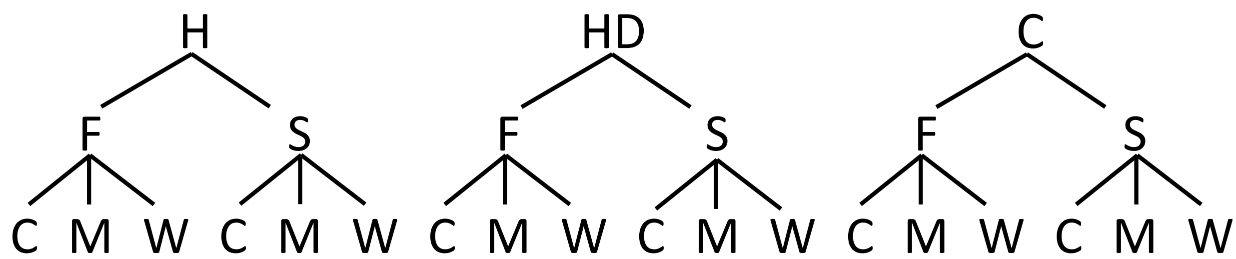
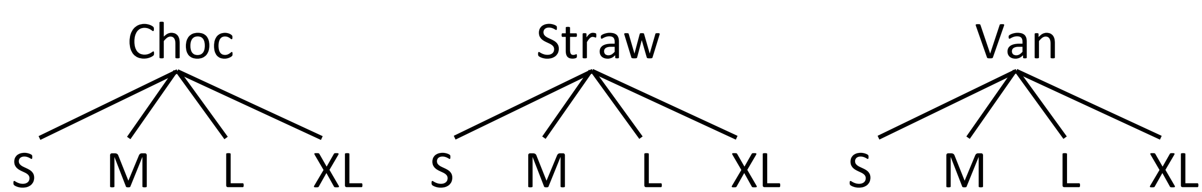
Fundamental Principle of Counting

How many ways can you order dinner if you can choose a sandwich (hamburger, hot dog, or chicken) a side (fries or coleslaw) and a drink (coffee, milk, or water)?



Sample Problem: Make a tree diagram to find the number of possible milkshakes if flavors are chocolate, vanilla, and strawberries, and the sizes are small, medium, large, and x-tra large.

Solution:



Fundamental Principle of Counting – If there are p ways to make a first choice and q ways to make a 2nd choice, then there are $p \cdot q$ possible combinations to make the 1st choice and then the 2nd choice.

Example: Sam has 5 shirts and 4 ties. There are 4·5=20 ways he can pair a shirt with a tie.

Sample Problems:

1. How many 3-digit area codes are possible if the first number can't be 0 or 1?

8	10	10

= 800

2. How many different license plates are possible in Minnesota? (3 letters and 3 numbers)

26	26	26	10	10	10

= 17,576,000

3. How many license plates (3 letters and 3 numbers) if there are no repeats?

26	25	24		10	9	8	=	11,232,000

4. How many ways can a family of 4 line up for a photo?

4	3	2	1	=	24

5. How many padlock combinations are possible if the combination consists of 3 numbers from 1 to 40?

40	40	40	=	64,000

6. How many combinations are possible if the combination consists of 3 numbers from 1 to 40 and no consecutive integers are the same?

39	40	39	=	60,840

7. How many ways can you seat 10 couples in a row of 10 chairs, assuming that each couple is seated together?

10	1	8	1	6	1	4	1	2	1	=	3840

8. How many different 2-digit numbers can be formed if the 1st digit is non-zero and even, and the 2nd digit is less than 7 but greater than 0?

4	6	=	24

9. A restaurant has 5 waiters, 4 cooks, 2 hostesses, and 7 kitchen staff. How many ways can you choose a cook and hostess?

4	2	=	8

10. How many ways can you choose a group of 3 people if exactly 1 is a hostess and 1 is a waiter?

2	5	11	=	110

Permutations

How many ways can a group of 3 students rearrange themselves in different ways?

3	2	1

= 6

How many ways can 5 roles in a play be assigned if 5 people try out?

5	4	3	2	1	=	120

<u>Factorial Notation</u> – When we want to multiply natural numbers from some number all the way to 1 we use factorial notation. The symbol is "!".

<u>Examples</u>:

$3! = 3\cdot2\cdot1=6$ $4!=4\cdot3\cdot2\cdot1=24$ $5!=5\cdot4\cdot3\cdot2\cdot1=60$

<u>Note</u>: By definition, $0! = 1$

Definition – When we want to arrange objects of a group in a particular order we use permutations.

When you find the number of permutations of a group of objects, when using all of the objects, we use ${}_nP_n$ which is read "the number of permutations of n objects taken n at a time."

$$ {}_nP_n = n(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1 $$

Sample Problems:

1. Find the number of permutations of the letters in the word LOST.

4	3	2	1	=	24

or $\quad {}_4P_4 = 4\cdot 3\cdot 2\cdot 1 = 24$

2. Find the number of ways that 6 books can be arranged on a shelf.

6	5	4	3	2	1	= 720

or ${}_6P_6 = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24$

Factorial Calculation Sample Problems:

1. 2(3!)

2. (3!)(4!)

3. 8(0!)

4. $\frac{6!}{4!}$

5. $\frac{8!}{5!3!}$

6. $\frac{12!}{9!3!}$

What if I wanted to find the number of permutations of the letters in LOST but only at a time?

From what we know,

4	3	= 12

We can use permutation notation for this. It looks like this:

$$_nP_r$$

which is read "The number of permutations of n objects taken r at a time."

The formula for $_nP_r$ is $_nP_r = \frac{n!}{(n-r)!}$

Let's look at our previous example:

$$_4P_2 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4\cdot 3\cdot \cancel{2}\cdot \cancel{1}}{\cancel{2}\cdot \cancel{1}} = 4\cdot 3 = 12$$

The purpose of the denominator is to get rid of all of the numbers to the right of the boxes!

Sample Problems: Calculate ${}_nP_r$.

1. $${}_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$$

$$= \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}}$$

$$= 6 \cdot 5 \cdot 4 \cdot 3$$

$$= 360$$

2. $${}_5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!}$$

$$= \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

$$= 5 \cdot 4$$

$$= 20$$

3. In a race of 6 people, how many ways can 1st, 2nd, and 3rd place be awarded?

6	5	4

= 120

$${}_6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 6 \cdot 5 \cdot 4 = 120$$

4. How many ways can you select a foreman and subforeman from a group of 12 jurors?

12	11

= 132

$${}_{12}P_2 = \frac{12!}{(12-2)!} = \frac{12!}{10!} = \frac{12 \cdot 11 \cdot 10!}{10!} = 12 \cdot 11 = 132$$

5. How many ways can gold, silver, and bronze be awarded if there are 7 teams competing?

$$ {}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 7 \cdot 6 \cdot 5 \ = 210 $$

6. Find the number of ways of arranging ten blocks on a book shelf if 6 are math books and 4 are history books and each category must be grouped?

We have 2 possibilities:

M	M	M	M	M	M	H	H	H	H

and

H	H	H	H	M	M	M	M	M	M

That turns into:

6	5	4	3	2	1	4	3	2	1

and

4	3	2	1	6	5	4	3	2	1

Notice that the numbers are the same in both, so we can multiply just one and then double it.

$$2 \cdot [6 \cdot 5 \cdot 4 \cdot 3 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1]$$

$$2 \cdot [6 \cdot 5 \cdot 4 \cdot 3 \cdot 1] \cdot [4 \cdot 3 \cdot 2 \cdot 1]$$

$$2 \cdot (6!) \cdot (4!)$$

$$2 \cdot \; {}_6P_6 \cdot \; {}_4P_4$$

$$2 \cdot 720 \cdot 24$$

$$34{,}560$$