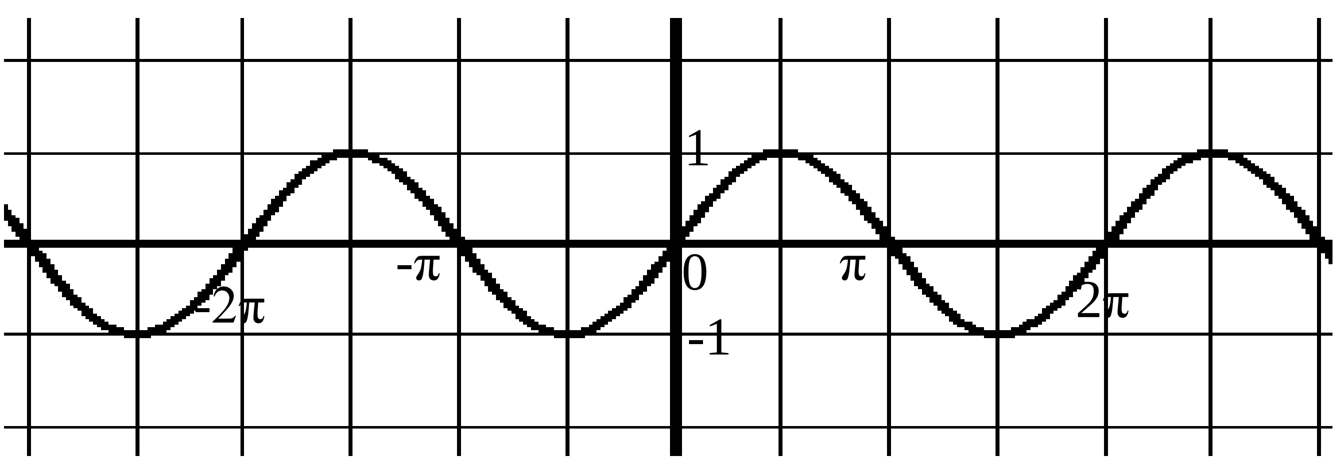
Graphs of Sine and Cosine Functions

Graph $y = \sin x$. This is the same as $y = \sin(x)$

x	y
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{5\pi}{6}$	$\frac{1}{2}$
π	0

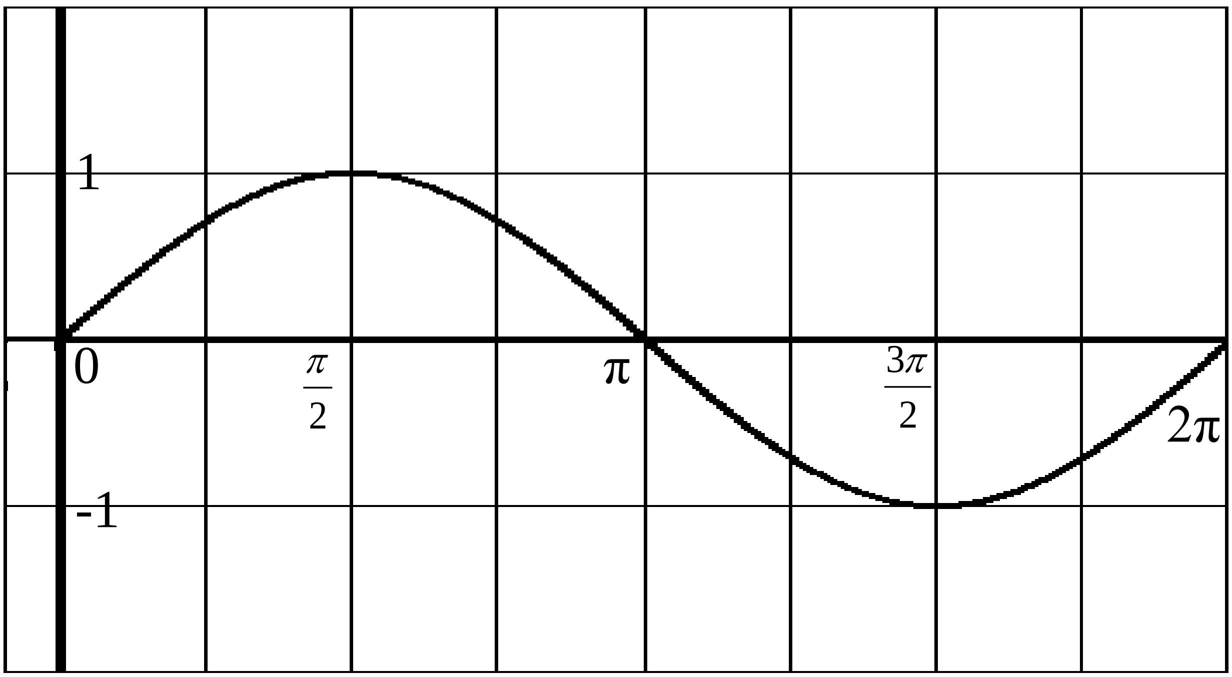
x	y
π	0
$\frac{7\pi}{6}$	$-\frac{1}{2}$
$\frac{5\pi}{4}$	$\frac{-\sqrt{2}}{2} \approx -0.7$
$\frac{4\pi}{3}$	$\frac{-\sqrt{3}}{2} \approx -0.87$
$\frac{3\pi}{2}$	-1
$\frac{2\pi}{3}$	$\frac{-\sqrt{3}}{2} \approx -0.87$
$\frac{7\pi}{4}$	$\frac{-\sqrt{2}}{2} \approx -0.7$
$\frac{11\pi}{6}$	$-\frac{1}{2}$
2π	0

x	y
0	0
$\frac{13\pi}{6}$	$\frac{1}{2}$
$\frac{9\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{7\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{5\pi}{2}$	1
$\frac{8\pi}{3}$	$\frac{\sqrt{3}}{2} \approx 0.87$
$\frac{11\pi}{4}$	$\frac{\sqrt{2}}{2} \approx 0.7$
$\frac{17\pi}{6}$	$\frac{1}{2}$
3π	0



The Domain of $y=\sin x$ is the set of all real numbers.
The Range of $y = \sin x$ is $-1 \le y \le 1$.

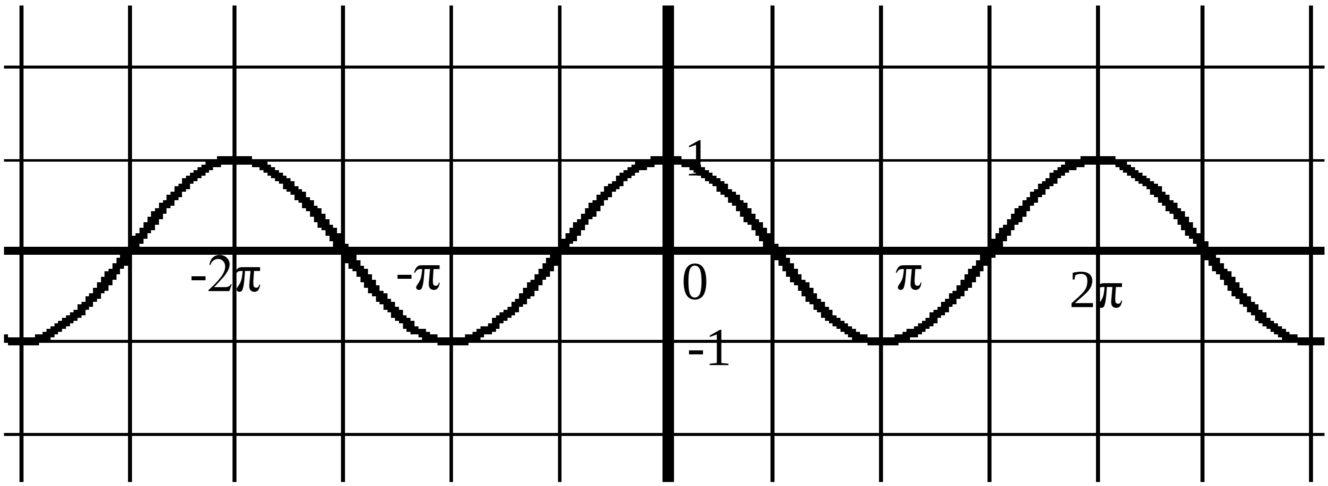
The portion of the graph of $y = \sin x$ that includes one period is called one cycle of the sine curve.



Every period of the sine curve has **5 key points**: the intercepts and a minimum and maximum point.

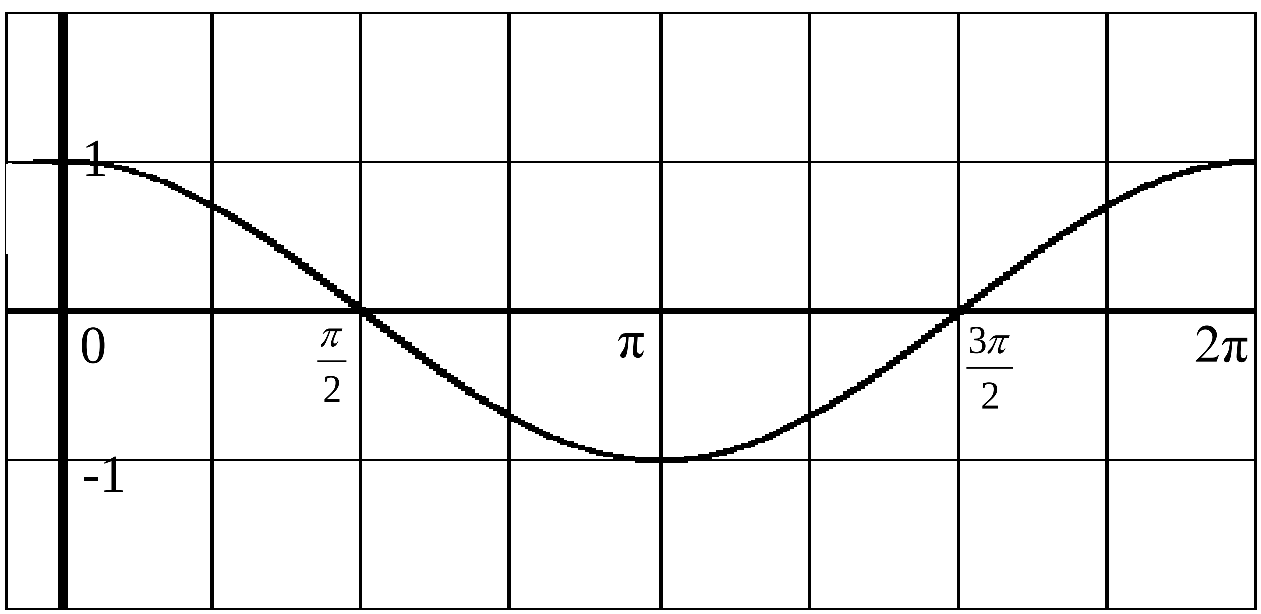
For one period of the sine curve, the x-intercepts occur at (0, 0), (π, 0), and (2π, 0). The maximum point is ($\frac{\pi}{2}$, 1) and the minimum point is ($\frac{3\pi}{2}$, -1).

Graph $y = \cos x$.



The Domain of $y = \cos x$ is the set of all real numbers.
The Range of $y = \cos x$ is $-1 \leq y \leq 1$.

The portion of the graph of $y = \cos x$ that includes one period is called one cycle of the cosine curve.



Every period of the cosine curve has **5 key points**: the intercepts and a minimum and maximum point.

For one period of the sine curve, the x-intercepts occur at ($\frac{\pi}{2}$, 0), and ($\frac{3\pi}{2}$, 0). The maximum point is (0, 1) and (2π, 0) and the minimum point is (π, -1).

**Both sine and cosine curves have a period of 2π. We consider the interval from 0 to 2π as the basic cycle.

Amplitude

On a graphing calculator, graph $y = \sin x$
$y = 2\sin x$
$y = 5\sin x$
$y = \frac{1}{2}\sin x$

What can you conclude?

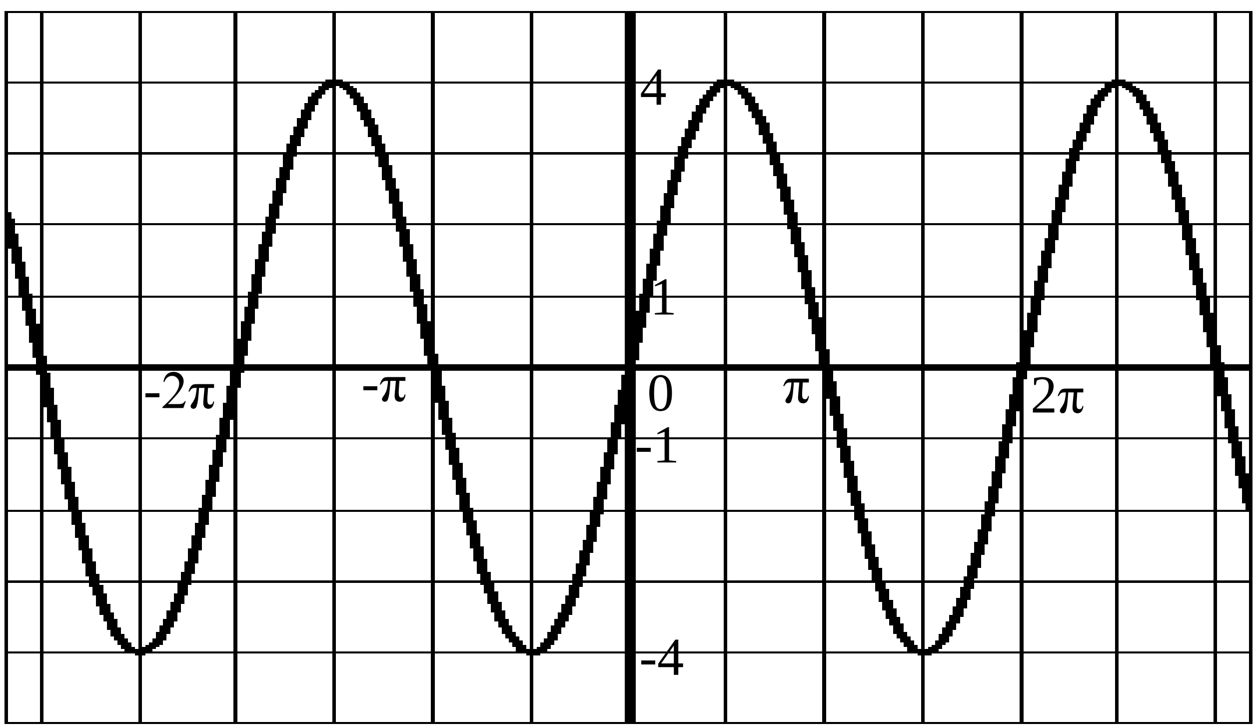
As the number being multiplied out front increases, the graph of $y = \sin x$ stretches vertically.

Definition: The amplitude of $y = a \sin x$ and $y = a \cos x$ represents half the distance between the maximum and minimum values of the function and is given by Amplitude = |a|.

***Note**: Just as with functions already studied, if *a* is a negative number, the graph of the function will be reflected over the x-axis.

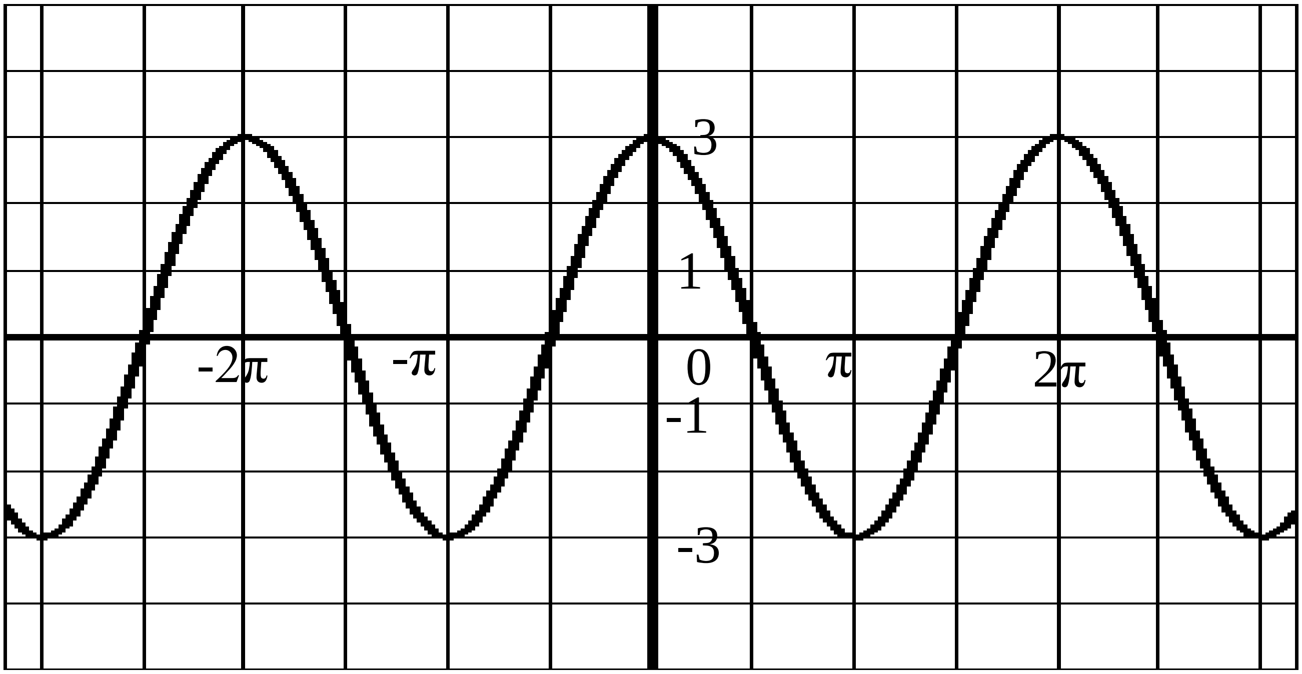
The amplitude of $y = \sin x$ is 1.
The amplitude of $y = 2 \sin x$ is 2.
The amplitude of $y = 5 \sin x$ is 5.
The amplitude of $y = ½ \sin x$ is ½ .
The amplitude of $y = -13 \sin x$ is 13. (*not* -13)

Graph $y = 4 \sin x$

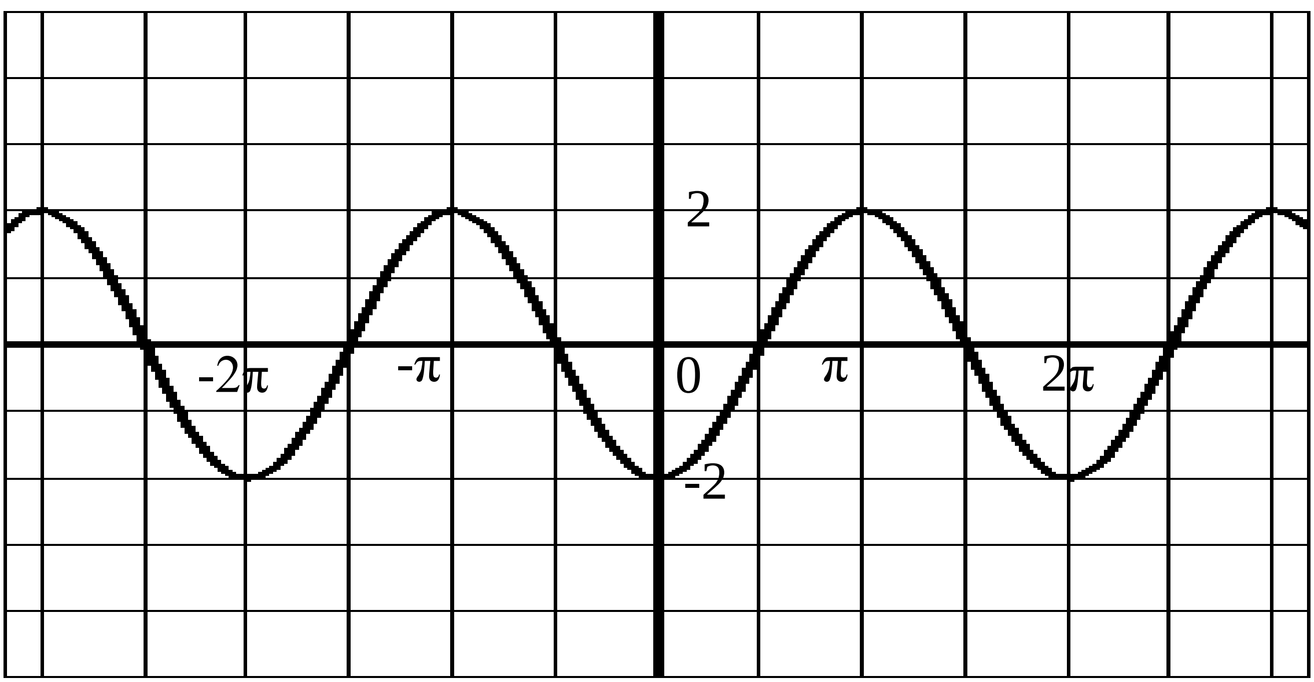


The period remains the same, but the amplitude changes.

Graph $y = 3 \cos x$



Graph $y = -2 \cos x$



Changing the Period of Sine and Cosine

On a graphing calculator graph: $y = \sin x$
$y = \sin 2x$

What do you notice?

The length of one cycle is half as long for $y = \sin 2x$*.*

Definition: Let b be a positive real number. The period of $y = a \sin bx$ and $y = a \cos bx$ is $2\pi/b$.

Example: Find the period of $y = \cos 6x$.

The period = $\frac{2\pi}{b} = \frac{2\pi}{6} = \frac{\pi}{3}$

Example: Find the period of $y = \sin \frac{x}{5}$.

The period = $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{5}} = 2\pi \cdot \frac{5}{1} = 10\pi$

Alternate Method of Finding the Period

For one cycle of $y = \sin x$ we have $0 \leq x \leq 2\pi$.

Another way to find the period of $y = \sin 2x$ is to put $2x$ between the 0 and 2π and solve for x in the middle.

For one period of $y = \sin 2x$ we have $0 \leq 2x \leq 2\pi$.

Divide through by 2.

$$\frac{0}{2} \leq \frac{2x}{2} \leq \frac{2\pi}{2}$$

We end up with $0 \leq x \leq \pi$ for the period of $y = \sin 2x$.

If we use the definition we get the period = $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$.

Example: Find the period of $y = \cos\frac{x}{4}$.

For cosine we have $0 \leq x \leq 2\pi$. Substitute $\frac{x}{4}$ for the x.

$$0 \leq \frac{x}{4} \leq 2\pi$$

$$0 \leq x \leq 8\pi$$

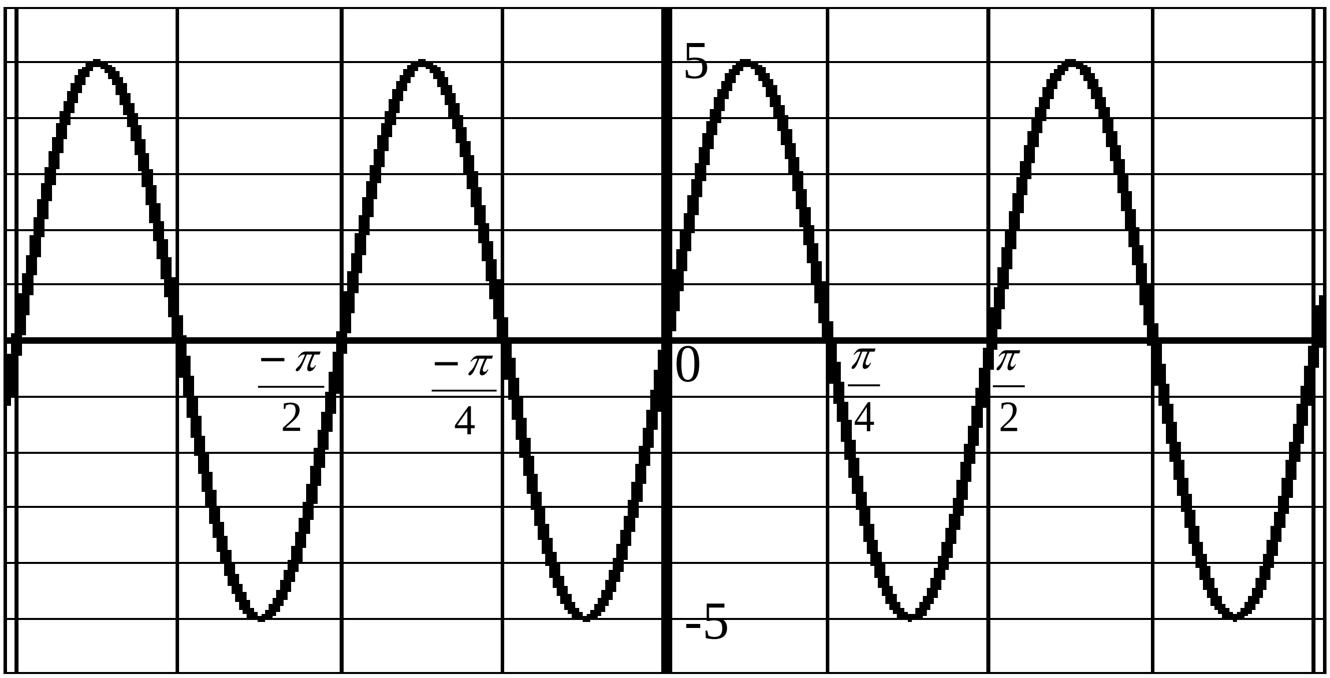
The period is from 0 to 8π, which is 8π units.

Graph $y = 5 \sin 4x$.

$$0 \le 4x \le 2\pi$$

$$0 \le x \le \frac{\pi}{2}$$

The period is $\frac{\pi}{2}$ and the amplitude is 5.



Note: Once you know the basic shape of the sine and cosine curves, it is basically a matter of making adjustments to the axes labels.

Horizontal Translation of Sine and Cosine Curves

The constant in c in the equations $y = a \sin(bx-c)$ and $y = a \cos(bx-c)$ create a horizontal translation (shift) of the basic sine and cosine curves.

On a graphing calculator graph: $y = \sin x$

$$y = \sin\left(x + \frac{\pi}{2}\right)$$

What do you notice?

The graph is the same shape and size as $y = \sin x$ but it is shifted horizontally.

Example: Graph y = cos(x - $\frac{\pi}{2}$)

Because the period for cosine is from 0 to 2π, we take

$$0 \le x - \frac{\pi}{2} \le 2\pi$$

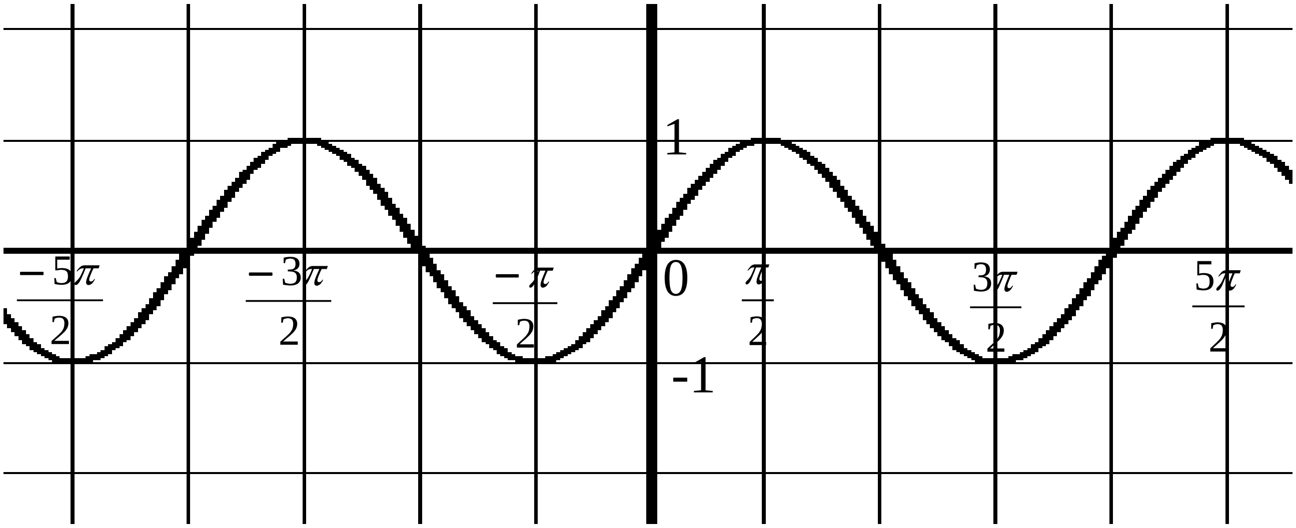
Add $\frac{\pi}{2}$ to all 3 parts of the inequality.

$$\frac{\pi}{2} \le x \le 2\pi + \frac{\pi}{2}$$

$$\frac{\pi}{2} \le x \le \frac{5\pi}{2}$$

This means that one period of our function goes from $\frac{\pi}{2}$ to $\frac{5\pi}{2}$ (which is 2π units long)

$$y = \cos(x - \frac{\pi}{2})$$



Example: Graph $y = 2\sin(4x + \pi)$

The amplitude is 2.

The period is $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$

One period of sine goes from 0 to 2π, so we have

$$0 \leq 4x + \pi \leq 2\pi$$

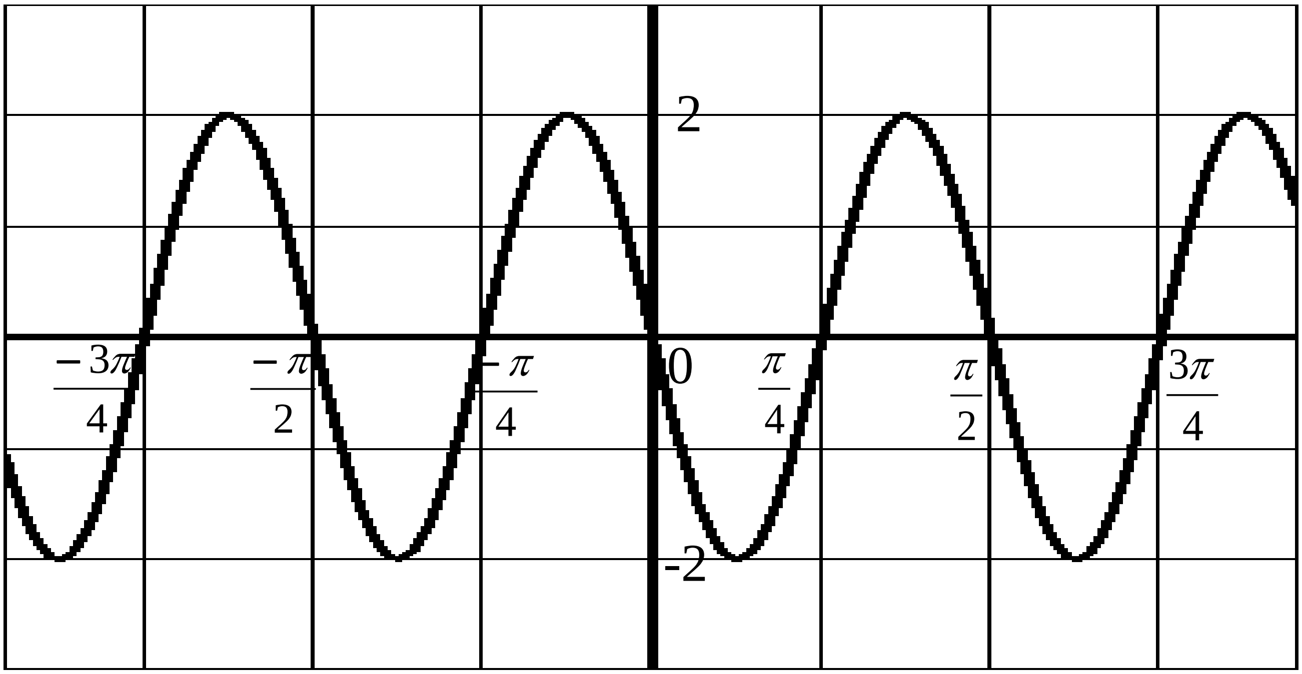
$$-\pi \leq 4x \leq \pi$$

$$\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$$

One period goes from $\frac{-\pi}{4}$ to $\frac{\pi}{4}$

(This is $\frac{\pi}{2}$ units long.)

$$y = 2\sin(4x + \pi)$$



Vertical Translations

On a graphing calculator graph: $y = \sin x$
$y = 2 + \sin x$

What do you notice?

The graph is the same shape and size as $y = \sin x$ but it is shifted vertically 2 units up.

Example : Graph $y = 3 + 5\sin(2x - \frac{\pi}{6})$

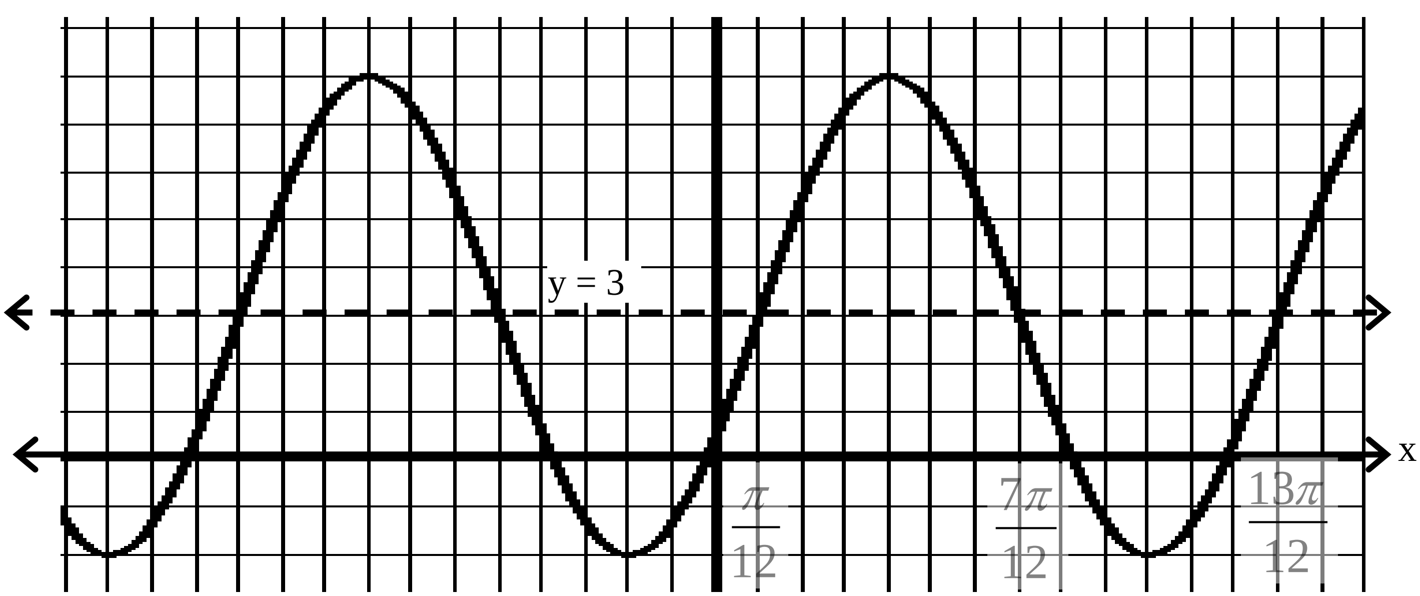
- The <u>amplitude</u> is 5.
- The <u>period</u> is $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$
- We have a <u>vertical shift</u> of 3 units up, so draw the line y=3 (dotted) as the graph will oscillate around that line instead of the x-axis.
- Find the <u>horizontal shift</u>. Since it is the sine curve we have

$$0 \le 2x - \frac{\pi}{6} \le 2\pi$$

$$\frac{\pi}{6} \le 2x \le \frac{13\pi}{6}$$

$$\frac{\pi}{12} \le x \le \frac{13\pi}{12}$$

One period goes from $\frac{\pi}{12}$ to $\frac{13\pi}{12}$
(This is π units.)



Example : Graph $y = 1 - \frac{1}{2}\sin(\frac{1}{2}x - \pi)$

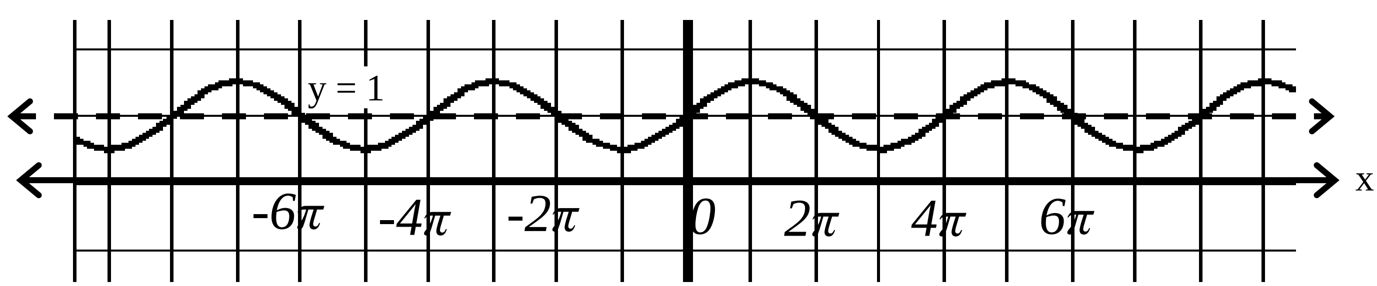
- The <u>amplitude</u> is $\frac{1}{2}$.
- Because there is a negative before the $\frac{1}{2}$, the graph is up-side-down (reflected over the x-axis.)
- The <u>period</u> is $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$
- We have a <u>vertical shift</u> of 1 unit up, so draw the line y=1 (dotted) as the graph will oscillate around that line instead of the x-axis.
- Find the <u>horizontal shift</u>. Since it is the sine curve we have

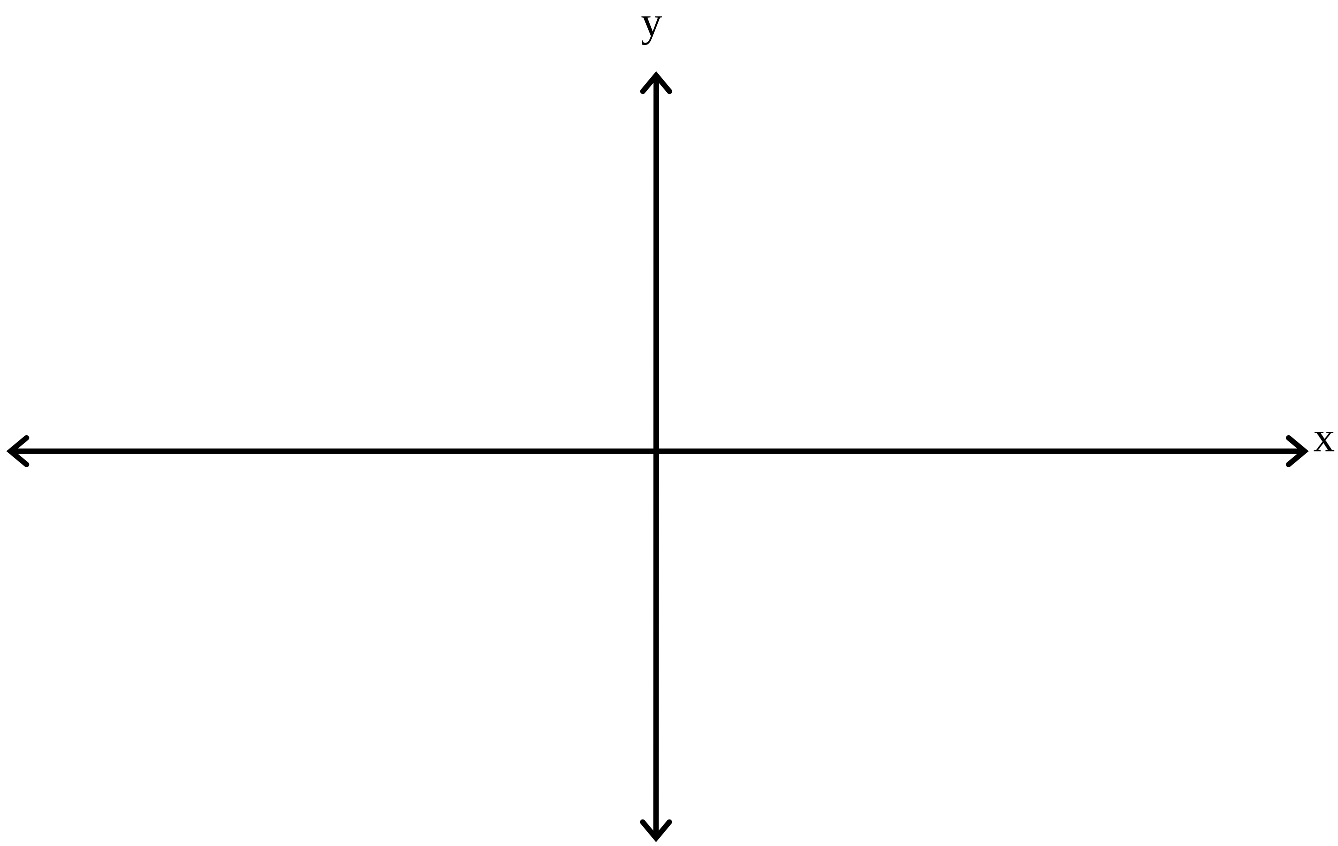
$$0 \le \frac{1}{2}x - \pi \le 2\pi$$

$$\pi \le \frac{1}{2}x \le 3\pi$$

$$2\pi \le x \le 6\pi$$

One period goes from 2π to 6 π.
(This is 4π units.)




y
x