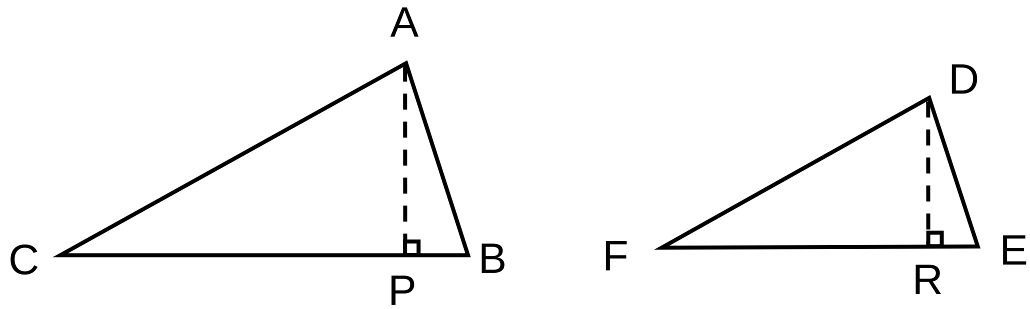
Section 13.4

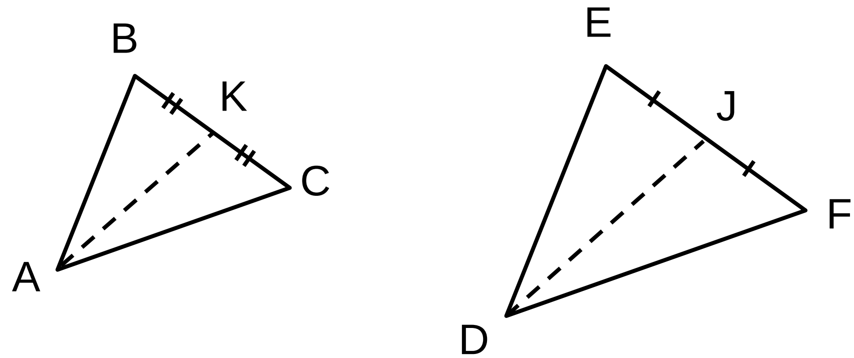
<u>Theorem 13.7</u>: In similar triangles the lengths of the altitudes extending to corresponding sides are in the same ratio as the lengths of the corresponding sides.



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AP}{DR}$$

altitudes

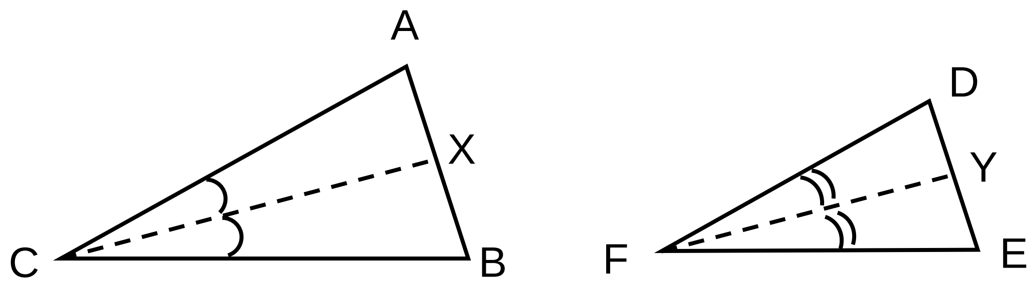
<u>Theorem 13.8</u>: In similar triangles the lengths of the medians extending to corresponding sides are in the same ratio as the lengths of the corresponding sides.



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AK}{DJ}$$

medians

<u>Theorem 13.9</u>: In similar triangles the lengths of the corresponding angle bisectors from the vertices to the points where they intersect the opposite sides are in the same ratio as the lengths of the corresponding sides.



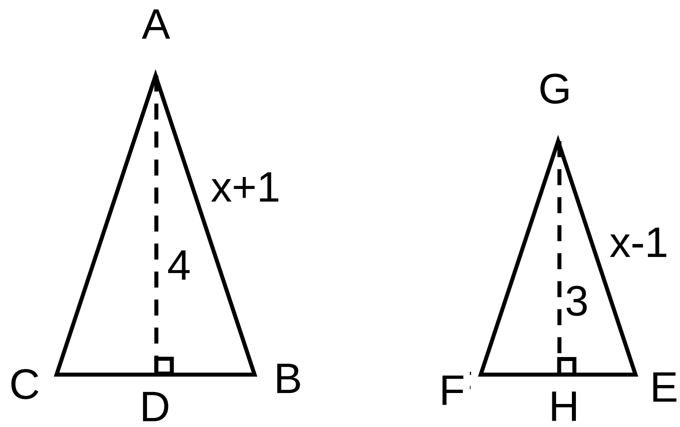
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{CX}{FY}$$

angle bisector segments

<u>Theorem 13.10</u>: In similar triangles the perimeters of the triangles are in the same ratio as the lengths of the corresponding sides.

<u>Theorem 13.11</u>: In similar triangles ratio of the areas of the triangles is equal to the square of the ratio of the lengths of the corresponding sides.

Sample Problem: Given $\Delta ABC \sim \Delta GEF$, $\overline{AD}$ and $\overline{GH}$ are altitudes. Find AB.

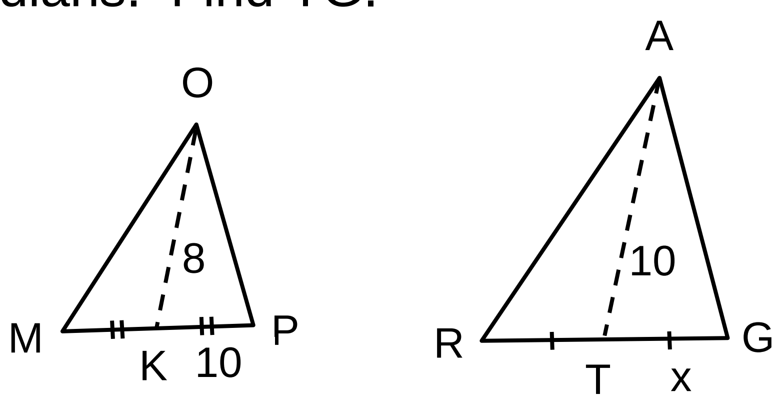


$$\frac{3}{4} = \frac{x-1}{x+1}$$

$3x+3 = 4x-4$

$x = 7 \Rightarrow$ So, $AB = x+1 = 8$

Sample Problem: Given $\Delta MOP \sim \Delta RAC$, $\overline{OK}$ and $\overline{AT}$ are medians. Find TG.



$MP = 2(10) = 20$

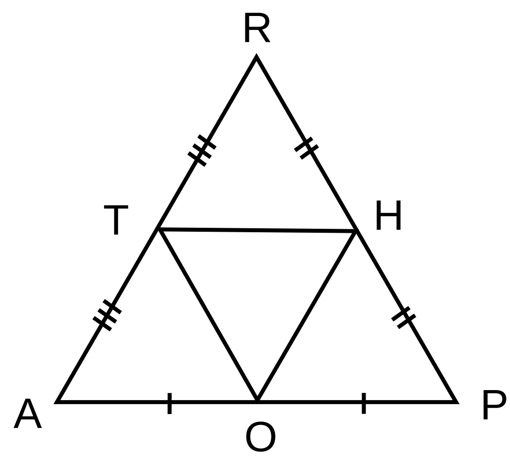
$RG = 2x$

$$\frac{8}{10} = \frac{20}{2x}$$

$16x = 200$

$x = 12.5$

Sample Problem: Given $\Delta ARP \sim \Delta HOT$, H, O, and T are midpoints and the area of ΔHOT is 24 sq. units. find the area of ΔARP if $\frac{HO}{AR} = \frac{1}{2}$

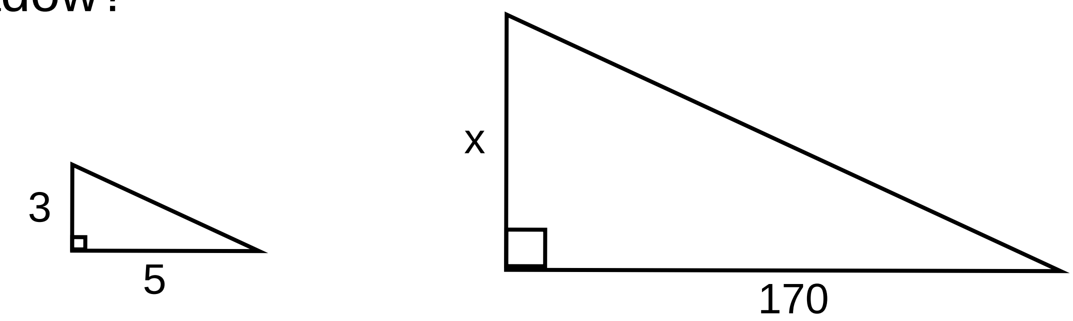


$$\frac{\text{Area } \Delta HOT}{\text{Area } \Delta ARP} = \left(\frac{1}{2}\right)^2$$

$$\frac{24}{A} = \frac{1}{4}$$

$$A = 96$$

Sample Problem: If a vertical yardstick casts a 5-foot shadow, how high is a building that casts a 170-foot shadow?



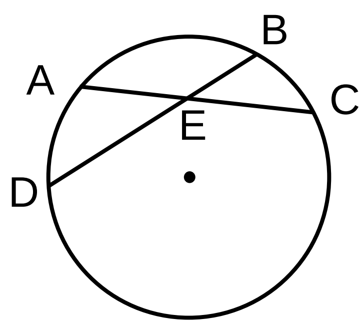
$$\frac{x}{3} = \frac{170}{5}$$

$$5x = 510$$

$$x = 102 \text{ ft.}$$

Section 13.6

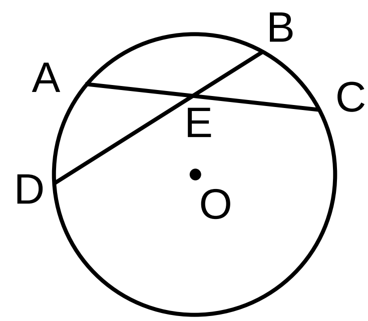
Theorem 13.12: If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



(AE)(EC) = (DE)(EB)

Theorem 13.12: If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

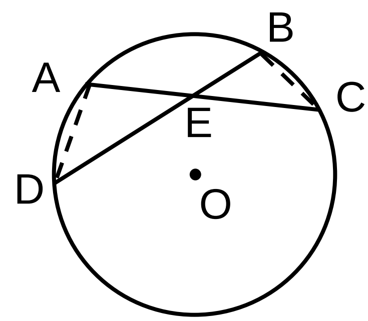
Given: Circle O with chords AC and BD that intersect at E



Prove: (AE)(EC) = (DE)(EB)

Statement	Reason
1.	1. Given
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.

<u>Theorem 13.12</u>: If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

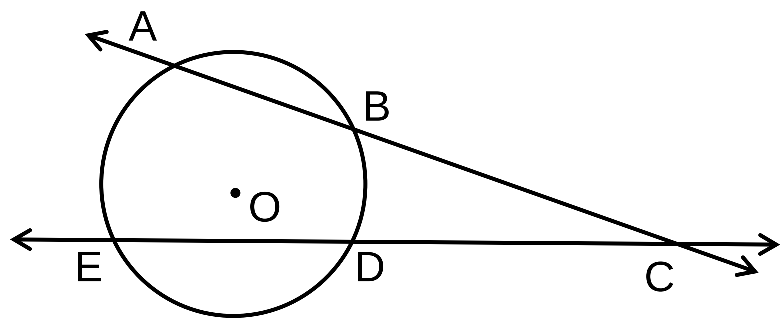


Given: Circle O with chords AC and BD that intersect at E

Prove: (AE)(EC) = (DE)(EB)

Statement	Reason
1. Circle O with chords $\overline{AC}$ and $\overline{BD}$ that intersect at E	1. Given
2. Draw $\overline{AD}$ and $\overline{BC}$	2. Auxiliary lines
3. $\angle AED \cong \angle BEC$	3. Vertical $\angle$ Thm
4. $\angle ADB \cong \angle BCA$	4. $\angle$'s inscribed in the same arc are $\cong$
5. $\Delta AED \sim \Delta BEC$	5. AA
6. $\frac{EB}{AE} = \frac{EC}{DE}$	6. Def. of similar Δ's
7. (AE)(EC) = (DE)(EB)	7. Mult. Prop. of Eq.
8. If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.	8. Law of Deduction

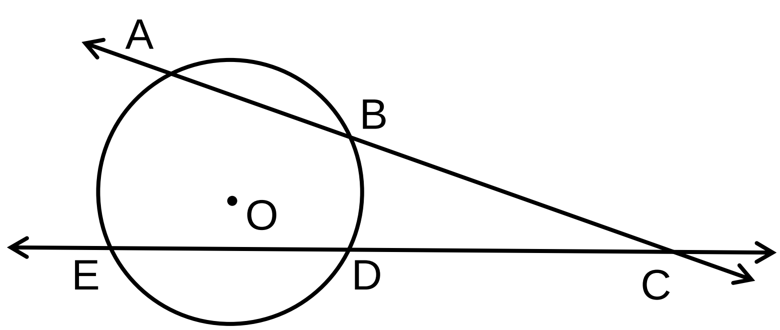
Look at:



$\overline{AC}$ is called a <u>secant segment</u> because it includes the chord and has one endpoint in the exterior of the circle.

$\overline{BC}$ is called an <u>external secant segment</u> because it is the portion of a secant segment that is external to the circle.

<u>**Theorem 13.13**</u>: If two secants intersect in the exterior of a circle, then the product of the lengths of one secant segment and its external secant segment is equal to the product of the lengths of the other secant segment and its external secant segment.

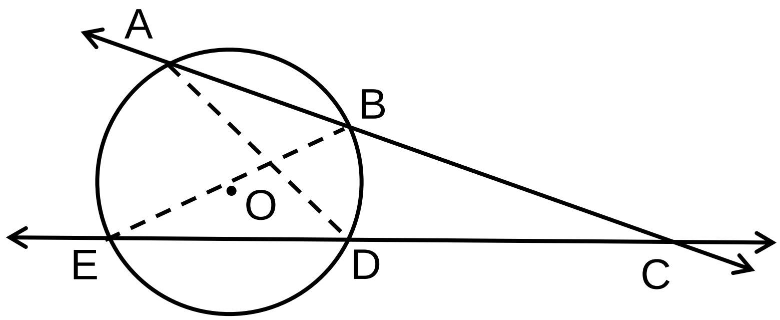


(AC)(BC) = (EC)(DC)

Proof:

Given: Secants $\overleftrightarrow{AB}$ and $\overleftrightarrow{ED}$ intersect at point C in the exterior of circle O

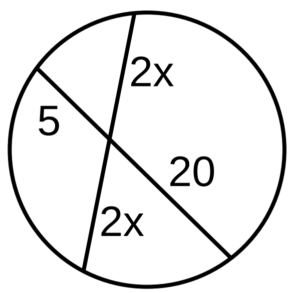
Prove: (AC)(BC) = (EC)(DC)



Statement	Reason
1. Secants $\overleftrightarrow{AB}$ and $\overleftrightarrow{ED}$ intersect at point C in the exterior of circle O	1. Given
2. Draw $\overline{BE}$ and $\overline{AD}$	2. Auxiliary lines
3. $\angle CAD \cong \angle CEB$	3. $\angle$'s inscribed in the same arc are $\cong$
4. $\angle ACE \cong \angle ACE$	4. Reflexive
5. $\Delta DAC \sim \Delta BEC$	5. AA
6. $\frac{BC}{DC} = \frac{EC}{AC}$	6. Def. of similar Δ's
7. (AC)(BC) = (EC)(DC)	7. Mult. Prop. of Eq.

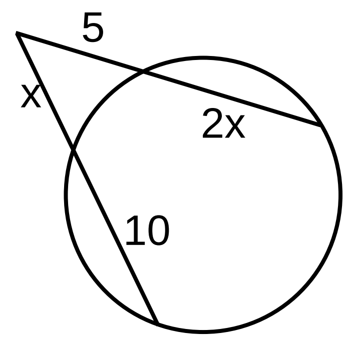
Sample Problems: Find x in the figures shown.

1.



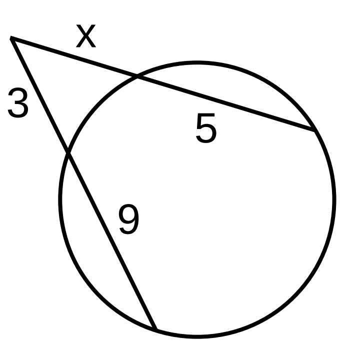
$$(2x)(2x) = 5(20)$$
$$4x^2 = 100$$
$$x^2 = 25$$
$$x = 5$$

2.



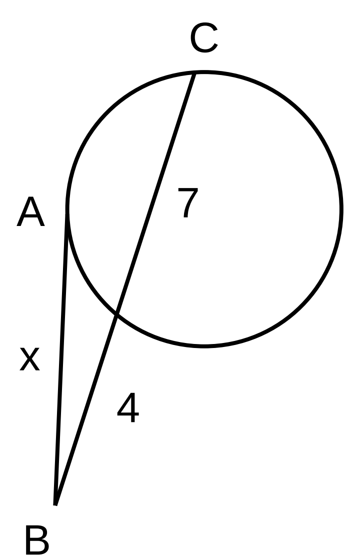
$$5(2x+5) = x(x+10)$$
$$10x + 25 = x^2 + 10x$$
$$x^2 = 25$$
$$x = 5$$

3.



$$x(x+5) = 3(12)$$
$$x^2 + 5x = 36$$
$$x^2 + 5x - 36 = 0$$
$$(x+9)(x-4) = 0$$
$$x = 4$$

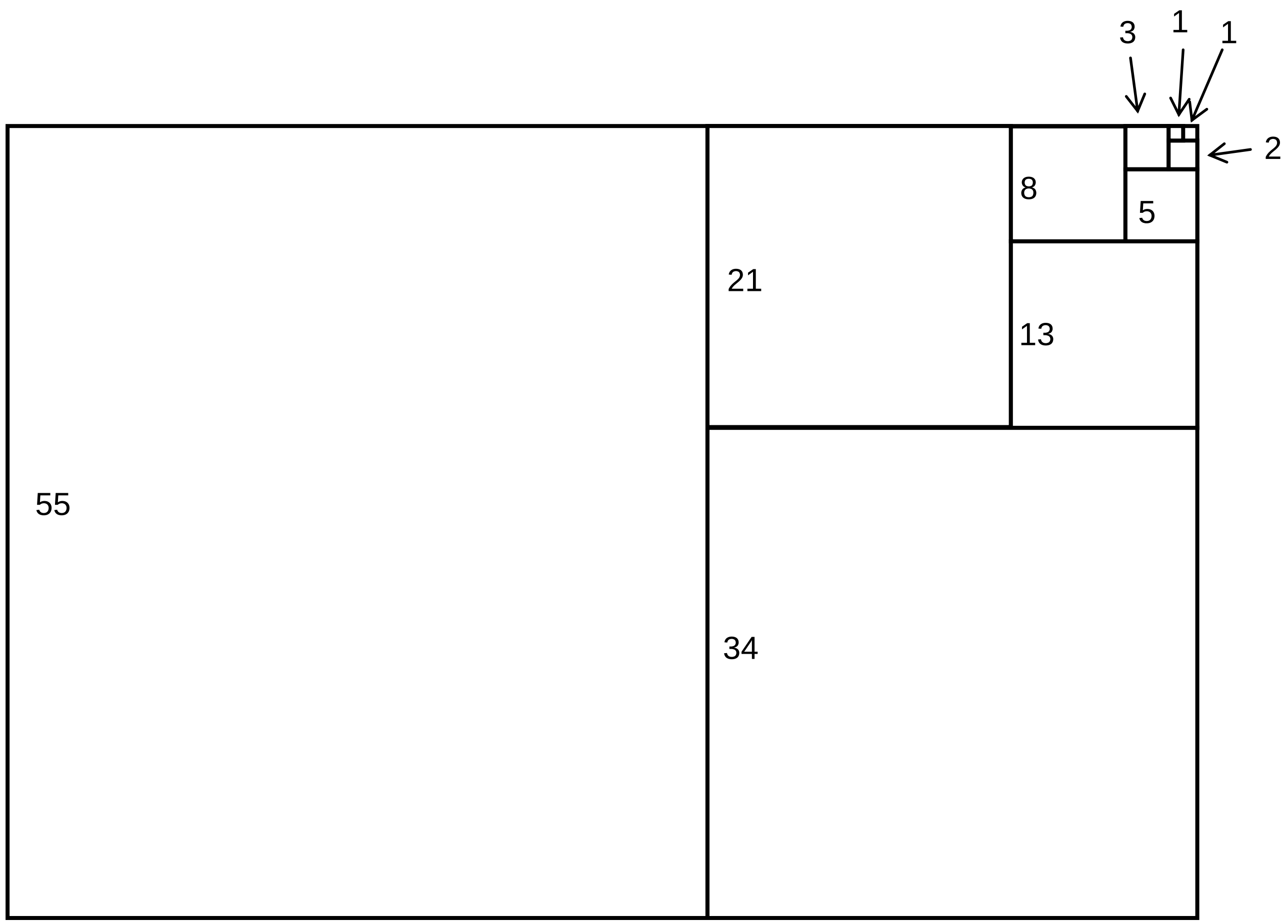
4.



$$x(x+0) = 4(11)$$
$$x^2 = 44$$
$$x = 2\sqrt{11}$$

(Treat the tangent as a secant)

A Golden Rectangle is a rectangle with the following characteristic: if a square is cut from one end of the rectangle, then the resulting rectangle has the same length to width ratio as the original.



The numbers 1,1,2,3,5,8,13,21,34,55,89,…
is called the Fibonacci sequence. The ratio of any term to the previous term approximates the Golden Ratio, and the approximation improves as the numbers get larger.

The Golden Ratio is approximately 1.618. If we figure it out algebraically, we get the exact value of

$$x = \frac{1 \pm \sqrt{5}}{2}$$

Chapter 13 Vocabulary:

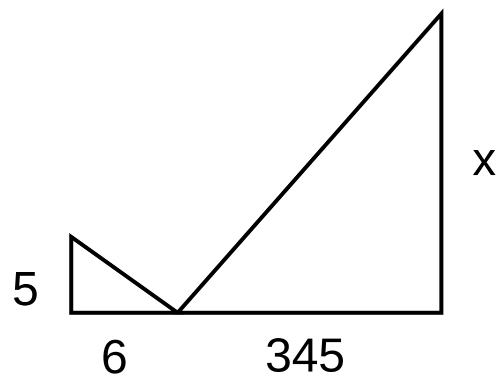
- AA Similarity Postulate
- cross multiplication
- external secant segment
- geometric mean
- Golden Ratio
- golden rectangle
- golden spiral
- proportion
- ratio
- SAS Similarity Theorem
- secant (line, plane, segment)
- segment (chord, circle)
- similar (figures, polygons)
- SSS Similarity Theorem

Notes on the test:

- Matching with terms
- Determining whether triangles are similar and why
- Calculations using similar triangle ratios
- Calculations based on the Golden Rectangle
- One Proof

Example:

Janet stands in front of the Capitol in Washington, D.C. She wants to know the building's height above the street level but cannot measure it directly. She puts a mirror on the ground 345 feet from the center of the rotunda in the Capitol and stands 6 feet from the mirror and sees the top of the Capitol reflected in it. The angle at which the image reflects is the same in both directions. What is the height of the building if her eyes are 5 feet from the ground?



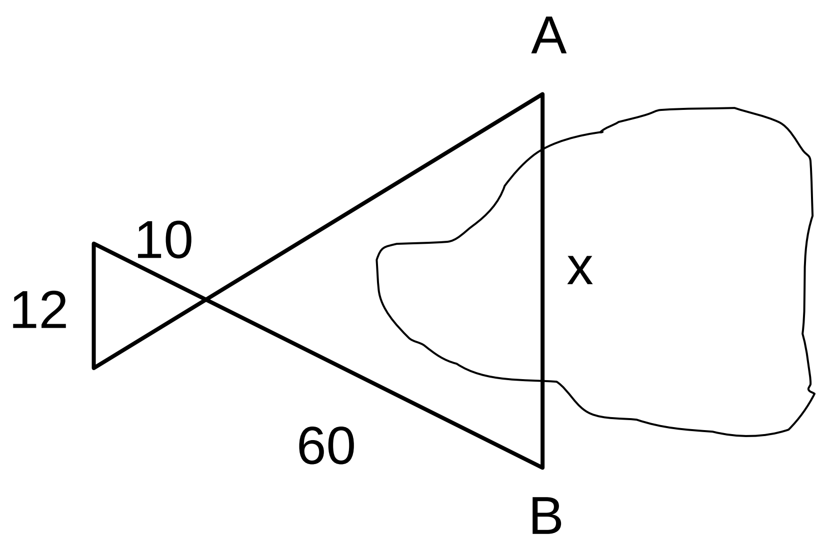
$$\frac{x}{5} = \frac{345}{6}$$

$$6x = 1725$$

$$x = 287.5$$

Example:

A state park is building a boardwalk trail for visitors through a wetland. The plan calls for the trail to cross part of the pond as it winds through the surrounding marsh and bog. How long must the bridge be to cross the pond at the points indicated?



$$\frac{x}{12} = \frac{60}{10}$$

$$\frac{x}{12} = 6$$

$$x = 72$$