section 4.6

Construction 4: Bisect an Angle

Given: $\angle ABC$

1. Place the point of the compass at B and mark an arc on both sides (rays) of the angle.

2. Move the point of the compass to the point of intersection between an arc and side. Mark an arc in the interior of the angle. Repeat this process at the other arc and side intersection. The interior arcs intersect at a point C.

3. Draw $\overrightarrow{BD}$ to form the angle bisector.

Construction 5: Copy an Angle

Given: $\angle ABC$

1. Draw a ray, $\overrightarrow{B'C'}$ with a straightedge.

2. Place the point of the compass on the given angle at B and construct an arc that intersects both sides of the given angle.

3. Without changing the compass, place its point on B′ and construct an arc that corresponds to the arc on the given angle.

4. Adjust the compass to measure the length between the two intersection points of the arc on the given angle.

5. Using this measurement, place the point of the compass at the intersection of the arc and the constructed ray. Draw a short arc that intersects the other arc. Label the point of intersection A′.

6. Connect B′ with A′ to form $\overrightarrow{B'A'}$ and thus $\angle A'B'C'$, which is congruent to $\angle ABC$.

Chapter 4 Vocabulary

acute angle
acute triangle
addition property
adjacent angles
Angle Addition Postulate
angle bisector
base of cone
base of cylinder
base of triangle
base of trapezoid
complementary angles
congruent angles
consecutive angles
consecutive sides
Continuity Postulate
degree measure
equilateral triangle
graph of an inequality
greater than
isosceles triangle
isosceles trapezoid
legs of trapezoid
legs of triangle
less than
linear pair
measure of an angle
multiplication property
obtuse angle
obtuse triangle
opposite angles
opposite sides
parallelogram
perpendicular lines
protractor
Protractor Postulate
rectangle
rhombus
right angle
right cone
right cylinder
right triangle
scalene triangle
square
straight
angle
supplementary angles
transitive property
trapezoid
vertical angles

Definition:

Reasoning is the step-by-step process that begins with a known fact or assumption and builds to a conclusion in an orderly, concise way. This is also called logical thinking.

Proverbs 4:7 "Though it cost you all you have, get understanding."

*****To understand, you must be able to reason effectively!!**

We will sometimes use tables and Venn diagrams to solve logic problems. Other times it will require trial and error.

Example:

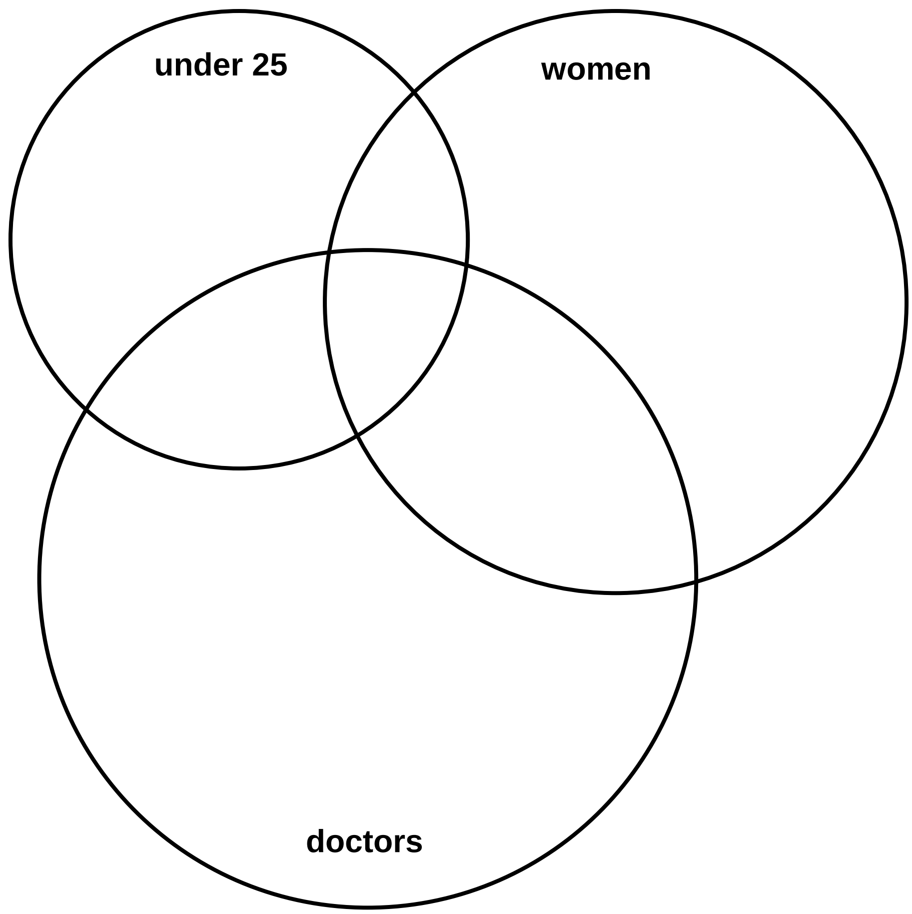
The last names of Fernando, Helena, and Jennifer are Grayson, Kraft, and Landers. Each person joined one of the U.S. Armed Forces – army, marines, or navy. Find each person's full name and armed service branch.

1. Landers is not a marine.
2. Grayson likes being in the army and tried unsuccessfully to talk Helena into joining.
3. The marine said he didn't like the basic training period.

	Grayson	Kraft	Landers	army	marines	navy
Fernando						
Helena						
Jennifer						
army						
marines						
navy						

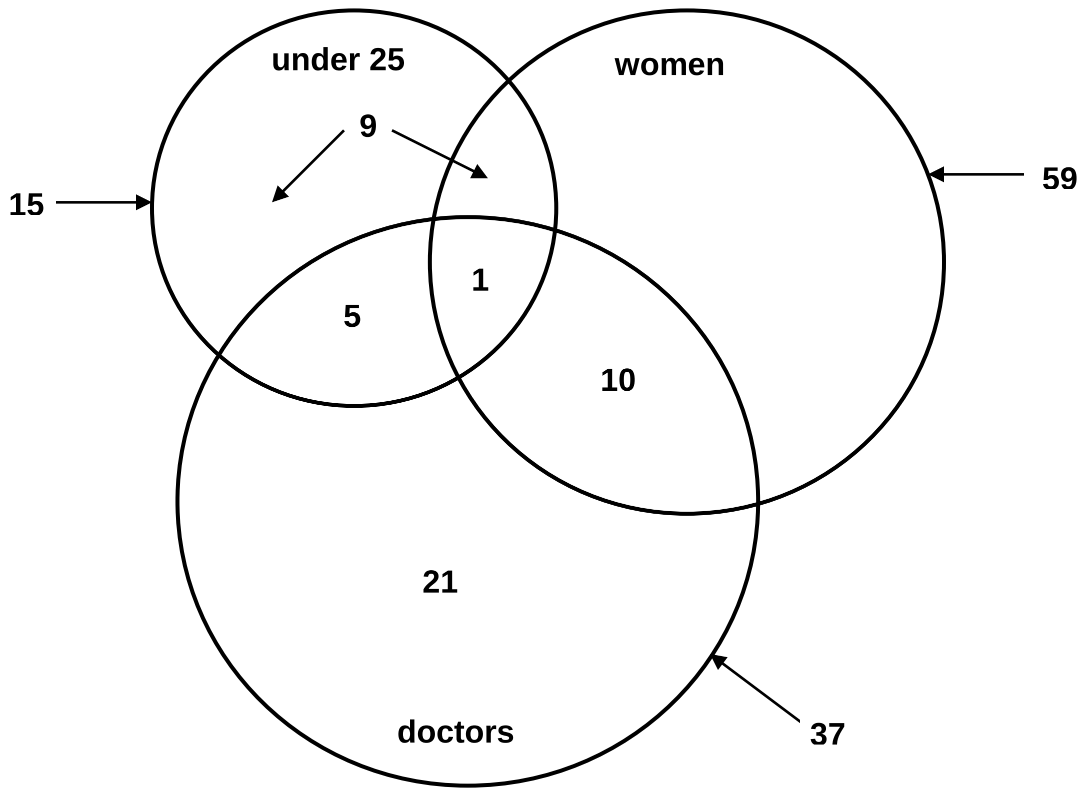
Venn Diagram Example:

Memorial Hospital employs 37 doctors and 59 women. Of the 15 employees under age 25, 9 are not doctors. Dr. Novak is the only woman doctor under the age of 25, though 10 other doctors are women. How many doctors are over 25?



Venn Diagram Example (with answers):

Memorial Hospital employs 37 doctors and 59 women. Of the 15 employees under age 25, 9 are not doctors. Dr. Novak is the only woman doctor under the age of 25, though 10 other doctors are women. How many doctors are over 25?



1. Put a 1 in the very center spot since Dr. Novak is the only one to fit all 3 categories.
2. If 9 of the 15 people under 25 are <u>not</u> doctors, then 6 of them must <u>be</u> doctors. There is already a 1 in the center spot, so the spot to the left of that must be 5, so that the overlap of “doctors” and “under 25” equals 6.
3. There are 11 women doctors. There is already a 1 in the center spot, so the spot to the right of that must be 10, so that the overlap of “women” and “doctors” equals 11.
4. We can fill in the “doctor” circle with 21, since the entire circle must equal 37 and we know that the 3 smaller spots are 5, 1, and 10. (37 – 5 – 1 – 10 = 21)
5. The number of doctors over 25 must be the amount in the “doctor” circle but not in the “under 25” circle. Thus it must be 21 + 10 = 31.
6. There are 31 doctors over 25.

*Note: Some problems don't use either the table or a Venn diagram, like #13 on page 165.

Look at:

Abraham Lincoln was assassinated.
10 – 3 = 6
x + 7 = 12
Jesus is God's Son.
2 · 11 = 22

Definition:

A statement is a sentence that is either true or false, but not both.

Question: Which of the above sentences are statements?

1,4,5 are true statements
2 is a false statement
3 is not a statement (until the value of x is defined)

Sample Problems: Determine whether the following are statements, and if so, whether they are true or false.

1. 3 + 5 = 6

statement, false

2. Why should we vote?

not a statement

3. Parallel lines intersect.

statement, false

4. $3x - 9 = 3(x - 3)$

statement, true

5. This sentence is false.

not a statement

6. $x^2 - 2x + 1 = (x-1)^2$

statement, true

7. $x + 4 = 10$

not a statement

8. Shut the door.

not a statement

**Mathematical reasoning and true conclusions are built on a series of statements.

Statements and Negations:

Statement: A cow is an animal.
Negation: A cow is not an animal.

Note: A statement and its negation must have opposite truth values (one is true and one is false.)

Notation: We use letters like **p** or **q** to represent statements. We use **~p** and **~q** for their negations.

Examples:

p: Parallel lines are coplanar. (true)
~p: Parallel lines are not coplanar. (false)

q: A year is a 12-month period. (true)
~q: A year is not a 12-month period. (false)

r: An apple is not a fruit. (false)
~r: An apple is a fruit. (true)

Universal Quantifiers:

All men are sinners.

"All" and "Every" are called <u>universal quantifiers</u>.

<u>Examples</u>:

All puppies are dogs.
Every girl is a female.
All animals breathe.
Every good gift is from God.

**The symbol we use for universal quantifiers is ∀. It is usually read "All", "Every" or "For every."

<u>Example</u>:

p: Female birds lay eggs.
∀p: All female birds lay eggs. <u>or</u> Every female bird lays eggs.

<u>Examples of universally quantified statements:</u>

All reptiles are cold-blooded.
Every test is graded.
No cow is a bull.
(For every cow, the cow is not a bull.)
No thief is honest.
(For every thief, the thief is not honest.)

**These last 2 are negative statements, but they are still considered universally quantified.

Existential Quantifiers

There exists a mammal that lays eggs.

Another type of quantifier is the existential quantifier, which implies "one or more."

**The symbol for the existential quantifier is ∃.
It is usually read "There exists a …"

Example:

p: Girls like to ride horses.
∃p: There exists a girl who likes to ride horses.

Examples of existentially quantified statements:

There is a boy in our class with a broken arm.
Some cats are gray.
(There exists a cat that is gray.)
There exists a bird with pink feathers.
Some reptiles are extinct.
(There exists a reptile that is extinct.)

Combination of symbols:

$\forall$ person $\exists$ a mother

For every person there exists a mother.

** One more math symbol: $\ni$ means “such that”

Example:

$$\forall\ x>0,\ \exists\ y<0 \ni x + y = 0$$

For every x>0, there exists y<0 such that x + y = 0
(definition of opposites)

Negating Statements with Quantifiers

****If the statement is not quantified, we usually just have to negate the predicate (verb).**

Example:

p: Water is a liquid.
$\sim$p: Water is not a liquid.

<u>Question</u>: What is the negative of the following:

Some people have blue eyes.

Many people say it is:

Some people don't have blue eyes.

Both of these statements are true. But negations are supposed to have opposite truth values. The correct negation is:

No people have blue eyes.

****To negate quantified statements:**

1. Negate the sentence.
2. Switch to the opposite quantifier.

Examples:

p: All flowers are pretty.
$\forall$ flowers are pretty.

~p: $\exists$ flowers that are not pretty.
Some flowers are not pretty.

To better understand this, let's look at some false sentences and think about how we would negate it to show what is true:

p: All Chinese men are too short to play basketball in the NBA.

Would we have to prove that <u>all</u> Chinese men are tall enough to play in the NBA to prove this false? No, we only need one!

~p: There is a man (Yao Ming) who is not too short to play in the NBA.

Using symbols, we get:

p: ∀ Chinese men are too short to play basketball in the NBA.

To negate this we change the quantifier and negate the sentence:

~p: ∃ a man (Yao Ming) who is <u>**not**</u> too short to play in the NBA.

Let’s look at a couple more:

q: There is a man who can breathe under water.

To negate this statement, we wouldn’t say:

“There is a man who can<u>not</u> breathe under water.”

You would read that and think “Of course there is a man who cannot breathe under water because <u>**no**</u> man can breathe under water!

The correct negation would be:

~q: No man can breathe under water.

Using symbols for these statements we get:

q: ∃ a man who can breathe under water.

To negate this we change the quantifier and negate the sentence:

~q: ∀ man, he can<u>not</u> breathe under water.

(For every man, he cannot breathe under water.)

Notice how it makes more sense to use the word “no?”

r: Some people will be saved by their works alone.

To negate this statement, we wouldn't say:

"Some people will <u>not</u> be saved by their works alone."

Because since the first statement is false, we need the negation to be true, and saying that *some* people won't be saved by their works alone is not true. It would lead us to believe that some will and some won't. The truth is that **<u>no one</u>** will be saved by works alone.

The correct negation would be:

~r: No person will be saved by their works alone.

In symbols:

r: $\exists$ people who will be saved by their works alone.

~r: $\forall$ person, he will <u>not</u> be saved by their works alone.

s: No mammals lay eggs.

How would I prove this to be false? By coming up with just one example to refute it.

We know that there really is a mammal that lays eggs. It is the duck-billed platypus. So to negate the statement we would say:

~s: There is a mammal who lays eggs.

In symbols it would look like this:

s: ∀ mammal does <u>not</u> lay eggs.

(Every mammal does not lay eggs.)

~s: ∃ a mammal who lays eggs.

Think of yourself as a lawyer trying to disprove a statement!

Now let's look at some general examples.

q: There exists a student who gets straight A's.
$\exists$ student who gets straight A's.

$\sim$q: $\forall$ students don't get straight A's.
All students don't get straight A's.

This is <u>ambiguous</u> and doesn't mean what we want it to mean. It could still mean that ***some* students ***do*** get straight A's. We should write:

No students get straight A's.

p: Some dogs have short hair.
$\exists$ a dog who has short hair.

$\sim$p: $\forall$ dog does not have short hair.
Every dog does not have short hair.
or
All dogs do not have short hair.

This is <u>ambiguous</u> because it could still mean that ***some* students ***do*** have short hair. We should write:

No dogs have short hair.

q: Some pies are not cherry.
$\exists$ a pie that is not cherry.

~q: $\forall$ pies are cherry.
All pies are cherry.

p: No square is a circle.
$\forall$ square, the square is not a circle.

~p: $\exists$ a square that is a circle.
There exists squares that are circles.
or
Some squares are circles.

<u>Common errors:</u>

p: Some A are B ~p: Some A are not B

p: Some dogs are small. ~p: Some dogs are not small.

**Both statements are true, so there's a problem!!!

~p should be: No dogs are small.

Negations are kitty-corner from each other.

	universal	existential
positive statement	All A are B	Some A are B
negative statement	No A are B	Some A are not B

Sample Problems: Negate the following.

1. p: Ripe tomatoes are red.

~p: Ripe tomatoes are not red.

2. p: Some dogs have fleas.

~p: No dogs have fleas.

3. p: All men are mortal.

~p: Some men are not mortal.

4. Green is a color.

~p: Green is not a color.

5. No boy has a football.

~p: Some boy has a football.

6. Some men are not tall.

~p: All men are tall.

Example on page 162:

		Captain	1st Mate	Cook	Mechanic
			brother to Mechanic		brother to 1st Mate
Phil's uncle	Bob				
	Jim				
Bob's nephew	Phil				
	Brent				

Example on page 162 with answers:

1. Bob and Phil can't be the captain because the captain has no relatives.
2. Brent has a relative so he can't be the captain.
3. Jim must be the captain. (Fill in the rest of the row with N's)

		Captain	**1st Mate**	**Cook**	**Mechanic**
			brother to Mechanic		brother to 1st Mate
Phil's uncle	**Bob**	**N**			
	Jim	**Y**	**N**	**N**	**N**
Bob's nephew	**Phil**	**N**			
	Brent	**N**			

4. Guess that Phil is one of the brothers. Then Phil is not the cook. Since Bob is Phil's uncle (not brother), that would mean Bob is the cook. But the cook is not the uncle of the mechanic (or his brother the 1st mate). So Phil is not one of the brothers and thus must be the cook. (Fill in the rest of the row and column with N's)

			brother to Mechanic		brother to 1st Mate
		Captain	**1st Mate**	**Cook**	**Mechanic**
Phil's uncle	**Bob**	**N**		**N**	
	Jim	**Y**	**N**	**N**	**N**
Bob's nephew	**Phil**	**N**	**N**	**Y**	**N**
	Brent	**N**		**N**	

5. Since the 1st mate is not the cook's uncle, then Bob can't be the 1st mate since he <u>is</u> the cook's uncle (because the cook is Phil). So Brent must be the 1st mate and Bob is the mechanic.

			brother to Mechanic		brother to 1st Mate
		Captain	**1st Mate**	**Cook**	**Mechanic**
Phil's uncle	**Bob**	**N**	**N**	**N**	**Y**
	Jim	**Y**	**N**	**N**	**N**
Bob's nephew	**Phil**	**N**	**N**	**Y**	**N**
	Brent	**N**	**Y**	**N**	**N**

6. Bob and Brent are brothers and Phil is Bob's nephew.