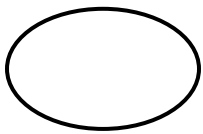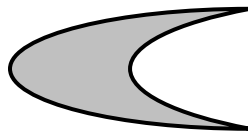section 2.5

Polygons and Convexity

Definitions:

<u>convex set</u> – has the property that any two of its points determine a segment contained in the set
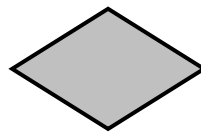
<u>concave set</u> – a set that is not convex

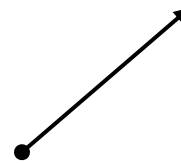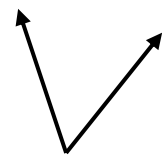concave   concave   convex   convex   concave

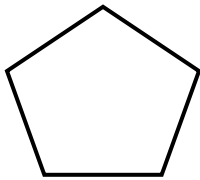## Definitions:

<u>polygon</u> – a simple closed curve that consists only of segments

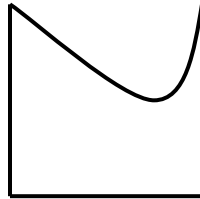<u>side of a polygon</u> – one of the segments that defines the polygon

<u>vertex</u> – the endpoint of the side of a polygon

1

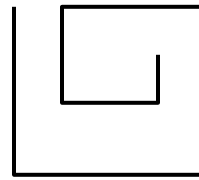<u>angle of a polygon</u> – an angle with two properties:

             1) its vertex is a vertex of the polygon

             2) each side of the angle contains a

                    side of the polygon

      polygon   not a   not a polygon

                       polygon   (called a polygonal curve)
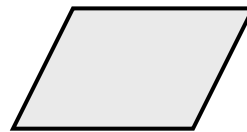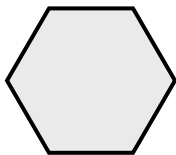
Definitions:

<u>polygonal region</u> – a polygon together with its interior
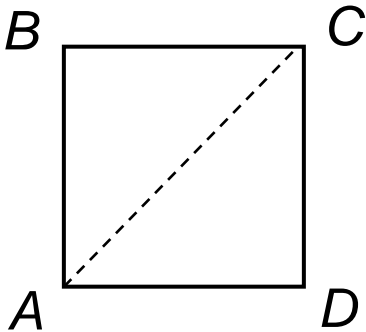
<u>equilateral polygon</u> – all sides have the same length

<u>equiangular polygon</u> – all angels have the same

                           measure

<u>regular polygon</u> – both equilateral and equiangular

      Example: A square is equilateral, equiangular,

               and regular.

<u>diagonal</u> – a segment that connects 2 vertices but is
not a side of the polygon

$\overline{AC}$ is a diagonal
$\overline{AB}$ is not a diagonal
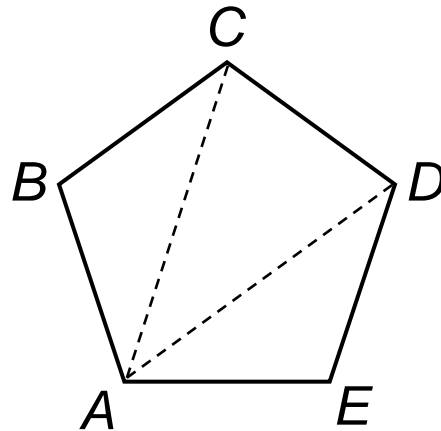
$\overline{AC}$ is a diagonal
$\overline{AD}$ is a diagonal
$\overline{AB}$ is not a diagonal

<u>Notation:</u> It does not matter which vertex you start
with, but the vertices must be listed in
order.

Above, we have square *ABCD* and pentagon *ABCDE.*

<u>interior of a convex polygon</u> – the intersection of the
interiors of is angles

<u>exterior of a convex polygon</u> – union of the exteriors
of its angles

Polygon Classification

Number of sides	Name of polygon
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
11	hendecagon
12	dodecagon
n	n-gon

Sample problem:

Classify each and tell whether it is concave or convex
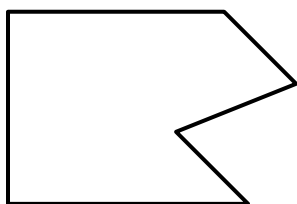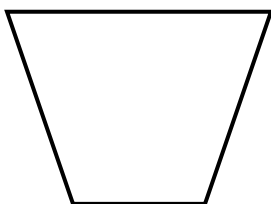
hexagon, concave   quadrilateral, convex   octagon, concave
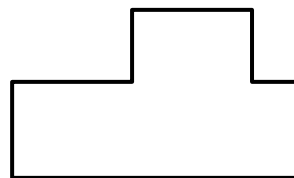
Question: If a diagonal of a polygon intersects the
exterior of the polygon, what can you
conclude?

answer: *it is concave*

Problem: Draw a polygon with a diagonal that
intersects both the interior and exterior
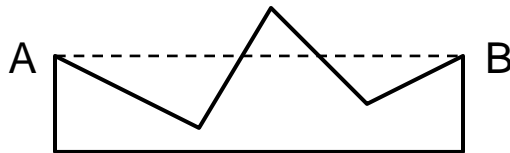
Subsets of Space

We have talked about subsets of planes (curves,
lines, polygons, regions, etc.) and now we will look at
the subsets of <u>space</u>.

Definitions:

<u>surface</u> – a connected set of points in space having
only the thickness of a point.

sphere – a surface in space consisting of the set of all
points at a given distance from a given point

center – the given point

radius – a segment that connects a point of the
sphere with the center
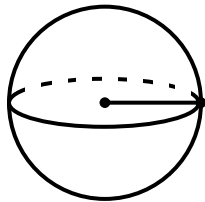
Definitions:

closed surface – surface with a finite size that divides
other points in space into an interior
and exterior

solid – the union of a closed surface and its interior

Sample Problem:

List balls that are spheres and those that are
spherical solids.

Spheres	**Spherical Solids**
basketballs	bowling balls
tennis balls	softballs
racquetballs	croquet balls
ping-pong balls	golf balls
soccer balls	medicine balls
volleyballs	billiard balls
beach balls	marbles
	Nerf balls
	pinballs

Definitions:

cone – the union of a region and all segments that connect the boundary of the region with a specific noncoplanar point called the vertex.

cylinder – the union of 2 regions of the same size and shape in different parallel planes, and the set of all segments that join corresponding points on the boundaries of the region.

lateral surface (face)

bases

lateral edge

7
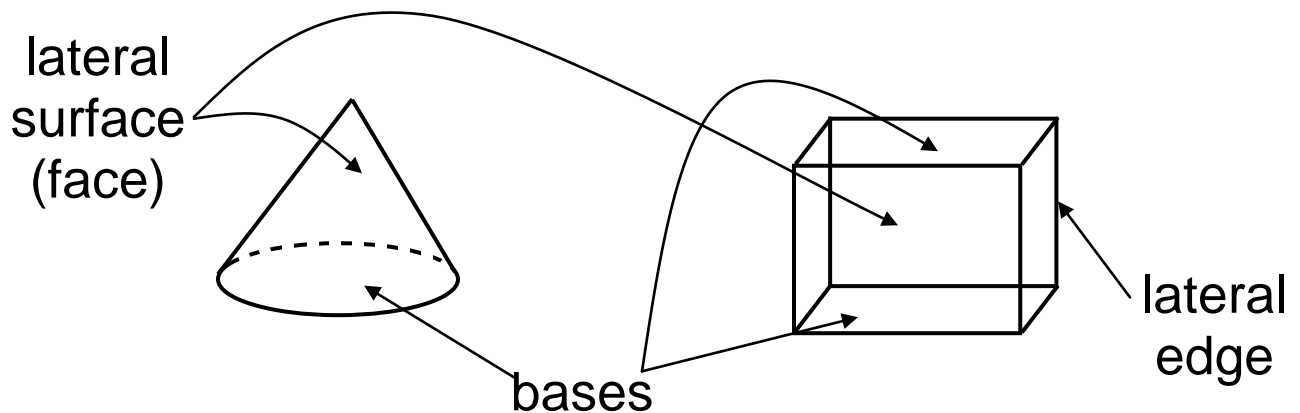
Types of cylinders and cones:

A cylinder or a cone is <u>circular</u> if each base is a circle.

A <u>prism</u> is a cylinder with polygonal regions as bases.

A <u>pyramid</u> is a cone with a polygonal region as its
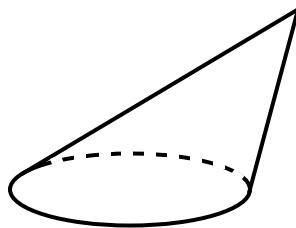base.

**Note: Cones and cylinders do not have to have
circular regions as their bases.

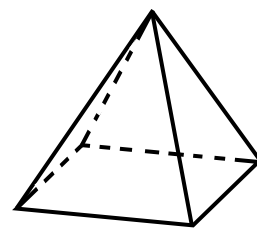**How to classify cylinders and cones:**

<u>right cone</u> – has vertex centered above the base

<u>oblique cone</u> – a cone that's not right.

oblique   right

<u>right prism</u> – segments forming the lateral surface
stand at right angles to the base

<u>oblique prism</u> – one that is not right
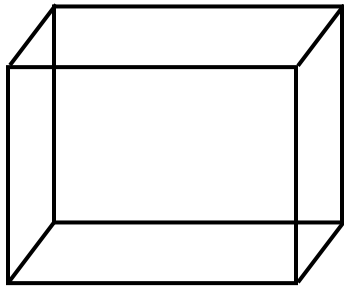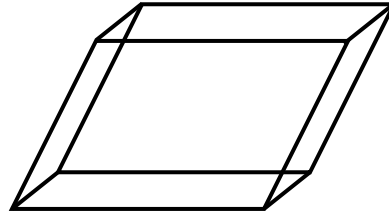
right   oblique

A pyramid or prism is <u>regular</u> if it is right and its base
is a regular polygon.

*You can <u>not</u> have a regular circular cone or cylinder.

Sample Problems:

1. Sketch a right prism that is not a regular prism.

2. Sketch a cone that is neither a circular cone nor a
    pyramid.

3. Sketch a solid cylinder that is not a right cylinder
    and has 4 lateral faces.

Polyhedra

Prisms and pyramids differ from spheres because they have flat faces. These closed surfaces are called polyhedra.

Definitions:

polyhedron – a closed surface made up of polygonal regions

face of a polyhedron – one of the polygonal regions that form the surface of the polyhedron

face of a polyhedron – one of the polygonal regions that form the surface of the polyhedron

Special Names of Polyhedra

Number of Faces	Names
4	tetrahedron
5	pentahedron
6	hexahedron
7	heptahedron
8	octahedron
10	decahedron
12	dodecahedron
20	icosahedron

Questions:

1. What is a polyhedron that is also a cone?

   a pyramid

2. What is a polyhedron that is also a cylinder?

   a prism

Definitions:

simple polyhedron – a polyhedron with no "hole" in it

regular polyhedron – a convex polyhedron having 2 properties:

      1. all faces are identical
      2. the same number of edges meet at each vertex

    Example of a regular polyhedron: dice

convex – the segment connecting any 2 points in the polyhedron is part of the polyhedron

Chapter 2 review:

<u>Note</u>: These words apply to several figures:

interior, exterior
right, oblique
side, vertex
regular
simple

<u>For the test:</u>

1. Identify figures, using proper symbols.
2. Identify plane figures. Be specific.
3. Identify space figures. Be specific.
4. Questions about definitions and theorems
5. Questions referring to figures given

Chapter 2 vocabulary:

angle
angle of a triangle
arc
bases (cone or cylinder)
between
boundary of a region
center (circle, sphere)
chord
circle
circular (cone or cylinder)
closed (curve, surface)
concave
cone
convex
cylinder
curve
decagon
decahedron
diagonal of a polygon
diameter
dodecagon
dodecahedron
edge (half-plane, polyhedron)
end point
equiangular
equilateral
exterior
face of a polyhedron
half-line
half-plane
hendecagon
heptagon
heptahedron
hexagon

hexahedron
icosahedron
interior
lateral (edge, face, surface)
n-gon
nonagon
oblique
octagon
octahedron
opposite (half-planes, rays)
origin
pentagon
pentahedron
polygon
polygonal region
polyhedron
prism
pyramid
quadrilateral
radius (circle, sphere)
ray
region
regular
right
segment
sides
simple (curve, closed curve)
simple polyhedron
solid
sphere
surface
tetrahedron
triangle
vertex