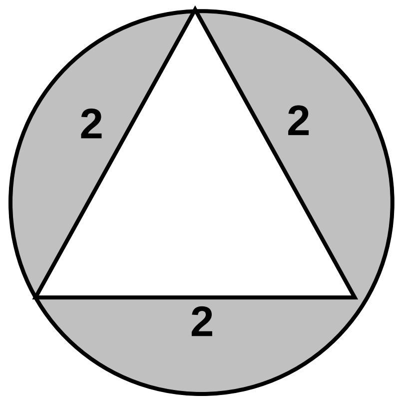
*******The length of the apothem gets closer to the radius of the circle, the perimeter gets closer to the circumference, and the area gets closer to the area of the circle.

$$\mathbf{a} \rightarrow \mathbf{r}$$
$$\mathbf{p} \rightarrow \mathbf{c}$$
$$\mathbf{A_{n\text{-}gon}} \rightarrow \mathbf{A_{circle}}$$

<u>Theorem 8.13</u>: The area of a circle is π times the square of the radius: $A = \pi r^2$

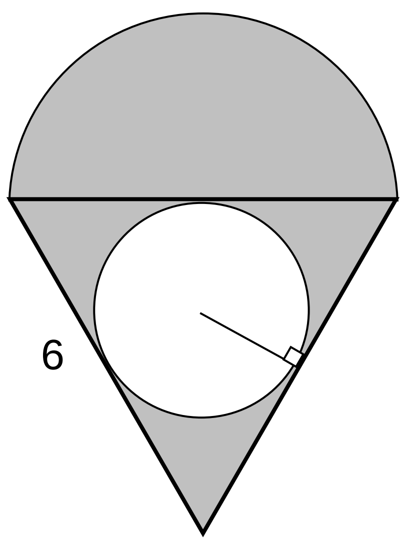
Sample Problems: Find the area of the shaded part:

1.



<u>Answer</u>: $\frac{4\pi}{3} - \sqrt{3}$

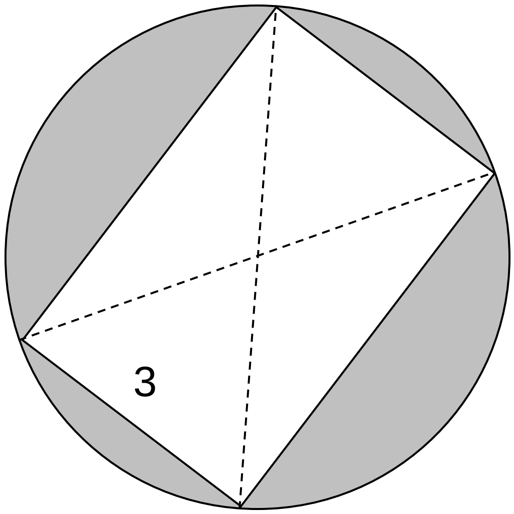
2.



The triangle is equilateral.

<u>Answer</u>: $\frac{3}{2}\pi + 9\sqrt{3}$

3.



The polygon is a rectangle and the radius of the circle is 5 units.

Answer: $25\pi - 3\sqrt{91}$

Summary of Formulas:

The area of an equilateral triangle: $A = \frac{\sqrt{3}}{4}s^2$

The height of an equilateral triangle: $h = \frac{\sqrt{3}}{2}s$

The apothem of an equilateral triangle: $a = \frac{1}{3}h$

The height of an equilateral triangle: $h = 3a$

The apothem of an equilateral triangle: $a = \frac{\sqrt{3}}{6}s$

The area of a regular polygon: $A = \frac{1}{2}ap$

The area of a circle: $A = \pi r^2$