section 3.1

Real Numbers

Sets of Numbers

N Natural Numbers: {1,2,3,…}

W Whole Numbers: {0,1,2,3,…}

Z Integers: {…-3,-2,-1,0,1,2,3,…}

Q Rational Numbers: {$\frac{p}{q}$ | p,q are integers and $q \neq 0$}
(i.e. numbers that can be written as simple fractions, including repeating decimals)

Ir Irrational Numbers: {all numbers that are not rational}
(i.e. non-repeating, non-terminating decimals)

R Real Numbers: {Rationals } U {Irrationals}

C Complex Numbers: {Reals} U {Imaginary numbers}

$$\mathbf{N} \subset \mathbf{W} \subset \mathbf{Z} \subset \mathbf{Q} \subset \mathbf{R} \subset \mathbf{C}$$

Properties of Real Numbers

Property	Addition	Multiplication
Commutative	a+b = b+a	ab = ba
Associative	(a+b)+c=a+(b+c)	(ab)c = a(bc)
Distributive		a(b+c) = ab+ac
Identity	a+0 = 0+a = a	a·1 = 1·a = a
Inverse	a+(-a) = 0	a·(1/a) = 1

Equality Properties	
Property	**Meaning**
Addition	If a=b, then a+c = b+c
Multiplication	If a=b, then ac=bc

More Equality Properties	
Reflexive	a=a
Symmetric	If a=b, then b=a
Transitive	If a=b and b=c, then a=c

Sample Problems: Name the Property

	Answers:
1. 3+4=4+3	*Commutative of Add.*
2. (3+4)+5 = 3+(4+5)	*Associative of Add.*
3. 3(4+5) = 3·4+3·5	*Distributive*
4. (8+2)+7 = (2+8)+7	*Commutative of Add.*
5. 7+(-7) = 0	*Additive Inverse*
6. 9·(7·10) = (7·10)·9	*Commutative of Mult.*

Definition:

An <u>equivalence relation</u> is a relation that is reflexive, symmetric, and transitive.

Example: Test the relation "is in the same family."

<u>Reflexive</u>: Bill is in the same family as Bill.

TRUE, so the relation is reflexive.

<u>Symmetric</u>: If Jan is in the same family as Joe, then Joe is in the same family as Jan.

TRUE, so the relation is symmetric.

<u>Transitive</u>: If Manda is in the same family as Chris and Chris is in the same family as Karen, then Manda is in the same family as Karen.

TRUE, so the relation is transitive.

Therefore, the relation "is in the same family" **<u>is</u>** an equivalence relation.

***<u>Equality</u> is another equivalence relation. It is one of the most important that we will study!!!!

Order of Operations

1. Grouping Symbols
2. Exponents
3. Multiply and Divide, left to right
4. Add and Subtract, left to right

Sample Problems: Simplify the following.

1. $|4-7|+2^3-(7+9\div 3)$

$$|-3|+2^3-(7+3)$$

$$3+2^3-10$$

$$3+8-10$$

$$11-10$$

$$1$$

2. $(6\div 3-5)^2+\sqrt{3^2+4^2}\div 5\cdot 4\div 2-1$

$$(-3)^2+\sqrt{25}\div 5\cdot 4\div 2-1$$

$$9+5\div 5\cdot 4\div 2-1$$

$$9+1\cdot 4\div 2-1$$

$$9+2-1$$

$$10$$

Properties:

<u>Substitution Property</u>: If a=b, then a can replace b in any mathematical statement.

<u>Trichotomy Property</u>: For any two real numbers a and b, exactly one of the following is true: $a=b$, $a>b$, or $a<b$.

Definition:

The absolute value of a number a is denoted as |a|, where:

$$|a| = \begin{cases} a \text{ if } a \geq 0 \\ -a \text{ if } a < 0 \end{cases}$$

*The absolute value is the distance that a number is from zero on the number line. Since distance is always positive, absolute value is always positive.

section 3.2

Segment Measure

Measurement refers to a basis of comparison.

- Early measurement units – a king’s forearm or foot
- Today’s measurement units – standardized, but not uniform (We still have 2 main systems – English and metric.)

Linear Measurement

Linear measurement is finding the length of a segment *or* the distance between 2 points.

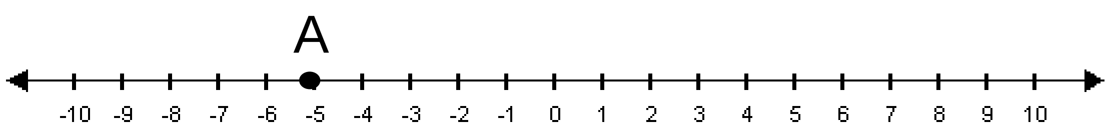
Ruler Postulate: Every point of a line can be place in correspondence with a real number.

(i.e. Every point on a line corresponds to a number on a ruler or a number line.)

Definition:

The coordinate of a point on a line is the number that corresponds to the point.

The graph of a number is the point on the number line that corresponds to it.



The *coordinate* of point A is -5.
Point A is the *graph* of the number -5

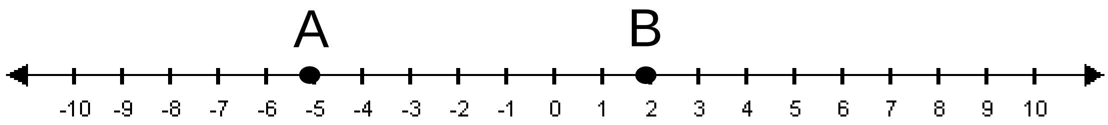
*When we measure length, we use a ruler. Number lines are our rulers for line.

<u>To Measure the distance on a number line, we can</u>

1. position zero at the left point and read the coordinate of the other point.
2. count the units
3. subtract

Definition: The <u>distance</u> between two points A and B is the absolute value of the difference of their coordinates. Distance between points A and B is denoted by AB, given by AB = |a-b|

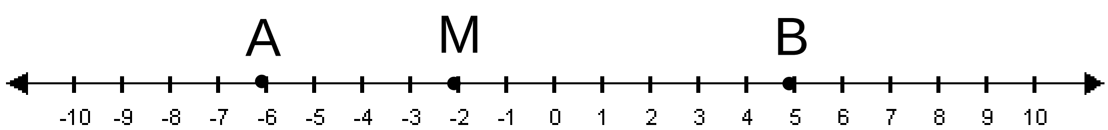
Note: Use capital letters to name points and lower-cased letters for the coordinates.



AB = | a-b | = | -5 – 2 | = | -7 | = 7
AB = | b-a | = | 2- (-5) | = | 7 | = 7

* It does not matter what order you subtract.

Betweenness



By looking at this number line we can see that M is between A and B. Now we can refine our definition of between.

Definition:

A point M is between A and B if AM + MB = AB

notation: A-M-B or B-M-A

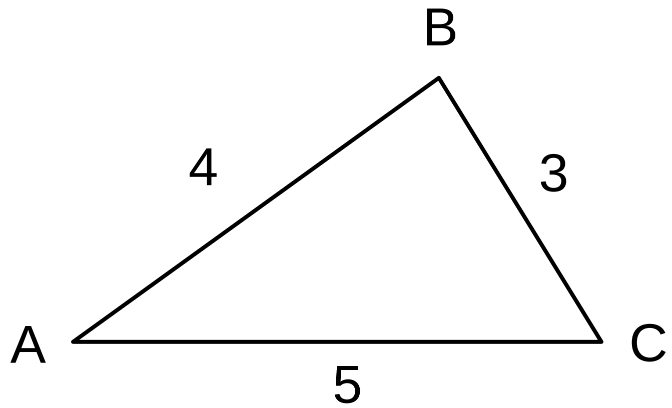
**Notice that if AM+MB=AB, then the points are collinear

Above, we see that

$$AM = |-6-(-2)| = |-6+2| = |-4| = 4$$
$$MB = |-2-5| = |-7| = 7$$
$$AB = |-6-5| = |-11| = 11$$

Then AM + MB = AB gives us 4 + 7 = 11

Look at the following triangle.



B is <u>not</u> between A and C since AB + BC ≠ AC

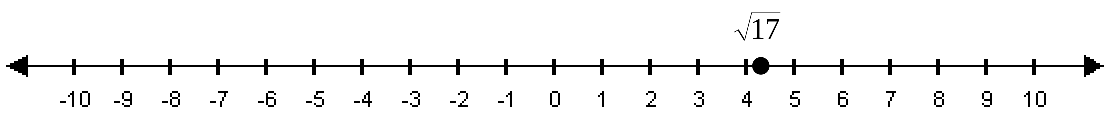
$4 + 3 \neq 5$

Sample Problems

1. Graph $\sqrt{17}$ on a number line.

$$\sqrt{16} < \sqrt{17} < \sqrt{25}$$

$$4 < \sqrt{17} < 5$$



2. What is the distance between 64 and -13?

$$|64-(-13)| = |77| = 77$$

3. Is A = -1 between B = -5 and D = 6?

(Check BA + AD = BD)

$$|-5-(-1)| + |-1-6| = |-5-6|$$
$$|-4| + |-7| = |-11|$$
$$4 + 7 = 11$$

Yes, A *is between* B *and* D.

<u>Completeness Postulate</u>: Given a ray, AB, and any positive real number *r*, there is exactly one point C on the ray so that AC = *r*.

(This postulate guarantees a point at every distance. There are no "holes" in a line.)

section 3.3

Segment Bisectors

Definition:

The midpoint of $\overline{AB}$ is M if A-M-B and AM = MB.

****Remember**: $\overline{AB}$ represents a *segment*
AB represents the *length* of $\overline{AB}$

<u>Midpoint Theorem</u>: If M is the midpoint of AB, then
AM = ½AB

Proof:

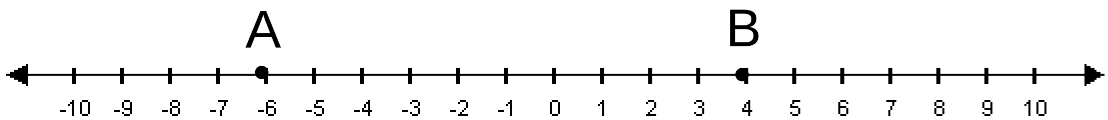
Statements	Reasons
1. M is the midpoint of $\overline{AB}$	1. Given
2. AM = MB and A-M-B	2. Definition of midpoint
3. AM + MB = AB	3. Definition of betweenness
4. AM + AM = AB	4. Substitution (step 2 into 3)
5. 2AM = AB	5. Distributive Property
6. AM = ½AB	6. Mult. Prop. of Equality

* Remember that theorems need to be proven, no matter how obvious they seem.

<u>Formula for finding the midpoint</u>:

If the midpoint of $\overline{AB}$ is M, then $m = \dfrac{a + b}{2}$

where m,a,and b are coordinates of M, A, and B.



The midpoint of $\overline{AB}$ is $\dfrac{-6+4}{2} = \dfrac{-2}{2} = -1$

Definitions:

A bisector of a segment is a curve that intersects the segment only at the midpoint.

Congruent segments are segments that have the same length.
The symbol is ≅ .

*Remember: When talking about sets, we know that equivalent sets have the same number of elements and equal sets have the same elements in each set.

- Equivalent segments are called congruent
- Segments consisting of identical sets of points are called equal.

Congruent segments have the same length, whereas equal segments share the same set of points.

- If $\overline{AB} = \overline{CD}$, then A and C must be the same points, and B and D must be the same points.

- If AB = CD, then the length of $\overline{AB}$ is the same as the length of $\overline{CD}$.

In summary:

- Segments $\overline{AB}$ and $\overline{CD}$ are not generally equal.

- If $\overline{AB} \cong \overline{CD}$, the segments are congruent and have the same length.

- If $\overline{AB} = \overline{CD}$, then the segments are identical sets of points.

- If AB = CD, the *length* of $\overline{AB}$ equals the *length* of $\overline{CD}$.

Sample Problems: True/False (with reasons)

1. If $\overline{AK} \cong \overline{BR}$, then AK = AR

   False. It should say AK = BR

2. If PQ = QW, then Q is the midpoint of PW.

   False. P, W, *and* W *may not be collinear*

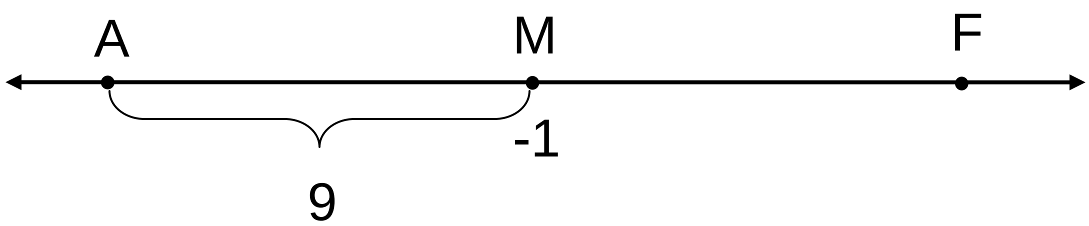
3. If S is the midpoint of $\overline{GP}$, then G-S-P.

   True. The midpoint of a segment is between the endpoints.

4. If BX + XC = BC, then X is the midpoint of BC.

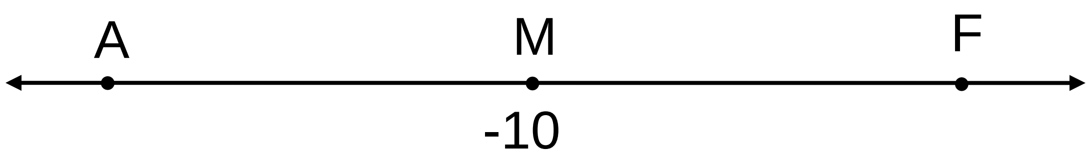
False. X *can be anywhere between* B *and* C.

Sample Problem: Suppose M is the midpoint of $\overline{AF}$ and that AM = 9. If M = -1, find the coordinates of A and F.



Solution: If AM = 9, then the coordinate of A must be -10, since M = -1. If we add 9 units to the right of M, we find that F must be 8.

Sample Problem: Suppose M is the midpoint of AF and that AM = 14. If m = -10, find the coordinates of A and F.



A = -10 – 14 = -24
F = -10 + 14 = 4

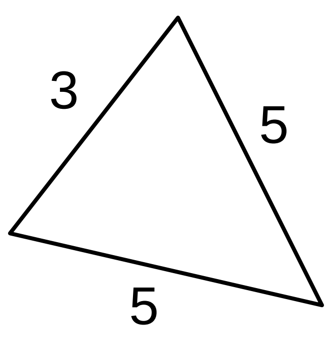
section 3.3

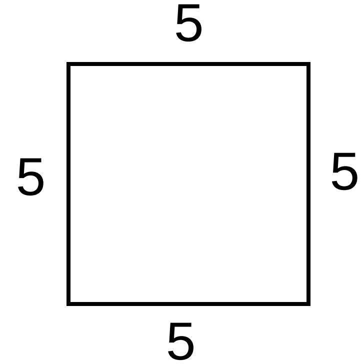
Perimeter and Circumference

Definition:

perimeter – the distance around a closed curve

"peri" means "around"
"metron" means "measure"





P = 3+5+5 = 13

P = 5+5+5+5 = 4(5) = 20

Theorem 3.2 The perimeter of a regular n-gon with sides of length s is $n \cdot s$.

Sample Problems:

1. The perimeter of a regular polygon is 108 cm and the sides are 12 cm each. What kind of polygon is it?

   a nonagon

2. The perimeter of a square is 64 cm. What is the length of each side?

   16 cm

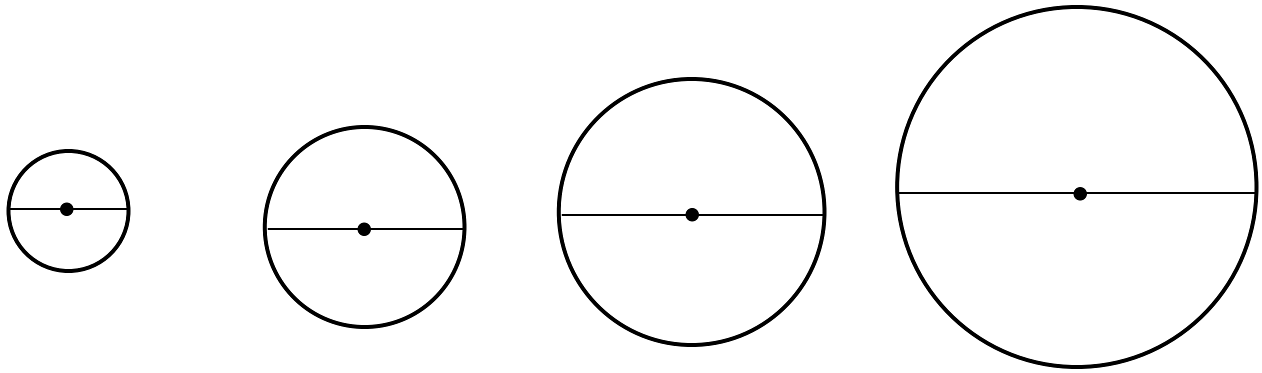
3. The perimeter of a parallelogram is 96 inches. If the long sides are 3 times as long as the short sides, what are its dimensions?

   12 in. and 36 in.

Definition:

Circumference is the distance around a circle.

"circum" means "around"



d = 2, C = 6.28

d = 3, C = 9.42

d = 7, C = 21.98

d = 12, C = 37.68

Find $\frac{C}{d}$ for each circle above.

***The ratio of $\frac{C}{d}$ is always the same number. We call this number pi or π.

$$\pi \; . \; 3.14159265\ldots$$

Since $\pi = \frac{C}{d}$ we get the following formulas:

<u>Formula for circumference</u>: C = πd

<u>*or*</u> C = $2\pi r$

where d is the diameter and r is the radius