section 11.1

Definition:

The volume of a solid is the number of cubic units needed to fill up the interior completely.

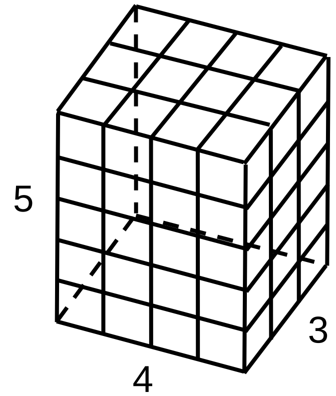
A cubic unit is a cube whose sides measure one unit.

Volume Postulate (11.1): Every solid has a volume given by a positive real number.

Congruent Solids Postulate (11.2): Congruent solids have the same volume.

Volume of Cube Postulate (11.3): The volume of a cube is the cube of the length of one edge: $V = e^3$

Volume Addition Postulate (11.4): If the interiors of two solids do not intersect, then the volume of their union is the sum of their volumes.

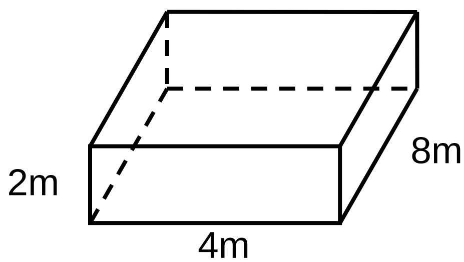


5 stacks of 12 cubes
(5)(12) = 60 cubes

Theorem 11.1: The volume of a rectangular prism is the product of its length, width, and height: V = lwh

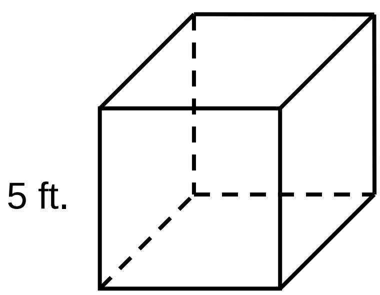
Sample Problems: Find the volume of the right rectangular prisms.

1.



$V = (2)(4)(8) = 64\ m^3$

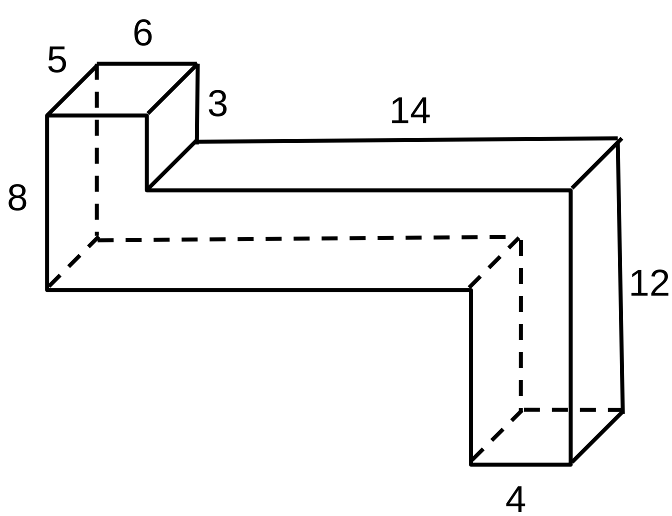
2.



$V = e^3$ or lwh

$V = 5^3 = 125\ ft.^3$

3.



Top = (3)(5)(6)=90

Middle=(5)(20)(5)=500

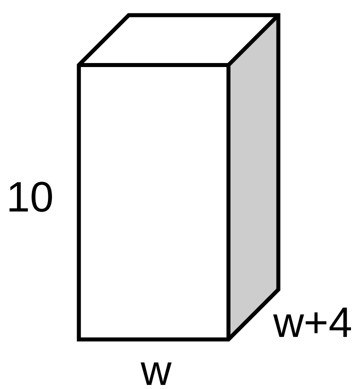
Bottom=(7)(4)(5) = 140

Total = 730 $units^3$

Sample Problem: How much concrete ($yd.^3$) will be needed for a two-lane road that is 30 ft. wide and 21 miles long? Assume the road is 1 ft. thick.

$$V = 30(110{,}880)(1) = 3{,}326{,}400 \text{ ft.}^3$$
$$3{,}326{,}400 \div 27 = 123{,}200 \text{ yd.}^3$$

Sample Problem: The volume of a right rectangular prism is 450 $inches^3$. The length of the base is 4 inches more than the width. The height of the prism is 10 inches. What are the dimensions of the prism?



$V = 10w(w+4)$
$450 = 10w^2 + 40w$
$10w^2 + 40w - 450 = 0$
$10(w+9)(w-5) = 0$
$w = -9$ or $w = 5$

width = 5, length = 9, height = 10

Section 11.2

Definition:

The <u>cross section</u> of a 3-dimensional figure is the intersection of the figure and a plane that passes through the figure and is perpendicular to the altitude.

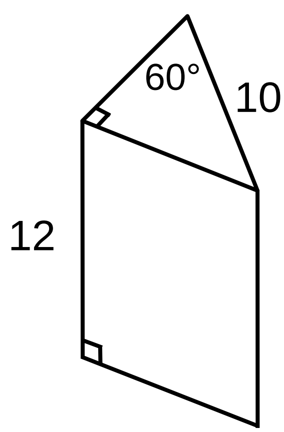
Cavalieri's Principle (Postulate 11.5): For any two solids, if all planes parallel to a fixed plane form sections having equal areas, then the solids have the same volume.

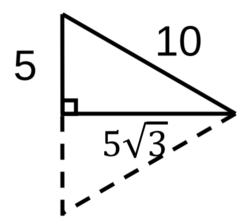
Theorem 11.2: A cross section of a prism is congruent to the base of the prism.

Theorem 11.3: The volume of a prism is the product of the height and the area of the base: V = BH

Sample Problems: Find the volume of each prism.

1.

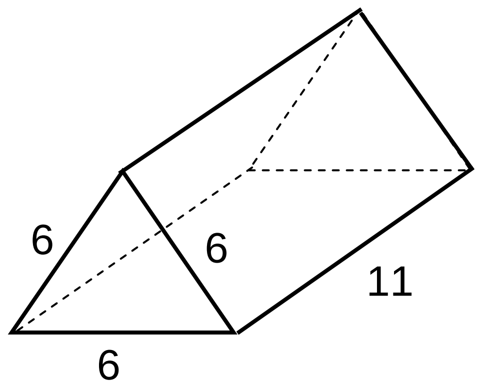




$$B = \frac{1}{2}(5)\left(5\sqrt{3}\right) = \frac{25\sqrt{3}}{2}$$

$$V = BH = (12)\frac{25\sqrt{3}}{2} = 150\sqrt{3}$$

2.



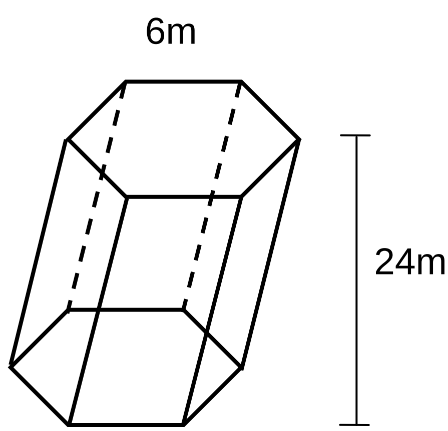
$B = \frac{\sqrt{3}}{4} \cdot 6^2$

$B = 9\sqrt{3}$

$V = BH = 9\sqrt{3} \cdot 11$

$V = 99\sqrt{3}$ cubic units

3.



Area = ½ ap

$a^2 = 6^2 - 3^2$

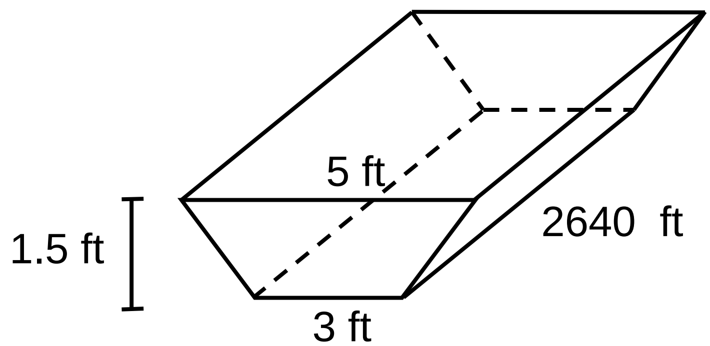
$a = 3\sqrt{3}$

$B = ½(3\sqrt{3})(36) = 54\sqrt{3}$

$V = BH = (24)54\sqrt{3} = 1926\sqrt{3} m^3$

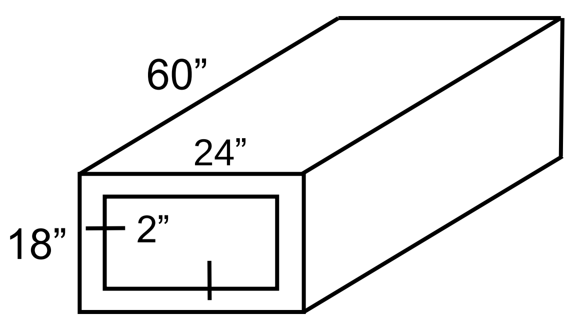
Note: The apothem of a regular hexagon is the only one you can find knowing only the side length because of the equilateral triangles that are formed.

Sample Problem: Find the capacity in gallons of a drainage ditch with a trapezoidal cross section that is ½ mile long. The water can flow 18 inches deep and the cross section at that depth has bases 3 feet and 5 feet. Assume 1 cubic foot of water contains 7.5 gallons.



B = ½ (3+5)(1.5) = 6
V = BH = 6(2640) = 15,840 cubic feet
(15,840)(7.5) = 118, 800 gallons

Sample Problem: Find the volume of fiberglass needed to construct the insulated heating duct.

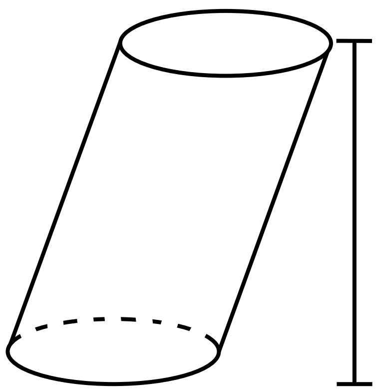


V = (18)(24)(60) – (20)(14)(60)
V = 9120 cubic inches

Theorem 11.4: The volume of a cylinder is the product of the area of the base and the height: $V = BH$. In particular, for a circular cylinder $V = \pi r^2 H$

Sample Problems: Find the volume. Assume bases are circular.

1.

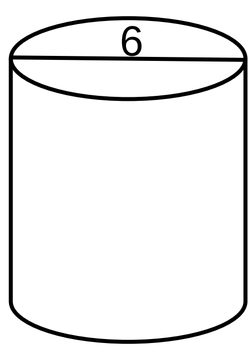


$C = 24\pi$

$C = 24\pi = 2\pi r$
$24\pi = 2\pi r$
$r = 12$

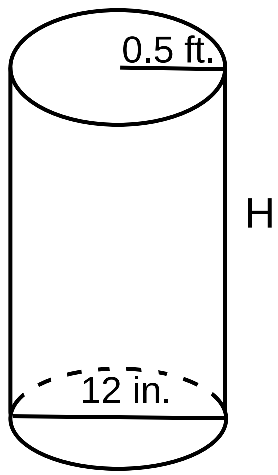
$V = BH$
$V = \pi r^2 H$
$V = \pi(144)(16) = 2304\pi$ cu. in.

2.



$V = \pi r^2 H = \pi(3^2)(11) = 99\pi$ cubic units

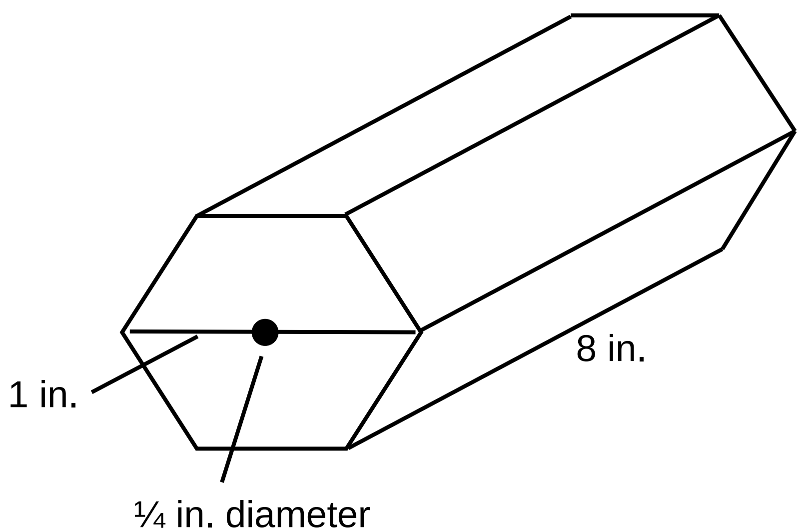
Sample Problem: What is the height of a 5-gallon cylindrical container if its diameter is 12 inches.
(1 cubic foot = 7.5 gallons)



5 gallons = 0.67 cubic feet

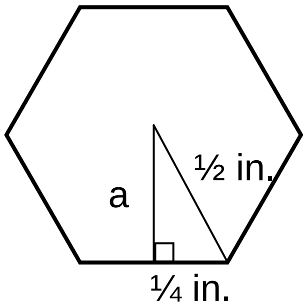
$V = \pi r^2 H$
$0.67 = \pi(0.5)^2 H$
$0.67 = .25\pi H$
$H \approx .85$ ft. or 10.2 in.

Sample Problem: A designer needs to know the weight of an aluminum part in the shape of a regular prism with a hexagonal base. It is 1 inch across the diameter of the base and 8 inches long. A ¼ - inch diameter hole is drilled through it lengthwise. Assume that the density of aluminum is 168.5 lb. per cubic ft.



Solution:

Volume of hexagonal prism: $V = BH = (\frac{1}{2}\,ap)H$



Use Pythag. Thm. to find a.

$(\frac{1}{2})^2 - (\frac{1}{4})^2 = a^2$

$a^2 = \frac{3}{16}$

$a = \frac{\sqrt{3}}{4}$

Volume of hexagonal prism: $V = \frac{1}{2}\,apH$

$V = \frac{1}{2}\left(\frac{\sqrt{3}}{4}\right)(6 \cdot \frac{1}{2})(8)$

$V = 3\sqrt{3}$

Volume of cylinder: $V = \pi r^2 H$

$V = \pi\left(\frac{1}{8}\right)^2 (8)$

$V = \frac{\pi}{8}$

Volume of figure $= 3\sqrt{3} - \frac{\pi}{8} \approx 4.8$ cubic inches

We need cubic feet. We know $1\text{ ft}^3 = 12^3\text{ in}^3 = 1728\text{ in}^3$

$4.8 \div 1728 = 0.0028\text{ ft}^3$

$0.0028\text{ ft}^3 \cdot 168.5\text{ lb/ft} \approx 0.47$ lb. or 7.5 oz.

Theorem 11.5: The volume of a pyramid is ⅓ the product of the height and area of the base:

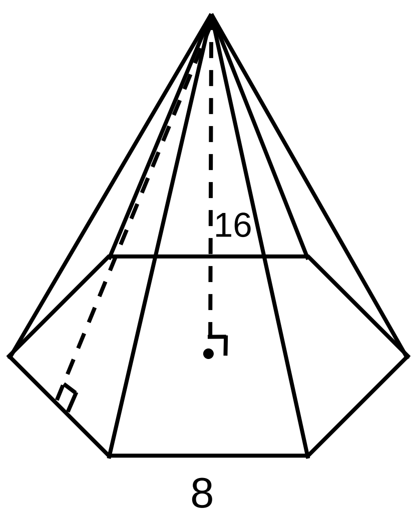
$$V = \tfrac{1}{3}BH$$

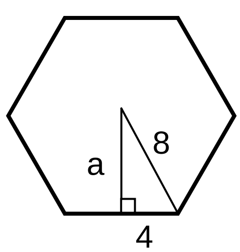
Theorem 11.6: The volume of a cone is ⅓ the product of the height and area of the base:

$$V = \tfrac{1}{3}\pi r^2 H$$

Sample Problems: Find the volume of each figure.

1.





$a = 4\sqrt{3}$ since it is a 30-60 triangle

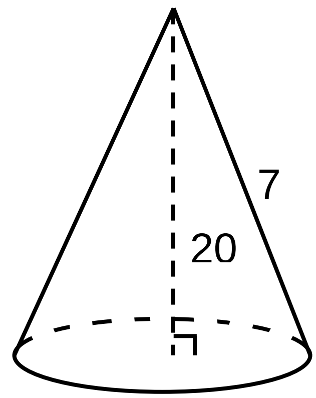
$p = 6 \cdot 8 = 48$

$B = \tfrac{1}{2}ap = \tfrac{1}{2}(4\sqrt{3})(48) = 96\sqrt{3}$

$V = \tfrac{1}{3}BH = \tfrac{1}{3}(96\sqrt{3})(16) = 512\sqrt{3}$

≈ 886.8 cubic units

2.



C = 16π

$C = 2\pi r$

$16\pi = 2\pi r$

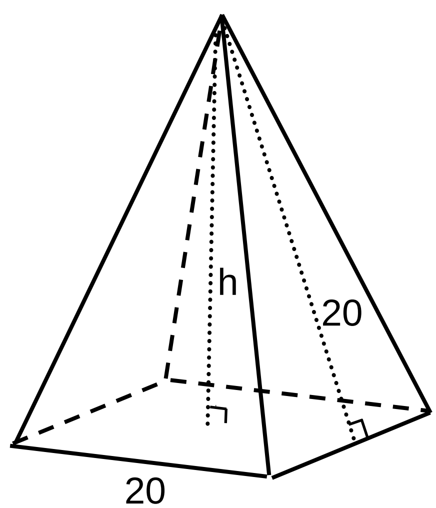
$r = 8$

$V = \frac{1}{3}\pi r^2 H$

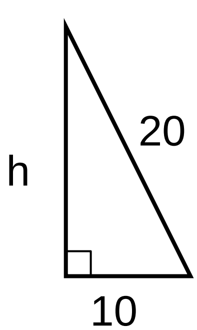
$V = \frac{1}{3}\pi (8^2)(20)$

$V = \frac{1280\pi}{3} \approx 1340.4$ un.^{3}

3.



base is a square



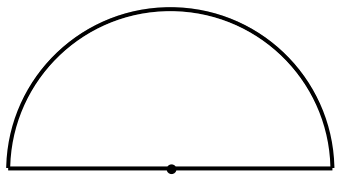
By Pythag. Thm. or 30-60 Δ, we know

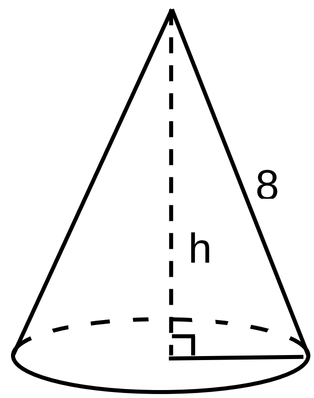
$h = 10\sqrt{3}$

$V = \frac{1}{3}(20^2)10\sqrt{3} = \frac{4000\sqrt{3}}{3}$) cm^3

≈ 2309.4 cm^3

Sample Problem: A piece of tin in the shape of a semicircle of radius 8 inches is rolled into a cone. Find the volume of the cone.





Circumference of semicircle:

$C_{semicircle} = ½ (2\pi r)$

$C_{semicircle} = ½ (2\pi)(8)$

$C_{semicircle} = 8\pi$

Circumference of cone:

$C_{cone} = C_{semicircle}$

$C_{cone} = 8\pi$

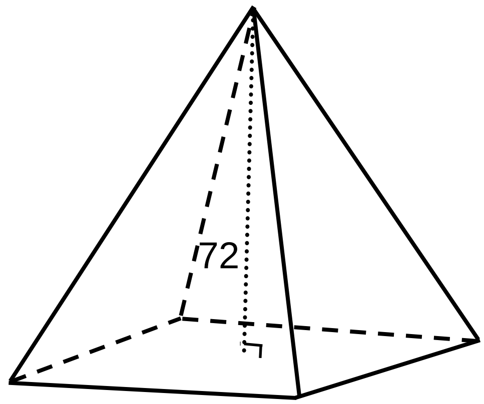
$8\pi = 2\pi r$

$r = 4$

Use Pythag. Thm or notice that it must be a 30-60 right Δ and h = $4\sqrt{3}$.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2 4\sqrt{3} = \frac{64\pi\sqrt{3}}{3} \approx 116.1 \text{ in.}^3$$

Sample Problem: The volume of a square Egyptian pyramid is 98,304 cubic meters and its height is 72 meters. What are the dimensions of the base?



$$V = \tfrac{1}{3} BH$$
$$98{,}304 = \tfrac{1}{3} B(72)$$
$$98{,}304 = 24B$$
$$B = 4096$$

Since the base is a square:

$$s^2 = 4096$$
$$s = 64 \text{ m}$$