Combinations

Let's say you have a group of 8 people and you need to choose a president and vice-president. How many ways can you do that?

8	7

= 56

Or $_8P_2 = \frac{8!}{6!} = 8 \cdot 7 = 56$

Let's say you have a group of 8 people and you want to find out how many ways 2 of them can be selected for a doubles tennis team.

8	7

= 56

<u>but</u>: Choosing Sam and Joe is the same doubles team as choosing Joe and Sam, since the order they are selected in doesn't matter.

**What we need to do to remove duplicates is divide by 2. That gives us:

8	7

= 56 ÷ 2 = 28

Sample Problems:

1. Find the number of ways the president, vice-president, and treasure can be chosen from a group of 8 people.

8	7	6	=	336

2. Find the number of ways a committee of 3 people can be chosen from a group of 8 people.

8	7	6	=	336

But the following ways would all be the same:

Sam, Joe, Tony	Sam, Tony, Joe	Joe, Sam, Tony
Joe, Tony, Sam	Tony, Sam, Joe	Tony, Joe, Sam

There are ${}_3P_3 = 3! = 6$ ways to arrange Tony, Joe, & Sam. So we need:

$$\frac{8 \cdot 7 \cdot 6}{3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = \frac{336}{6} = 56$$

**Whether order is important or not make a huge difference!!!!

- When order is important we use <u>permutations</u>.
- When order is <u>not</u> important we use <u>combinations</u>.

${}_nC_r$ is read "The combination of n objects taken r at a time"

<u>Formula</u>: ${}_nC_r = \frac{n!}{r!(n-r)!}$

<u>Example</u>: Look at our committee chosen from 8 people:

$${}_8C_3 = \frac{8!}{3!\,(8-3)!} = \frac{8!}{3!\,5!} = 56$$

The $\frac{8!}{5!}$ is the permutation. The 3! Takes care of the duplicates.

Sample Problems:

1. $_{11}C_4 =$

Solution: $\frac{11!}{4!7!} = \frac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{(4\cdot 3\cdot 2\cdot 1)(7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1)} = \frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2\cdot 1} = 330$

2. $_{7}C_3 =$

Solution: $\frac{7!}{3!4!} = \frac{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(4\cdot 3\cdot 2\cdot 1)} = \frac{7\cdot 6\cdot 5}{3\cdot 2\cdot 1} = 35$

Are the following permutations or combinations?

1. The number of ways to choose 2 students from a group of 40 to be lunch helpers?

combination

2. The number of ways to choose a greeter and MC for the spring showcase from 200 students.

permutation

3. The number of ways to put 3 letters and 3 numbers on a license plate

permutation

4. The number of ways to choose the girls on a softbasll team

combination

5. The number of ways to pick 4 shirts to pack for a trip

combination

6. The number of ways to make a seating chart for class

permutation

7. The number of ways to set your batting order

permutation

Sample Problems: Write the answer using combination or permutation notation but do not solve.

1. How many ways can you choose 5 out of 7 flower types for a bouquet?

$$_{7}C_{5}$$

2. How many ways can a leadoff hitter and cleanup hitter be chosen from a group of 12 ballplayers?

$$_{12}C_{2}$$

3. Find the number of ways of choosing two co-chairs from a list of 14 candidates?

$$_{14}C_{2}$$

4. Find the number of ways of selecting a committee of 6 men and 6 women from a group of 30 men and 25 women.

$$_{30}C_{6} \cdot {}_{25}C_{6}$$

5. How many ways are there to divide 18 students into 3 equal teams?

$$_{18}C_6 \cdot {}_{12}C_6 \cdot {}_6C_6$$

Ways to choose team 1 | Ways to choose team 2 | Ways to choose team 3

Section 10.8

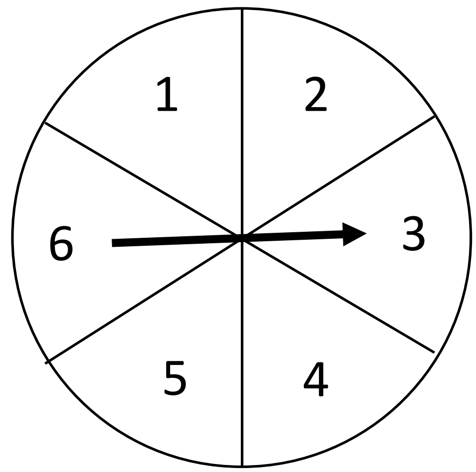
Probability

- Spinning a spinner is called an experiment.
- The set of all possible outcomes is the sample space.
- Any single spin in the experiment is called a trial.
- A possible result (outcome) of a trial is called an event.
- The probability of an even, P(E), is a ratio that describes the likelihood of the event occurring.

Formula: The probability of event E is

$$P(E) = \frac{\#\, desired\ outcomes}{\#\, possible\ outcomes}$$

Sample Problems:



1. P(3)

$$P(3) = \frac{1}{6}$$

2. P(odd)

$$P(odd) = \frac{3}{6} = \frac{1}{2}$$

3. P(x<5)

$$P(3) = \frac{4}{6} = \frac{2}{3}$$

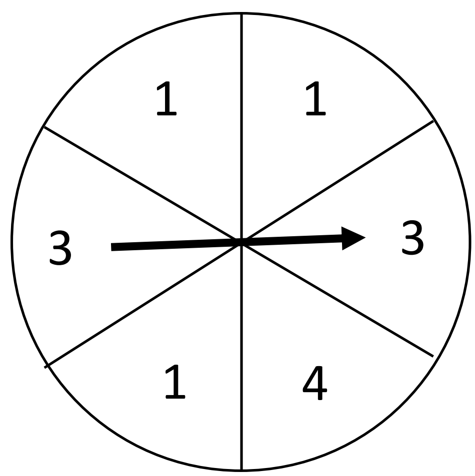
4. P(7)

$$P(3) = 0$$

5. P(x≤6)

$$P(\text{x} \le 6) = \frac{6}{6} = 1$$

Sample Problems:



Answers:

1. P(4) $\frac{1}{6}$

2. P(1) $\frac{3}{6} = \frac{1}{2}$

3. P(3) $\frac{2}{6} = \frac{1}{3}$

Is it possible to spin a 3 and a 4 in a single spin?

No. These 2 events are mutually exclusive.

Is it possible to spin a 3 and an odd in a single spin?

Yes. A spin of 3 and it fits both.
These 2 events are not mutually exclusive.

Sample Problems: Consider a bag of Legos. Which of the following are mutually exclusive?

1. Drawing a red or blue

   Mutually exclusive

2. Drawing a blue or square

   Not mutually exclusive because there is a blue square

3. Drawing a square or 8-pin

   Mutually exclusive

4. Drawing a yellow or prime color

   Not mutually exclusive because yellow is a prime color

Comment on notation:

Probabilties can be listed as a fraction, decimal, or %.

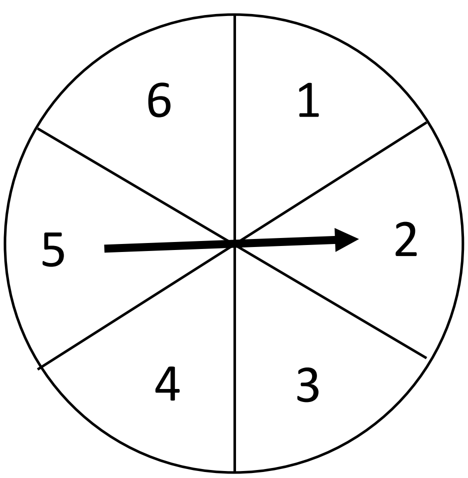
**In your textbook it says to write probabilities as both a fraction and decimal. You can just write them as fractions.

Probability of Mutually Exclusive Events

Formula: If A and B are mutually exclusive events, then

P(A or B) = P(A) + P(B)

Sample Problems: Find the following probabilities using the spinner:



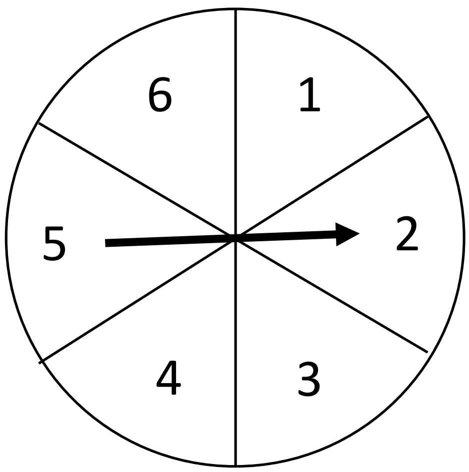
1. P(2 or odd)

$$\frac{1}{6}+\frac{3}{6}=\frac{4}{6}=\frac{2}{3}$$

2. P(3 or 5)

$$\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$$

Example of an Event that is ***Not*** Mutually Exclusive



P(3 or odd)

Spinning a 3 fits both conditions so we need to reason through to the answer.

There are 3 spins that are either a 3 or an odd, so

P(3 or odd)= $\frac{3}{6} = \frac{1}{2}$

Sample Problem: Find P(even or x > 3)

$$\text{P(even or x } > \ 3) = \frac{4}{6} = \frac{2}{3}$$

Sample Problem: Consider a bag that contains 4 yellow, 3 red, 4 blue, 4 black, 1 white, and 1 gray Lego. Find the probabilities.

1. P(Y)

$$P(Y) = \frac{4}{17}$$

2. P(W)

$$P(W) = \frac{1}{17}$$

3. P(red or blue)

$$P(red\ or\ blue) = \frac{7}{17}$$

4. P(black or square)

$$P(black\ or\ square) = \frac{5}{17}$$

5. P(square or yellow)

$$P(yellow\ or\ square) = \frac{6}{17}$$

Independent and Dependent Events

Consider a bag full of ping pong balls numbered 1 through 6.

What is the probability of drawing a 2, putting it back, then drawing a 3?

$$P(2\ and\ then\ 3) = \frac{1}{36}$$

One draw (2,3) gives us what we want.

There are 6 ways to draw the 1st time and 6 ways to draw the 2nd time, so there are 6x6=36 ways to draw 2 balls.

$$P(2\ and\ then\ even) = \frac{3}{36}$$

(2,2), (2,4), & (2,6) give us what we want.

6x6=36 ways to draw 2 balls.

*If 2 events are independent, they have no effect on each other. Another example would be flipping a coin and rolling a die.

Now, let's say we draw one ball out but don't replace it. What is the probability of drawing a 2 and then a 3?

One draw (2,3) gives us what we want.

$$P(2\ then\ 3\ without\ replacement) = \frac{1}{6 \cdot 5} = \frac{1}{30}$$

There are 6 ways to draw the 1st time and 5 ways to draw the 2nd time, so there are 6x5=30 ways to draw 2 balls.

(2,4), & (2,6) give us what we want.

$$P(2\ then\ even\ without\ replacement) = \frac{2}{30} = \frac{1}{15}$$

6x5=30 ways to draw 2 balls.

There's an easier way to calculate these probabilities:

For <u>independent events</u> A and B,

$$P(A \ and \ B) = P(A) \cdot P(B)$$

***Multiply the individual probabilities together!

Look again at the 2 previous problems:

$$P(2 \ then \ 3 \ without \ replacement) = \frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$$

$$P(2 \ then \ even \ without \ replacement) = \frac{1}{6} \cdot \frac{2}{5} = \frac{1}{15}$$

<u>New Notation:</u>

P(A|B) stands for "the probability of A, given B has occurred"

For <u>dependent events</u> A and B,

$$P(A \ and \ B) = P(A) \cdot P(B|A)$$

Sample Problems:

The names of 10 club members, 4 boys and 6 girls, are placed in 2 hats. Jack is one of the boys and Sally is one of the girls. Suppose names will be drawn to select a boys' representative and girls' representative.

1. Are the events independent or dependent?

   independent

2. What is the probability that Jack and Sally will be chosen as representatives?

$$\frac{1}{4}\cdot\frac{1}{6}=\frac{1}{24}$$

3. What is the probability that neither Jack nor Sally will be chosen?

$$\frac{3}{4}\cdot\frac{5}{6}=\frac{5}{8}$$

4. What is the probability that Sally will be chosen but Jack will not?

$$\frac{1}{8}$$

Suppose all of the 10 names (4 boys and 6 girls) are put in the same hat and that the 1st name drawn is president and the 2nd name drawn is vice president. Jack is one of the boys and Sally is one of the girls.

1. Are the events independent or dependent?

$$dependent$$

2. Find P(Jack, then a boy)

$$P(Jack) \cdot P(boy|Jack) = \frac{1}{10} \cdot \frac{3}{9} = \frac{1}{30}$$

3. Find P(girl other then Sally, then a boy)

$$\frac{5}{10} \cdot \frac{4}{9} = \frac{2}{9}$$

4. Find P(boy, then girl)

$$\frac{4}{10} \cdot \frac{6}{90} = \frac{4}{15}$$

5. Find P(girl, then boy)

$$\frac{6}{10} \cdot \frac{4}{9} = \frac{4}{15}$$

6. Find P(1 boy and 1 girl) [order does not matter]

Solution:

What we are really asking for is P(boy, then girl or girl, then boy) These are mutually exclusive events so we add the 2 probabilities:

$$\frac{4}{15} + \frac{4}{15} = \frac{8}{15}$$

P(girl, then boy)
$=\frac{6}{10} \cdot \frac{4}{9} = \frac{4}{15}$

P(boy, then girl)
$=\frac{4}{10} \cdot \frac{6}{9} = \frac{4}{15}$