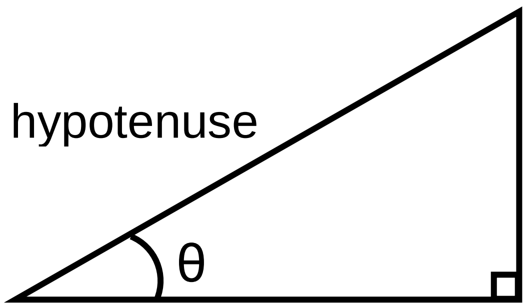
Right Angle Trigonometry

Look at the right triangle:



The six trigonometric functions, **sine, cosine, tangent, cotangent, secant, and cosecant,** can be defined as follows.

Let θ be an acute angle of a right triangle. Then

$$\text{sine}\,\theta = \sin\theta = \frac{\text{length of side opposite }\theta}{\text{length of hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

$$\text{cosine}\,\theta = \cos\theta = \frac{\text{length of side adjacent to }\theta}{\text{length of hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

$$\text{tangent}\,\theta = \tan\theta = \frac{\text{length of side opposite }\theta}{\text{length of side adjacent to }\theta} = \frac{\text{opp}}{\text{adj}}$$

$$\text{cosecant}\,\theta = \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\text{secant}\,\theta = \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\text{cotangent}\,\theta = \cot\theta = \frac{\text{adj}}{\text{opp}}$$

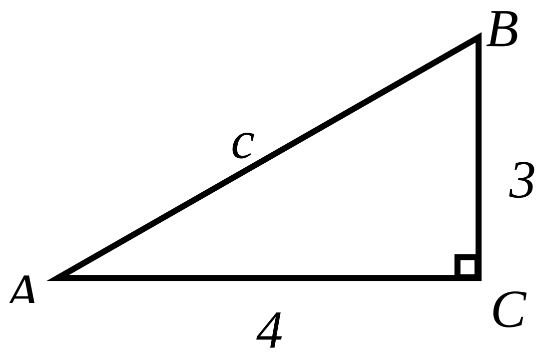
In general, we have:

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta = \frac{\text{hyp}}{\text{opp}}$
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}}$
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$

**Notice that <u>sine</u> and <u>cosecant</u> are reciprocals,
<u>cosine</u> and <u>secant</u> are reciprocals,
and <u>tangent</u> and <u>cotangent</u> are reciprocals.

SOHCAHTOA!!!

Example: Find the hypotenuse and then find all 6 trigonometric values for both angles.



1. Find the hypotenuse by the Pythagorean Theorem.

$$c^2 = 3^2 + 4^2$$
$$c^2 = 9 + 16$$
$$c^2 = 25$$
$$c = 5$$

2.

$$\sin A = \frac{opp}{hyp} = \frac{3}{5} \qquad \sin B = \frac{opp}{hyp} = \frac{4}{5}$$
$$\cos A = \frac{adj}{hyp} = \frac{4}{5} \qquad \cos B = \frac{adj}{hyp} = \frac{3}{5}$$
$$\tan A = \frac{opp}{adj} = \frac{3}{4} \qquad \tan B = \frac{opp}{adj} = \frac{4}{3}$$
$$\csc A = \frac{hyp}{opp} = \frac{5}{3} \qquad \csc B = \frac{hyp}{opp} = \frac{5}{4}$$
$$\sec A = \frac{hyp}{adj} = \frac{5}{4} \qquad \sec B = \frac{hyp}{adj} = \frac{5}{3}$$
$$\cot A = \frac{adj}{opp} = \frac{4}{3} \qquad \cot B = \frac{adj}{opp} = \frac{3}{4}$$

Notice that:

sin A = cos B
cos A = sin B

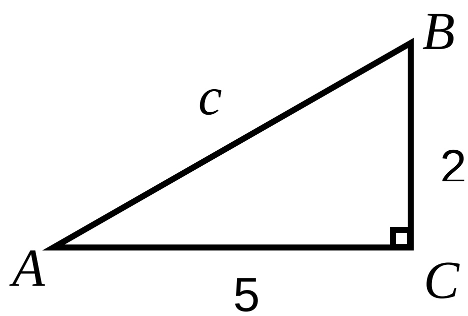
and

sec A = csc B
csc A = sec B

<u>Note</u>: Sine and Cosine of complementary angles are =.
Secant and Cosecant of complementary angles are =.
Tangent and Cotangent of complementary angles are =.

Example: sin 60° = cos 30°

Example: Find the hypotenuse and then find the sine, cosine and tangent for both angles.



1. Find the hypotenuse by the Pythagorean Theorem.

$$c^2 = 2^2 + 5^2 \qquad c^2 = 29$$
$$c^2 = 4 + 25 \qquad c = \sqrt{29}$$

2.

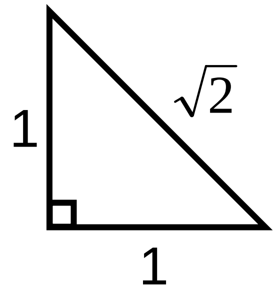
$$\sin A = \frac{opp}{hyp} = \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{2\sqrt{29}}{29} \qquad \sin B = \frac{opp}{hyp} = \frac{5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$$

$$\cos A = \frac{adj}{hyp} = \frac{5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{5\sqrt{29}}{29} \qquad \cos B = \frac{adj}{hyp} = \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$

$$\tan A = \frac{opp}{adj} = \frac{2}{5} \qquad \tan B = \frac{opp}{adj} = \frac{5}{2}$$

Special Right Triangles

45°- 45° Triangle:



$$c^2 = 1^2 + 1^2$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

(If the 2 angles are equal, then the 2 legs must be equal. You can use whatever you want for the legs as long as they are equal.)

$$\sin 45° = \frac{opp}{hyp} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

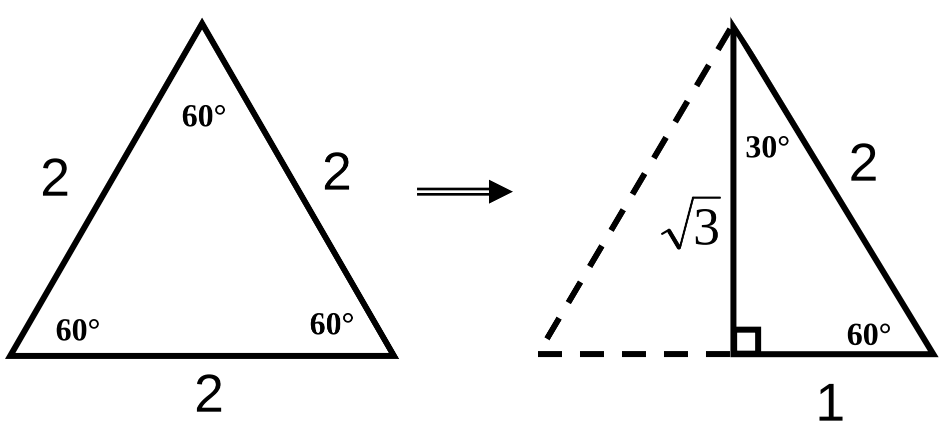
$$\cos 45° = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45° = \frac{opp}{adj} = \frac{1}{1} = 1$$

Memorize these!!!!

30°- 60° Triangle:

Start with an equilateral triangle. It does not matter what the lengths of the sides are.



$$2^2 = 1^2 + b^2$$
$$4 = 1 + b^2$$
$$b^2 = 3$$
$$b = \sqrt{3}$$

$$\sin 30° = \frac{opp}{hyp} = \frac{1}{2} \qquad \sin 60° = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$$

$$\cos 30° = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} \qquad \cos 60° = \frac{adj}{hyp} = \frac{1}{2}$$

$$\tan 30° = \frac{opp}{adj} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \qquad \tan 60° = \frac{opp}{adj} = \frac{\sqrt{3}}{1}$$

Memorize these!!!!

Cofunctions of Complementary Angles are Equal

If θ is an acute angle, then

$\sin(90° - \theta) = \cos\theta$ *or* $\sin(\frac{\pi}{2} - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$ *or* $\cos(\frac{\pi}{2} - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$ *or* $\tan(\frac{\pi}{2} - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$ *or* $\cot(\frac{\pi}{2} - \theta) = \tan\theta$

$\sec(90° - \theta) = \csc\theta$ *or* $csc(\frac{\pi}{2} - \theta) = \sec\theta$

$\csc(90° - \theta) = \sec\theta$ *or* $\csc(\frac{\pi}{2} - \theta) = \sec\theta$

Remember our definitions of the 6 Trig Functions:

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta = \frac{\text{hyp}}{\text{opp}}$
$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}}$
$\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$

From these we get the following Identities:

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

*Note: We write $\sin^2\theta$ instead of $(\sin\theta)^2$.

Example: If θ is an acute angle such that $\cos\theta = 0.3$, find $\sin\theta$.

Solution:

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta + (0.3)^2 = 1$$
$$\sin^2\theta + 0.09 = 1$$
$$\sin^2\theta = 1 - 0.09$$
$$\sin^2\theta = 0.91$$
$$\sin\theta = \sqrt{0.91} = \sqrt{\frac{91}{100}} = \frac{\sqrt{91}}{10}$$

Example: If θ is an acute angle such that sin θ = 0.6, find the following:

(a) cos θ

$$\sin^2\theta + \cos^2\theta = 1$$
$$(0.6)^2 + \cos^2\theta = 1$$
$$0.36 + \cos^2\theta = 1$$
$$\cos^2\theta = 1 - 0.36$$
$$\cos^2\theta = 0.64$$
$$\cos\theta = 0.8$$

(b) tan θ

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8} = 0.75$$

Example: If θ is an acute angle such that tan θ = 4, find the following:

(a) cot θ

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{1}{4}$$

(b) sec θ

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + 4^2 = \sec^2\theta$$

$$1 + 16 = \sec^2\theta$$

$$17 = \sec^2\theta$$

$$\sec\theta = \sqrt{17}$$

Evaluating Trig Functions with a Calculator

*Before using your calculator to evaluate trig functions, make sure the MODE is set to match the angle unit you are using (DEGREE or RADIAN).

DEGREE Mode

Example: Set in Degree Mode and find:

(a) cos 15.3°

[COS] 15.3 [ENTER] ≈ 0.9646

(b) sin 24° 32′

[SIN] (24° 32′) [ENTER] ≈ .4152

or [SIN] (24 + $\frac{32}{60}$) [ENTER] ≈ .4152

(c) sec 46°

Remember: $\sec 46^\circ = \dfrac{1}{\cos 46^\circ}$

[COS] (46) [x^{-1}] [ENTER] ≈ 1.4396 (must close ( ) after 46)

or 1 / [COS] 46 [ENTER] ≈ 1.4396

More examples: Find the following.

a) tan 45.67° answer: 1.0237
b) csc 65° answer: 1.1034
c) sec 40° 32′ 23″ answer: 1.3159

RADIAN Mode

Example: Set in Radian Mode and find:

a) $\sin\frac{\pi}{4}$

[SIN] ($\pi/4$) [ENTER] ≈ .7071

b) $\sec\frac{5\pi}{3}$

[COS] ($5\pi/3$) [x^{-1}] [ENTER] ≈ 2.0000

Mixed Modes

Example: Find sin 34° while in RADIAN mode.

[SIN] (34 [ANGLE] [°]) [ENTER] ≈ .5592

Example: Find sin $\frac{\pi}{3}$ while in DEGREE mode.

[SIN] (($\pi/3$) [ANGLE] [r]) [ENTER] $\approx$ .8660

<u>Finding the angle when given the Trig value</u>

If sin θ = .5, we know that θ must be 30°.

On the calculator: [2^{nd}] [SIN] (.5)[ENTER] = 30°.

Use [SIN] if you know the angle and want the sine.
Use [SIN^{-1}] if you know the sine and want the angle.

Example: Solve for θ in the following:

a) sin θ = 0.8145 answer: 54.5°
b) cos θ = 0.9848 answer: 10.0°
c) tan θ = 0.3125 answer: 17.4°

<u>Application – Solving Right Triangles</u>

When solving a right triangle, you are usually given one side of a right triangle and one of the acute angles and are asked to find one of the other sides, or you are given two sides and are asked to find one of the acute angles.

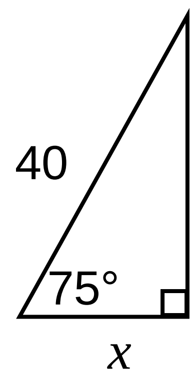
Example: Solve for x in the following right triangle.

Look at what values you are given and how they are related.

$$\cos x = \frac{adj}{hyp}$$

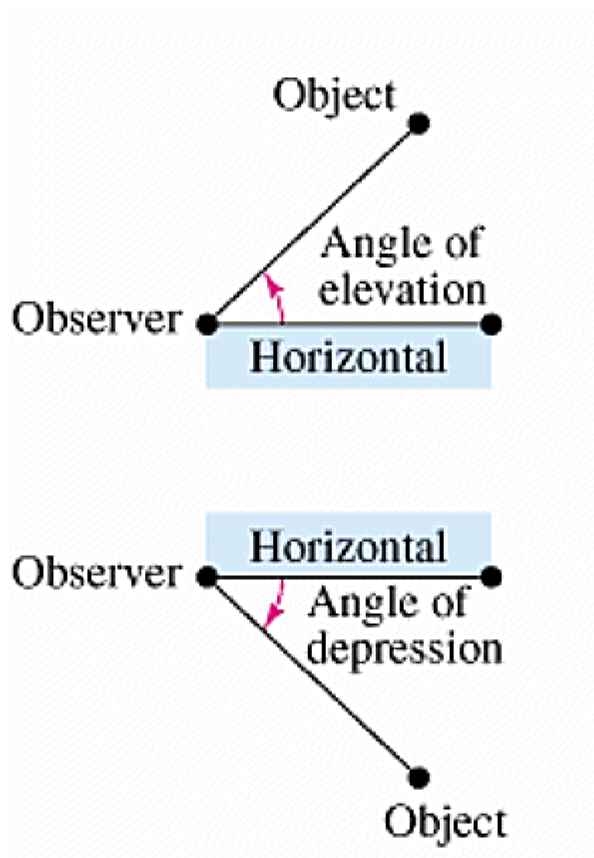
$$\cos 75° = \frac{x}{40}$$

$$x = 40 \cos 75° \approx 10.35$$

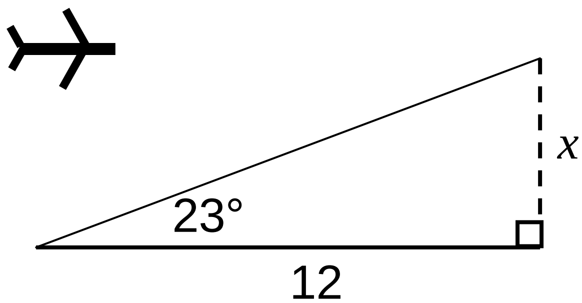


- The term **angle of elevation** means the angle from the horizontal upward to an object.

- The term **angle of depression** means for objects that lie below the horizontal, the angle from the horizontal downward to an object.



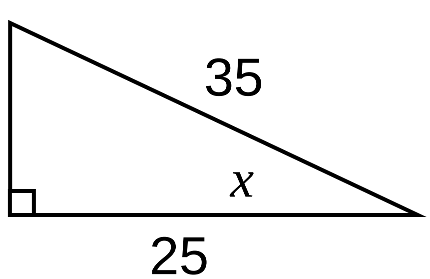
Example: A plane flying over level ground will pass directly over a radar antenna. It is 12 miles on the ground from the antenna to the point directly under the plane and the angle of elevation from that point on the ground to the top of the antenna is 23°. Find the altitude at which the plane is flying.



$$\tan 23° = \frac{x}{12}$$
$$x = 12\tan 23° \approx 5.09$$

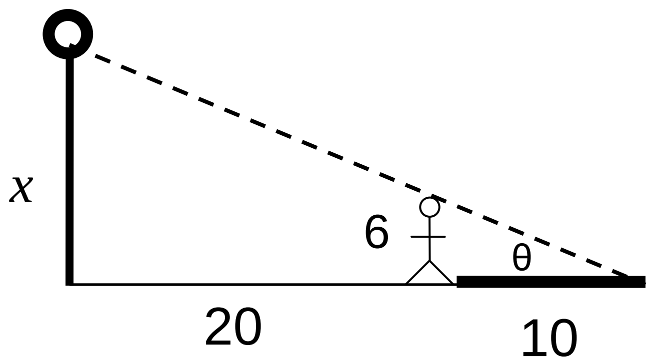
The plane is flying at 5.09 miles.

Example: If a rope tied to the top of a flagpole is 35 feet long, then what angle is formed by the rope and the ground when the rope is pulled to the ground, 25 feet from the base of the pole?



$$\cos x = \frac{25}{35} = 0.7143$$
$$x = \cos^{-1}(0.7143) \approx 44.4°$$

Example: A six-foot man standing 20 feet from a street light casts a 10-foot shadow. How tall is the street light?



We know from the smaller triangle that $\tan\theta = \frac{6}{10} = 0.6$.

Using the larger triangle we get:

$$\tan\theta = \frac{x}{30}$$

$$0.6 = \frac{x}{30}$$

$$x = 30(0.6) = 18$$