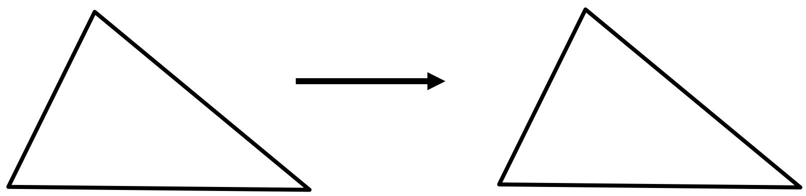
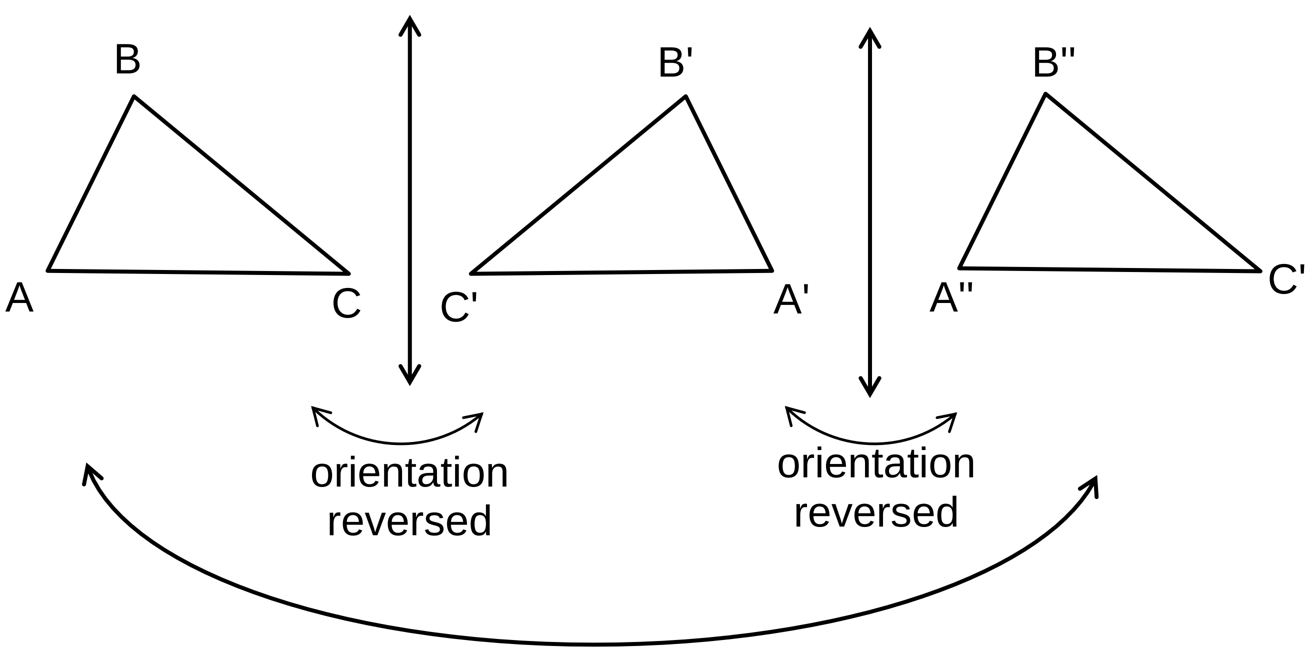
section 12.2

Translation:



A translation can be defined in terms of reflections:



orientation preserved in double reflection

Note: The lines must be parallel.

When we perform two or more transformations on a geometric figure, it is called a <u>composition</u> of transformation.

If we call the first reflection R and the second reflection P, then we have:

$$R(\Delta ABC) = \Delta A'B'C' \quad \text{and } P(\Delta A'B'C') = \Delta A''B''C''$$

We put these together as a composition:

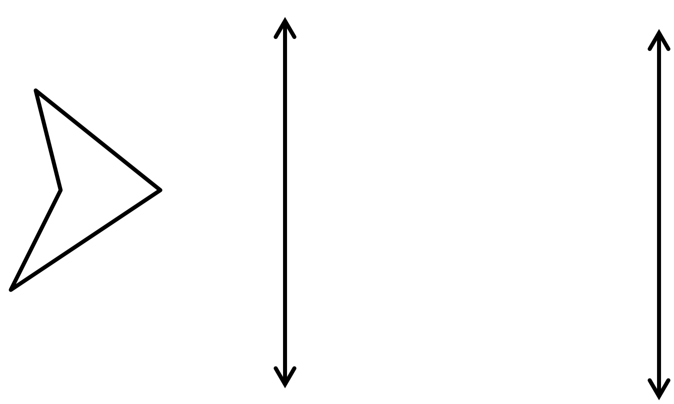
$$P(R(\Delta ABC)) = \Delta A''B''C''$$

<u>Notation</u>: $P \circ R = P(R(\Delta ABC))$

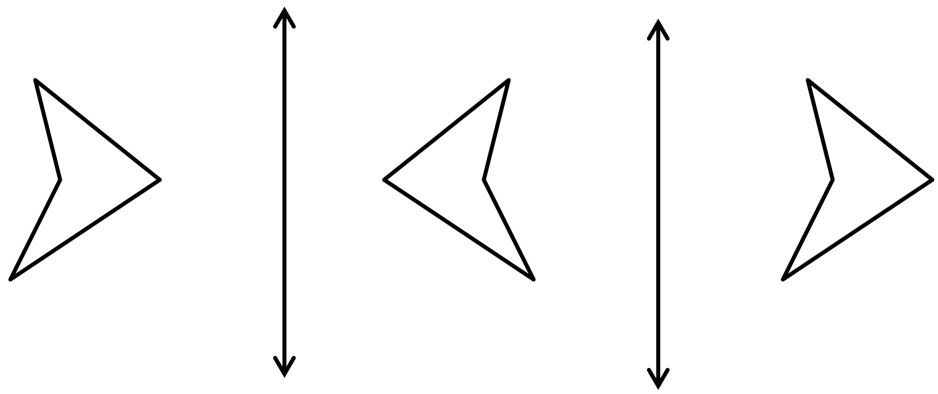
<u>Definition</u>:

A <u>translation</u> is a transformation formed by the composition of two reflections in which the lines of reflection are parallel lines.

<u>Sample Problem</u>: Translate the figure by doing 2 reflections with parallel lines.

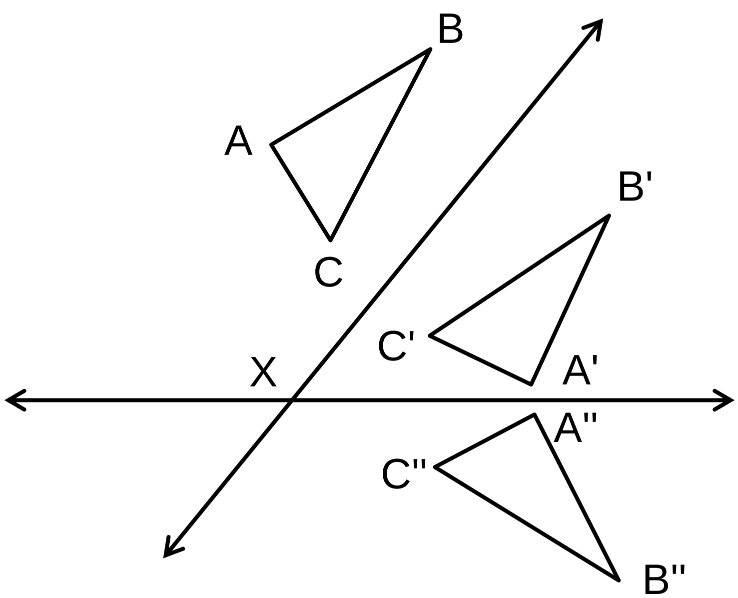


Solution:



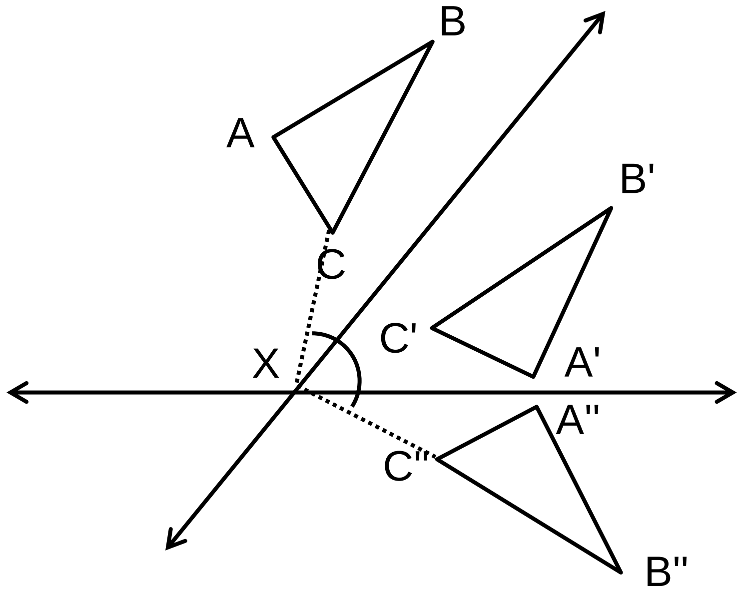
Rotations:

If we reflect a figure over two lines that are not parallel we get a transformation called a rotation.



The center of rotation is the intersection of the two mirror lines.

The angle that the rotation takes to the new position is called the <u>magnitude</u> of the rotation.



m∠CXC'' = 100° so 100° is the magnitude of rotation

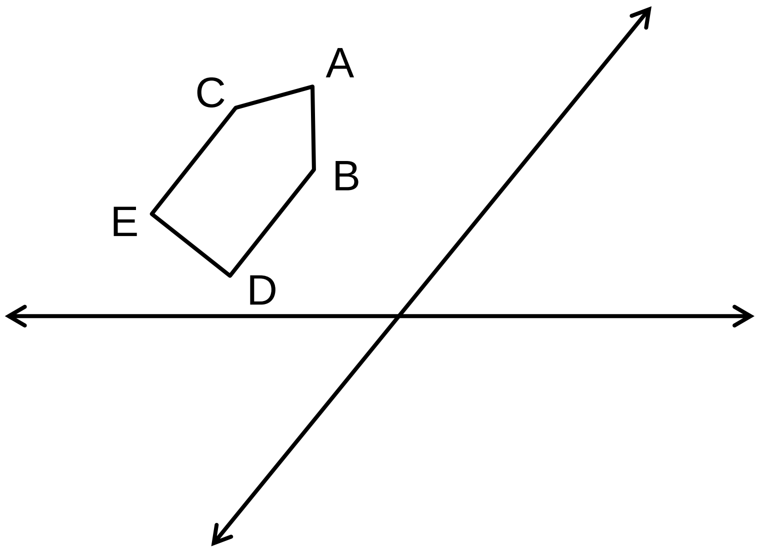
<u>Note</u>: The acute angle that the lines of reflection make is always half of the magnitude.

The acute angle formed by the lines above is 50°

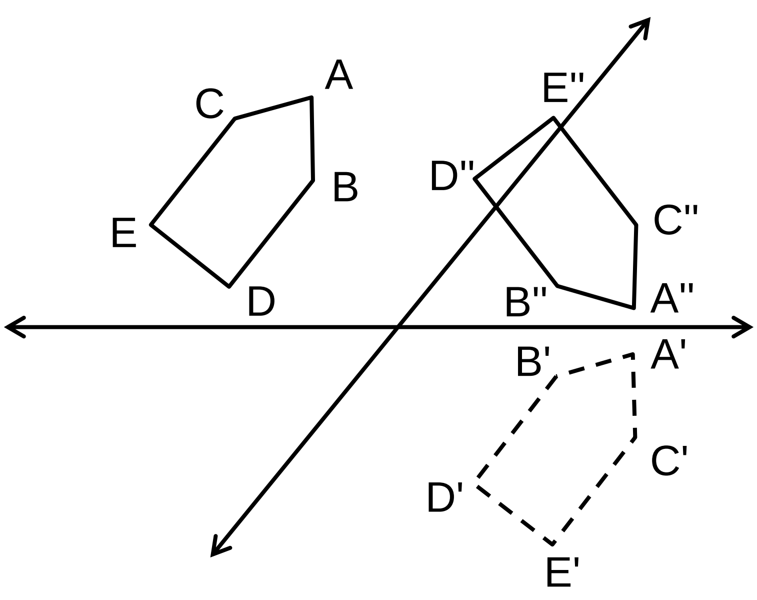
<u>**Definition**</u>:

A <u>rotation</u> is a transformation formed by the composition of two reflections in which the lines of reflection intersect.

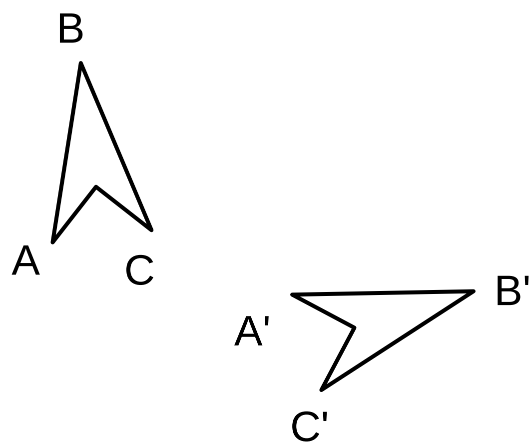
Sample Problem: Rotate the figure through the two intersecting lines.



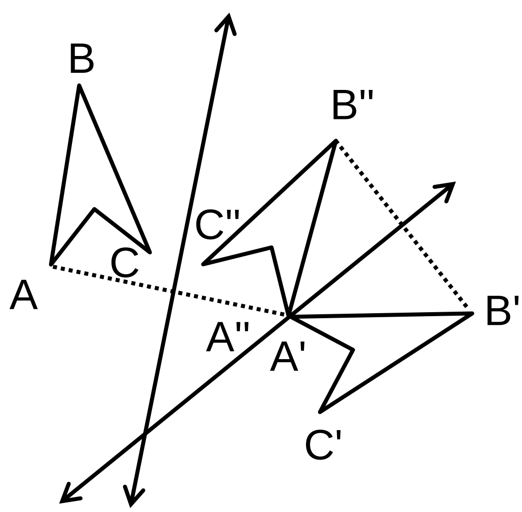
Solution:



<u>Sample Problem</u>: Find the center of rotation.



Solution:



1) Draw the ⊥ bisector of $\overline{AA'}$
2) Reflect the figure over the line.
3) Draw the ⊥ bisector of $\overline{B'B''}$

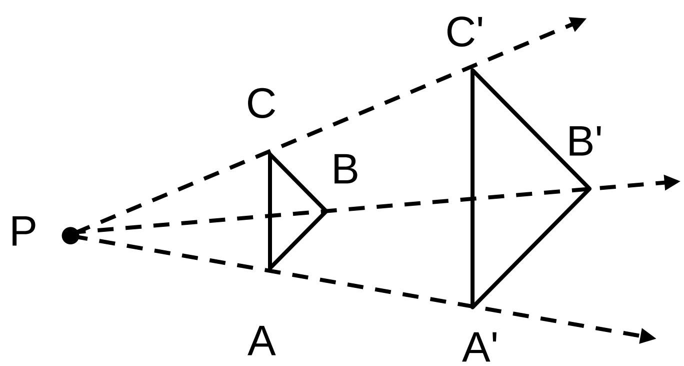
Any transformation that maps each point of a figure onto itself is called an <u>identity transformation</u>.

<u>Examples</u>: A rotation of 360°
A composition of rotating clockwise 30° and then counterclockwise 30°

Enlarging or contracting an image is called dilation.

Definition:

Dilation is a transformation that expands or contracts the points of the plane in relation to a fixed point, P.



PC = 1”
PC' = 2”
PB = 1.2”
PB' = 2.4
PA = 1
PA' = 2

ΔABC has been enlarged in relation to P

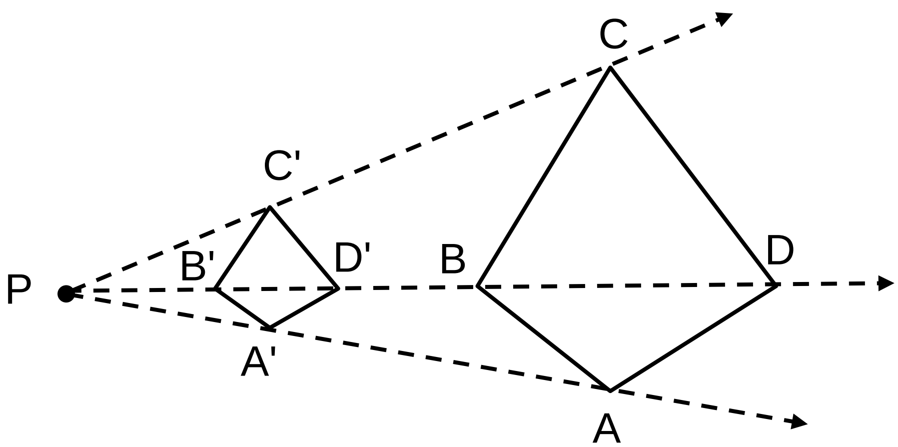
$\frac{PA'}{PA} = \frac{2}{1} = 2$ $\frac{PB'}{PB} = \frac{2.4}{1.2} = 2$ $\frac{PC'}{PC} = \frac{2}{1} = 2$

Definition:

The ratio of the image to the preimage length is called the scale factor of the dilation, denoted k.

$$k = \frac{\text{image length}}{\text{preimage length}}$$

**For any dilation, the image of P is P (the fixed point) and for any other point R, the image R' is on $\overrightarrow{PR}$ so that PR' = k(PR)



PC' = 3 cm
PB' = 2 cm
PA' = 3 cm
PD' = 4 cm
PC = 9 cm
PB = 6 cm
PA = 9 cm
PD = 12 cm

$$\frac{PA'}{PA} = \frac{3}{9} = \frac{1}{3} \qquad \frac{PB'}{PB} = \frac{2}{6} = \frac{1}{3}$$

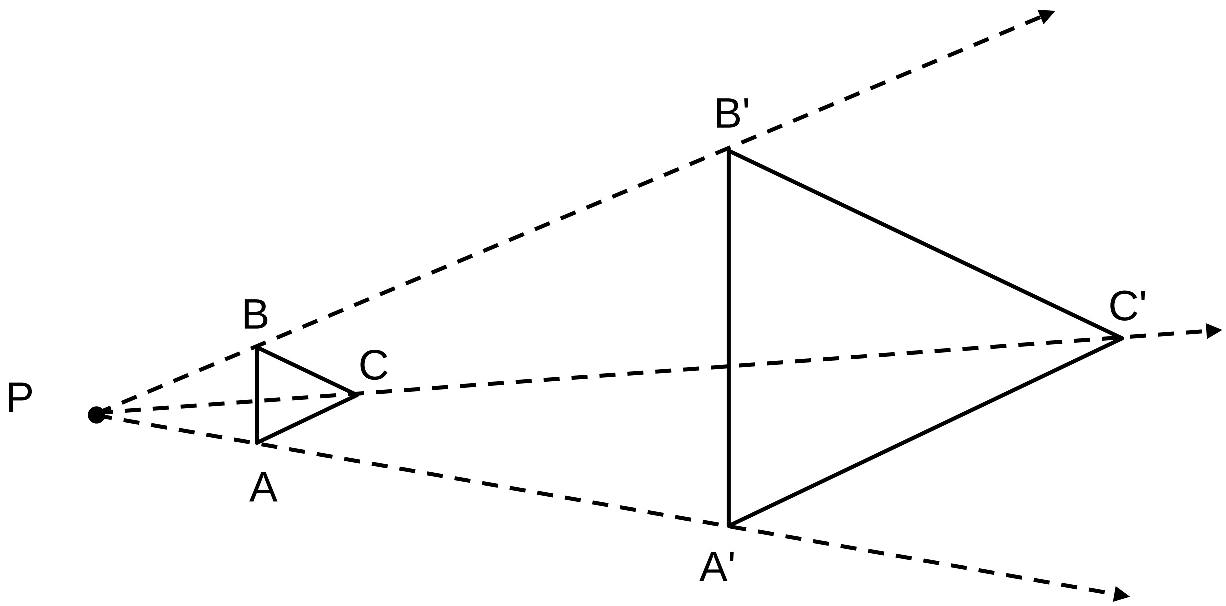
$$\frac{PC'}{PC} = \frac{3}{9} = \frac{1}{3} \qquad \frac{PD'}{PD} = \frac{4}{12} = \frac{1}{3}$$

The scale factor is ⅓.

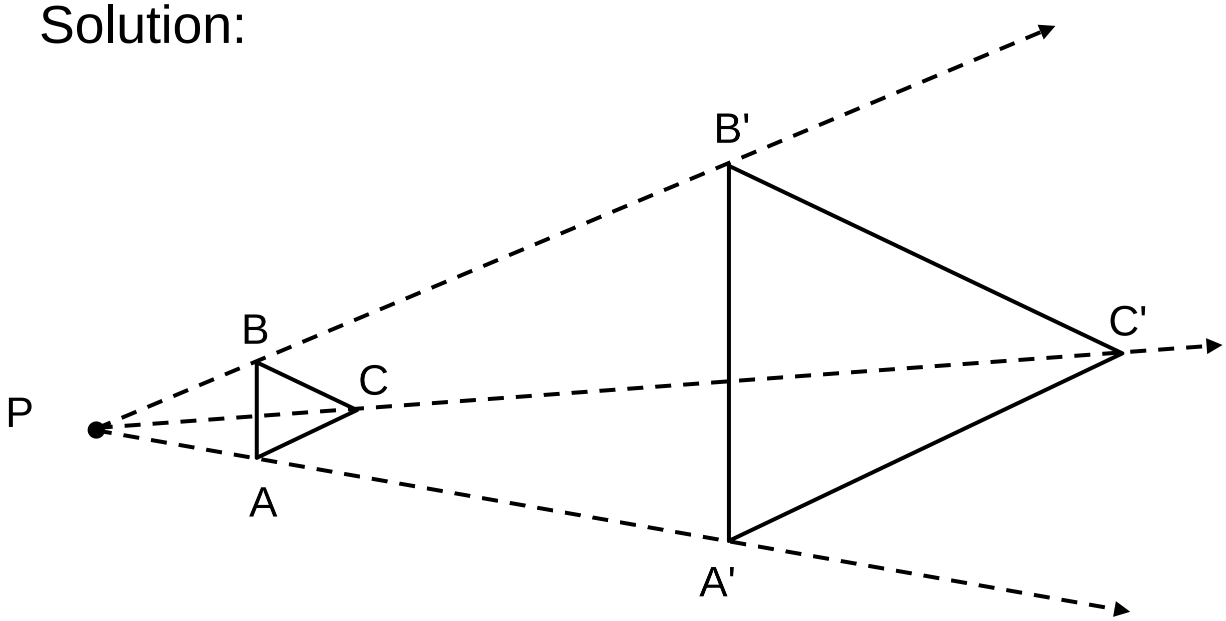
For scale factor k,

- If k > 1, then the dilation is an enlargement (expansion)
- If k = 1, then the dilation is an identity transformation (the size of the figure does not change
- If 0 < k < 1, then the dilation results in a reduction (contraction)

Sample Problem: Classify the dilation and find the scale factor.



Solution:



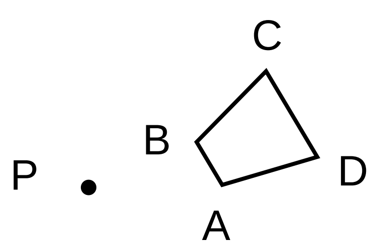
scale factor = $\frac{PB'}{PB} = \frac{8}{2} = 4$ (enlargement, since 4>1)

Find m∠A and m∠A'

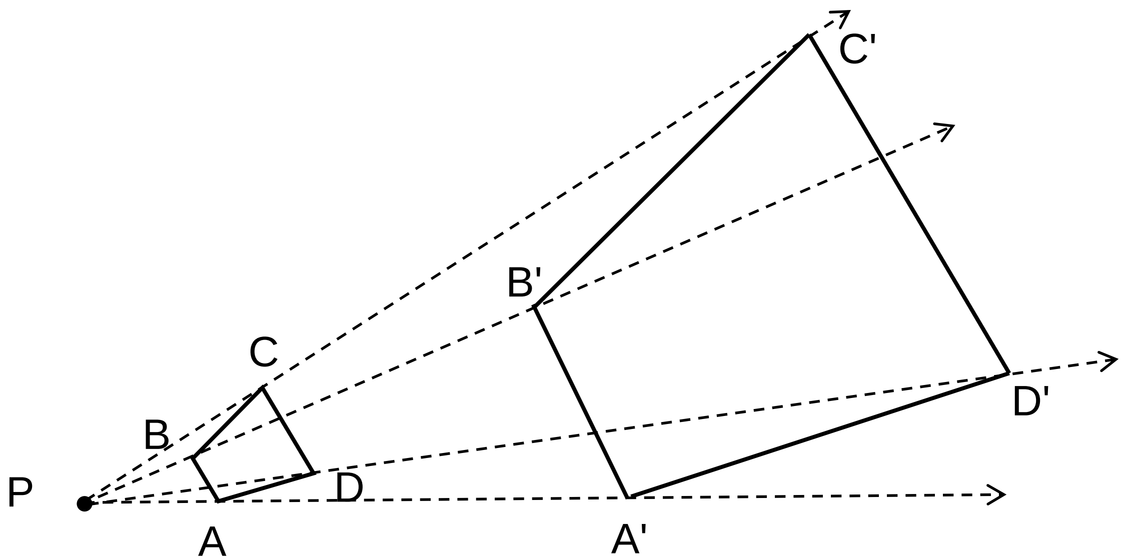
Are they congruent? *Yes*

**In a dilation, corresponding angles will have the same measure. This means the figures have the same *shape* even though they are not the same *size*. We say that the figures are <u>similar</u>.

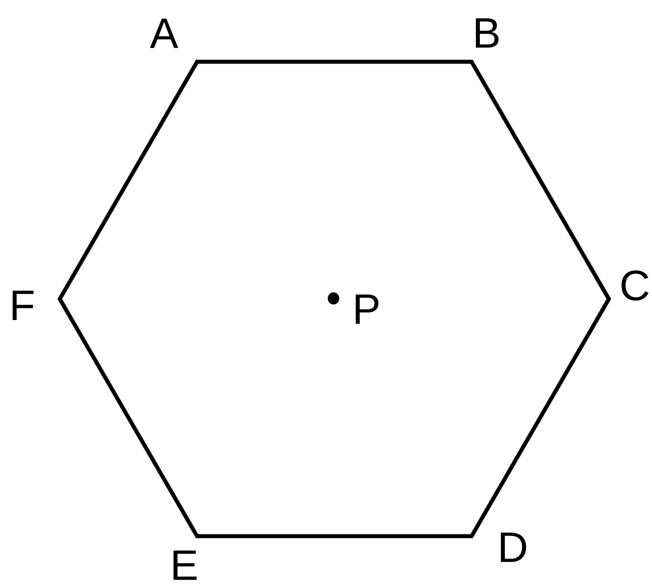
<u>Sample Problem</u>: Find the image of the dilation using the scale factor of k = 3.



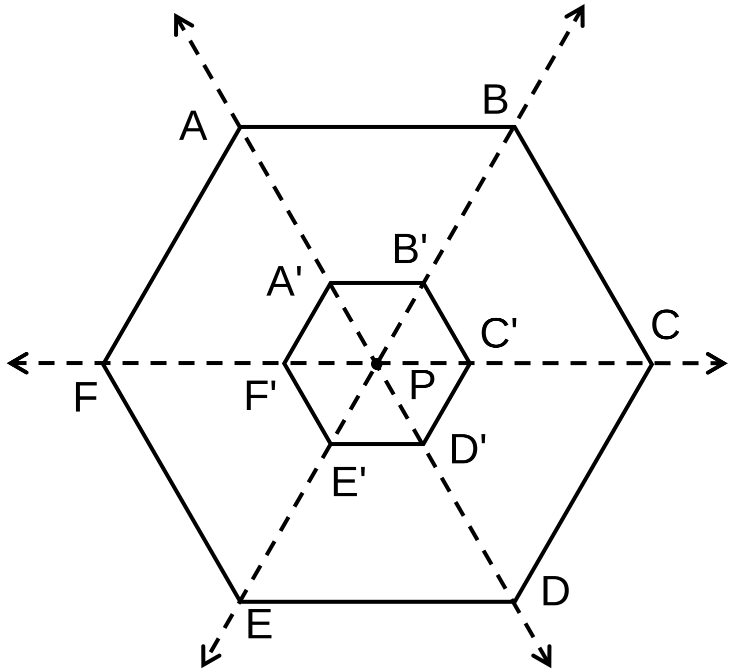
Sample Problem: Find the image of the dilation using the scale factor of k = 3.



Sample Problem: Find the image using the scale factor of k = ⅓ using regular hexagon ABCDEF with radius 3.6 cm



Solution:



1. Draw dilation lines
2. Use scale factor:
   PB' = ⅓ (PB) =
   = ⅓ (3) = 1
3. All dilation lengths are 1 since it is a regular hexagon.

Section 12.4

Invariance means “not varied” or “constant”

*Some transformations have invariant qualities. If the preimage and the image of a transformation always share a certain characteristic, the transformation <u>preserves</u> that characteristic.

<u>Example</u>: In a reflection, distance is invariant.

<u>Example</u>: In a dilation, angle measure is invariant.

Transformations studied so far: reflections
rotations
translations
dilations

Which of these preserves distance?

reflections, rotations, translations

**If a transformation preserves distance, then it is called an isometry.

Definition:

An isometry is a transformation that preserves distance.

Greek "isos" means "equal"
"metron" means "measure"

Isometries we have studied: reflection
translation
rotation
identity transformations
composite of isometries

6 Invariant Properties of Isometries:

1. Distance is preserved
2. Collinearity of points is preserved
3. Betweenness is preserved
4. Angle measure is preserved
5. Parallelism is preserved
6. Triangle congruence is preserved.

Isometry Theorem (Thm. 12.1): Every isometry can be expressed as a composition of at most 3 reflections.

*A transformation that preserves *shape* is a similarity
*A transformation that preserves *size* and *shape* is an isometry.

Dilations do not preserve distance so they are not isometries. They do preserve angle measure so they preserve shape but not size.

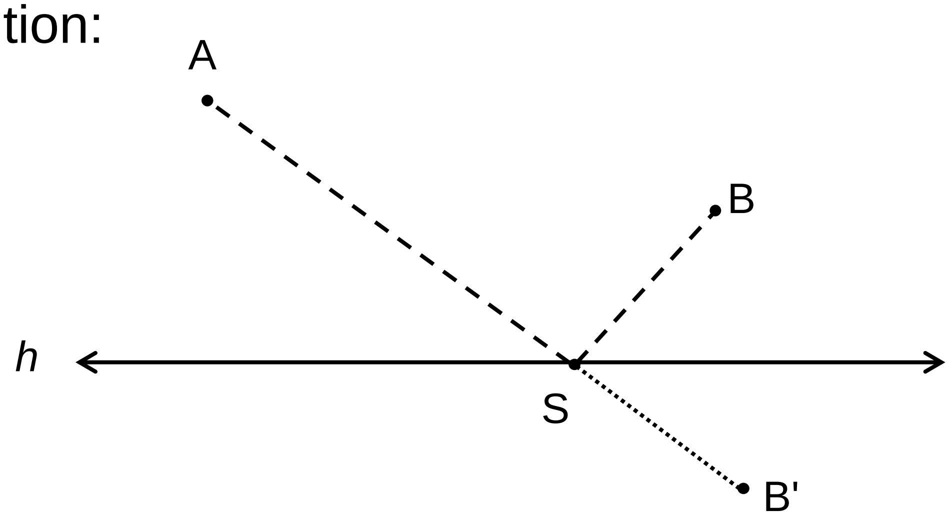
Properties of Dilation:

1. A dilation preserves collinearity of points.
2. A dilation preserves betweenness.
3. A dilation preserves angle measure
4. A dilation preserves parallel lines

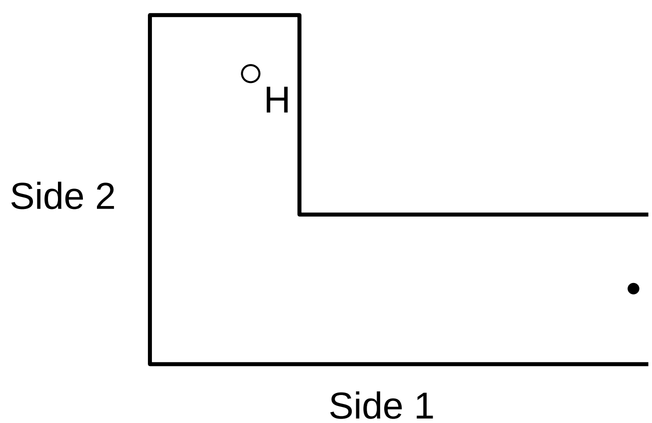
Note: Similarities and isometries are not the only types of transformation.

Example 1: A beam will be sent from satellite A to satellite B by being sent to the earth and reflected by a booster station to satellite B. The satellite engineers are trying to place the booster station in the spot where the total distance that the beam travels will be the shortest. If the booster station must be located somewhere along line h, what is the best location?

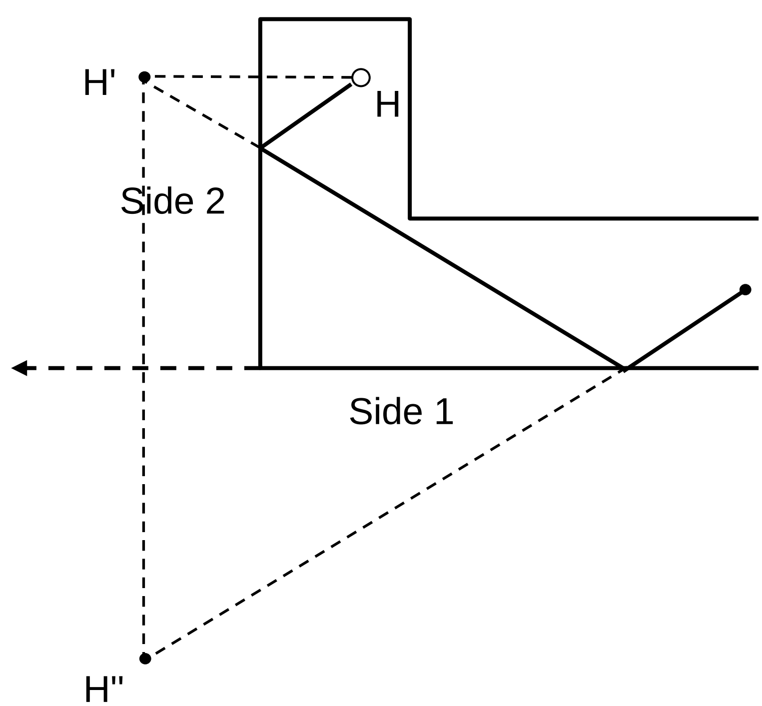
Solution:



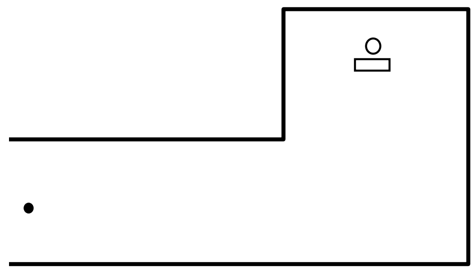
Example 2: What spots on sides 1 and 2 would you aim for so that you will make a hole in one on this miniature golf green?



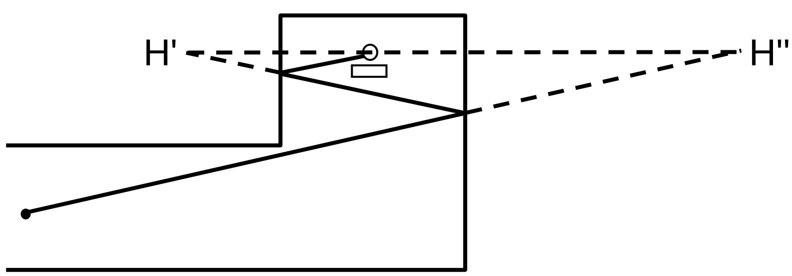
Solution:



Sample Problem: Draw the shot that would produce a hole in one by hitting exactly two parallel sides.



Solution:



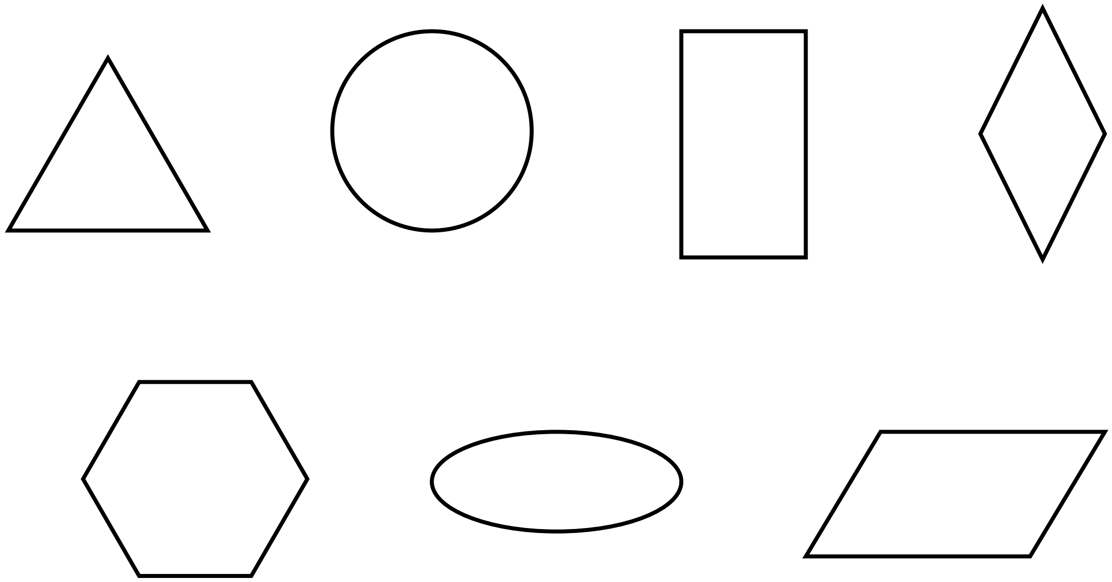
Section 12.6

Definition:

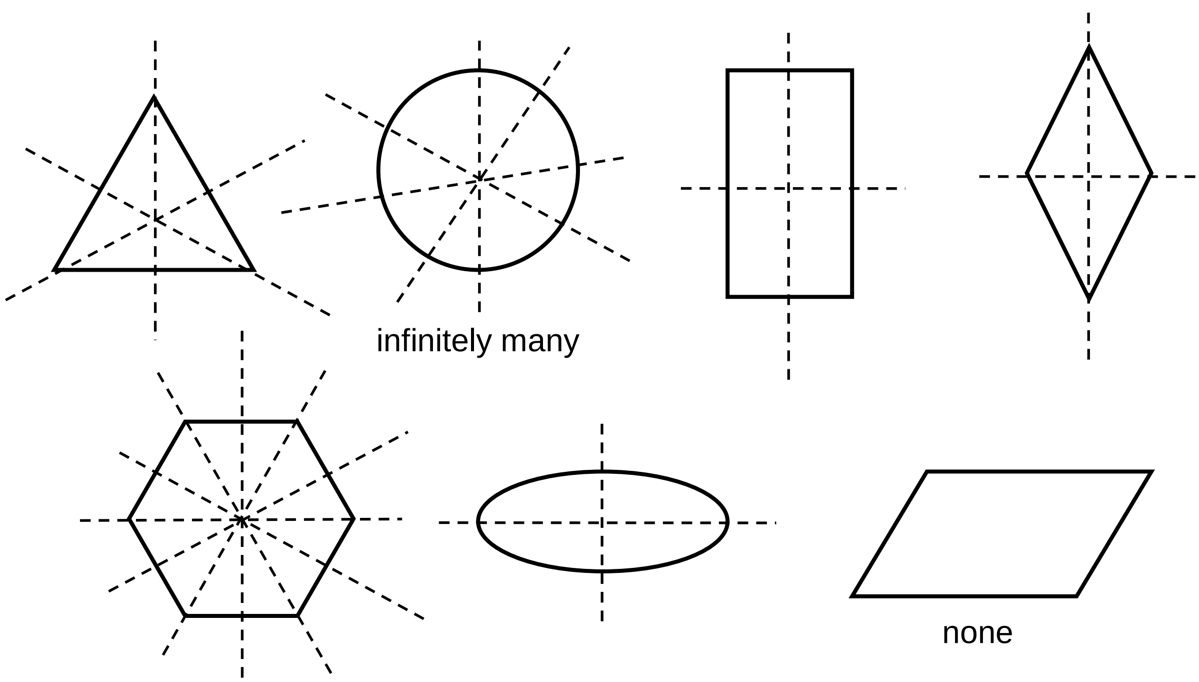
A figure has line symmetry when each half of the figure is the image of the other half under some reflection in a line.

The line of reflection is called the axis of symmetry.

Sample Problem: Draw the axes of symmetry.



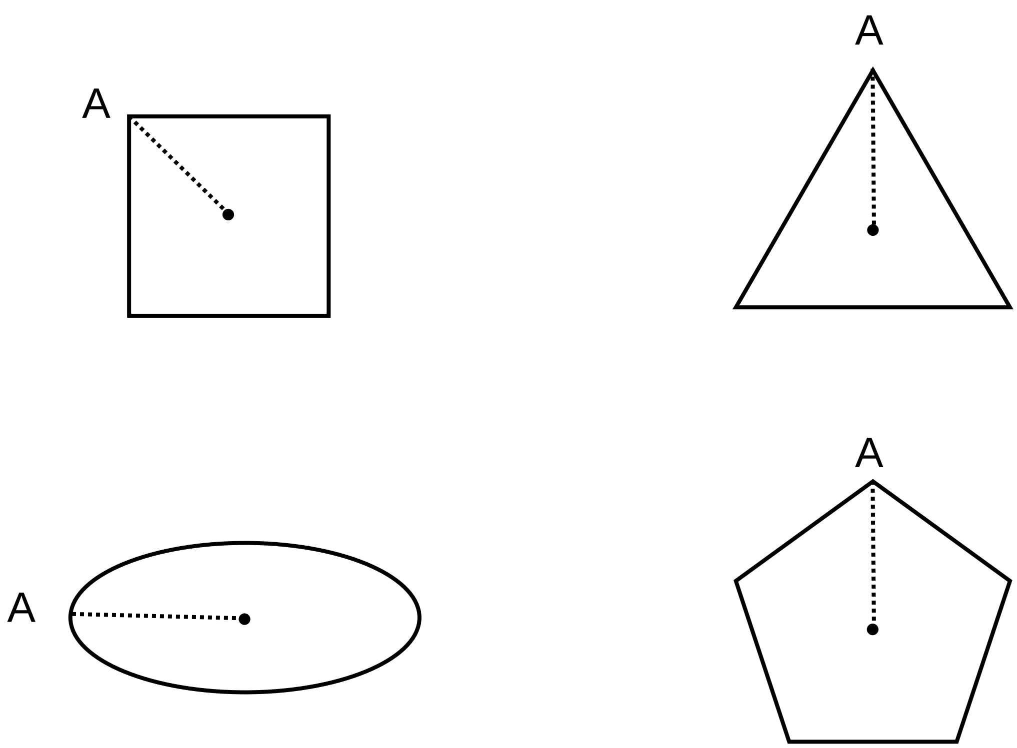
Solution:



Rotation Symmetry:

<u>Definition</u>:

A figure has <u>rotational symmetry</u> when the image of the figure coincides with the figure after a rotation. The magnitude of the rotation must be less than 360°.



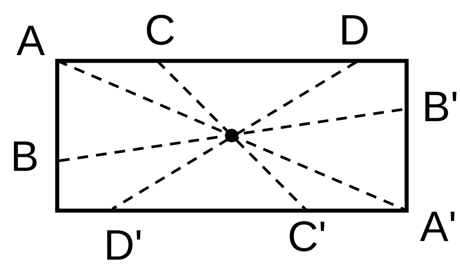
Point Symmetry:

Rotational symmetry of 180° is also called point symmetry. You can “reflect” each point of the figure through the center of the rotation to obtain another point on the figure.

Definition:

A figure has point symmetry with respect to point P if the reflection of every point on the figure through P is also a point on the figure.

Example:



Every figure that has 180° rotational symmetry also has point symmetry.