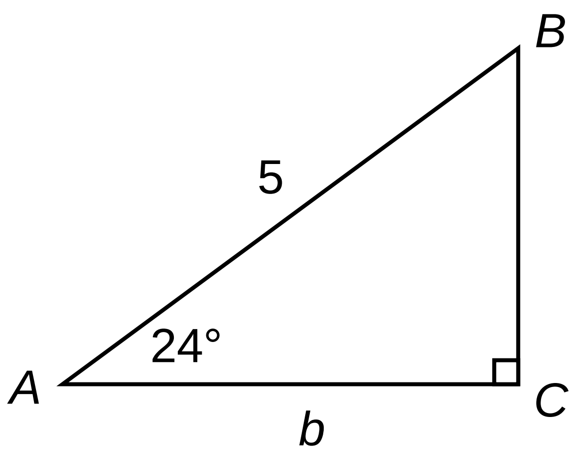
Applications and Models

Applications Involving Right Triangles

For these problems, angles will be labeled with the uppercase letters *A, B,* and *C* (*C* is the right angle), and their opposite sides with the lowercase letters *a, b,* and *c* (*c* is the hypotenuse).

Example: If a right triangle has $A = 24°$ and $c = 5$, find b.



$$\cos A = \frac{adj}{hyp}$$

$$\cos 24° = \frac{b}{5}$$

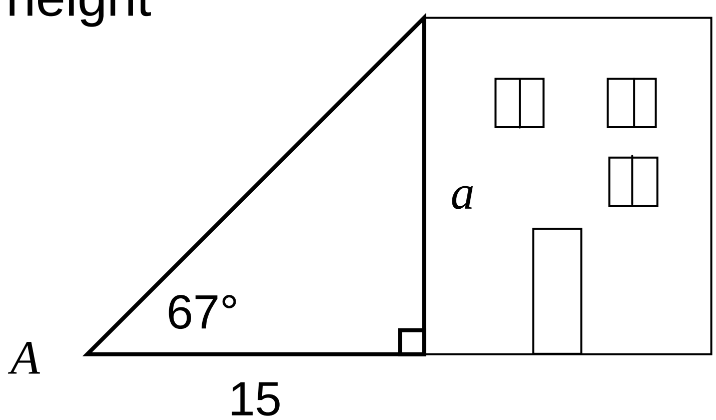
$$5\cos 24° = b$$

$$b \approx 4.6$$

Example: A man at ground level measures the angle of elevation to the top of a building to be 67°. If, at this point, he is 15 feet from the building, what is the height of the building?

1. Find: The height of the building

2. Let a = building height



3.

$$\tan A = \frac{opp}{adj}$$

$$\tan 67° = \frac{a}{15}$$

$$15 \tan 67° = a$$

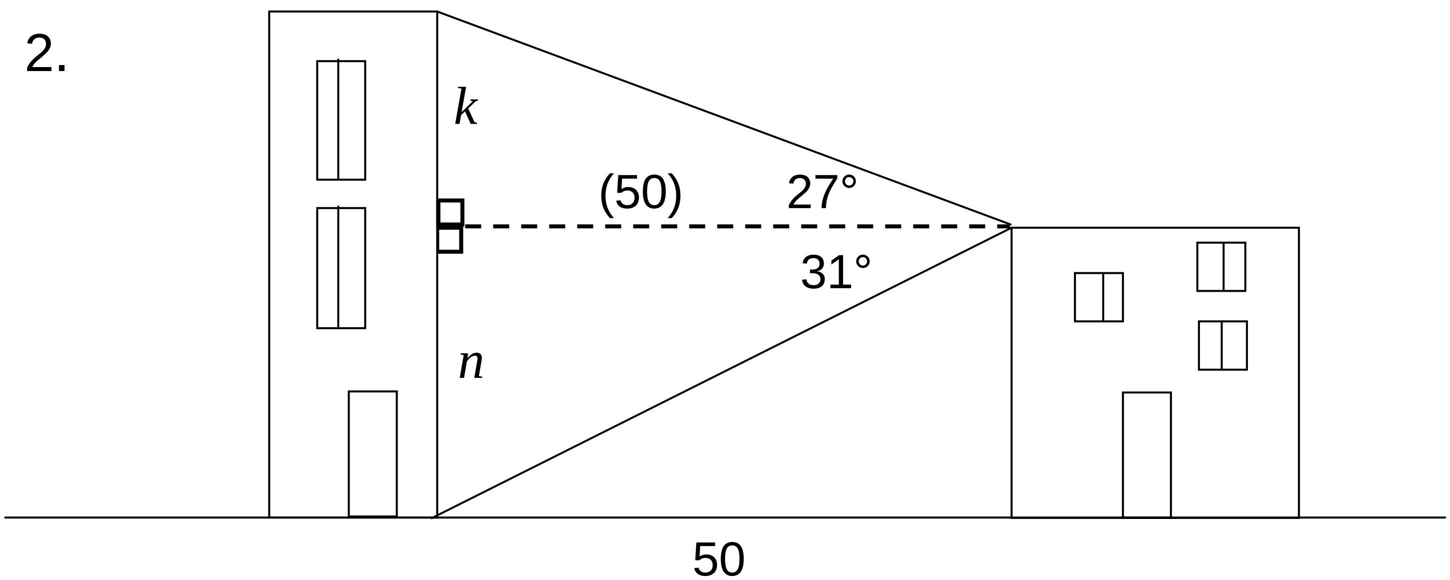
4. a ≈ 35.3 feet

5. Therefore, the building is approx. 35.3 feet tall.

Example: The same man now stands atop a building. He measures the angle of elevation to the building across the street to be 27° and the angle of depression (to the base of the building) to be 31°. If the two buildings are 50 feet apart, how tall is the taller building?

1. Find: the height of the taller building.

2.



Let n = the height of the 27° angle triangle
Let k = the height of the 31° angle triangle

3.

$$\tan 27° = \frac{k}{50} \qquad \tan 31° = \frac{n}{50}$$

$$k = 50\tan 27° \qquad n = 50\tan 31°$$

$$k \approx 25.5 \qquad n \approx 30.0$$

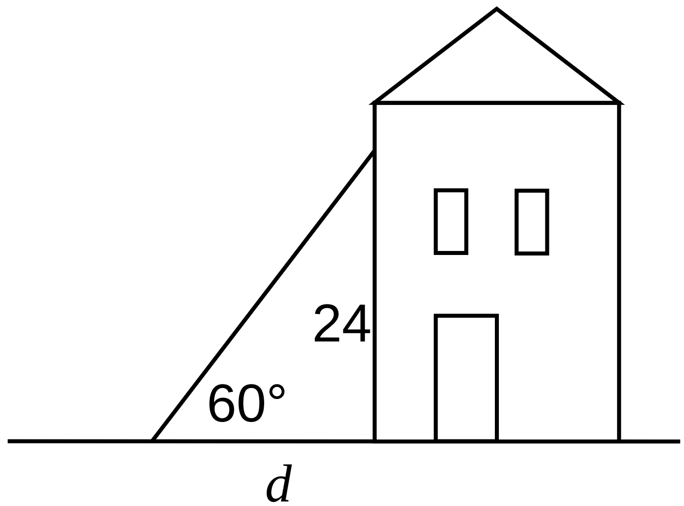
4. 25.5 + 30.3 = 55.8

5. Therefore the building is 55.5 feet tall.

Example: A ladder leaning against a house reaches 24 feet up the side of the house. The ladder makes a 60° angle with the ground. How far is the base of the ladder from the house?

1. Find: The distance from the ladder base to the house.

2. Let d = distance



3.

$$\tan 60° = \frac{24}{d}$$

$$d \tan 60° = 24$$

4.

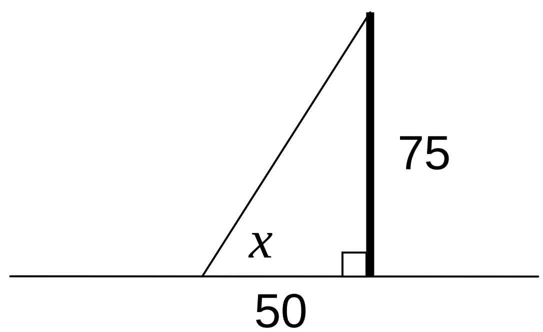
$$d = \frac{24}{\tan 60°} \approx 13.9$$

5. Therefore, the ladder is 13.9 feet from the base of the building.

Example: An amateur radio operator erects a 75-foot vertical tower for an antenna. Find the angle of elevation to the top of the tower at a point on level ground 50 feet from its base.

1. Find: the angle of elevation

2. Let x = the angle of elevation



3.
$$\tan x = \frac{75}{50}$$
$$\tan x = 1.5$$
$$\arctan(\tan x) = \arctan(1.5)$$

4.
$$x \approx 56.3$$

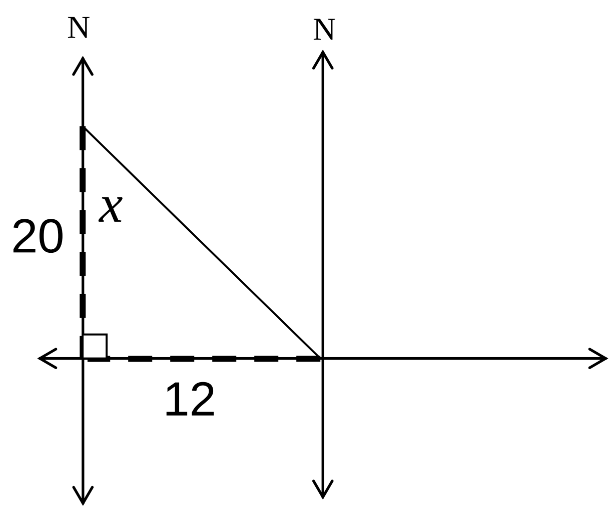
5. Therefore, the angle is approximately 56.3°.

In surveying and navigation, directions are generally given in terms of bearings. A bearing measures the acute angle that a path or line of sight makes with a fixed **north-south** line. For example, the bearing S 30°E means 30° east of south.

Example: A ship leaves port and sails 12 miles due west. It then turns and sails due north for 20 miles. At this point, at what bearing should the ship sail to get back to port?

1. Find: the bearing the ship should sail

2. Let x = the angle of bearing



3.

$$\tan x = \frac{12}{20}$$

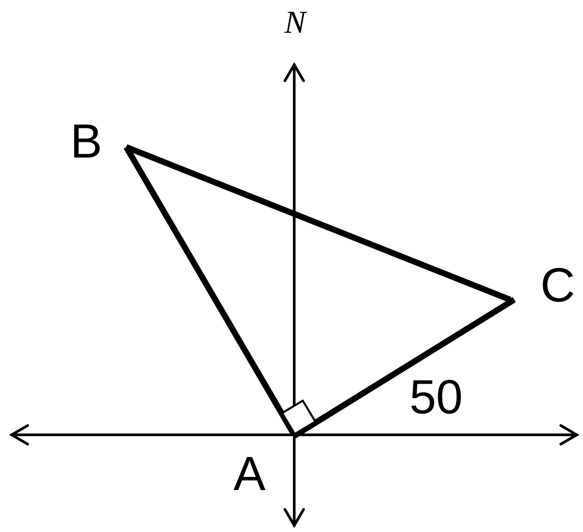
$$\tan x = 0.6$$

$$\arctan(\tan x) = \arctan(0.6)$$

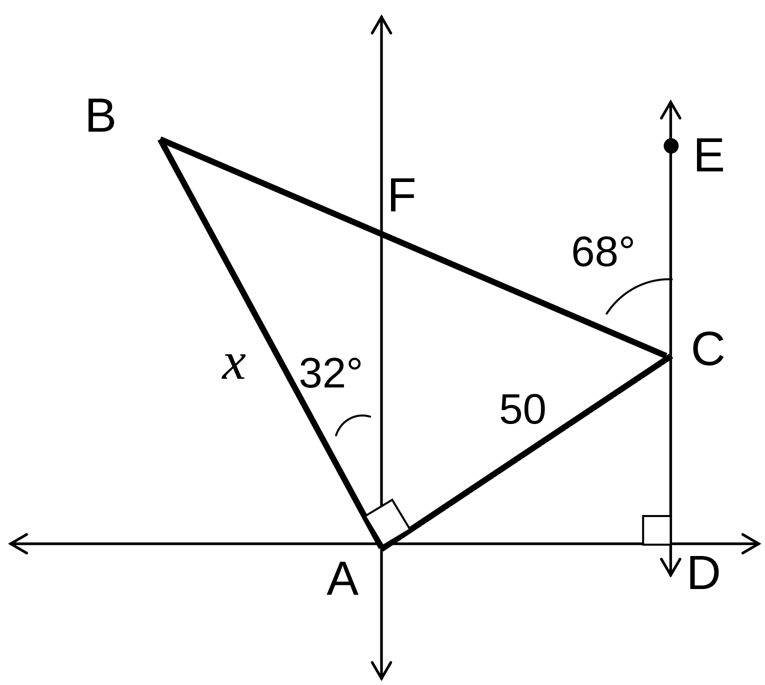
4. $x = 30.9°$

5. Therefore the course should be S 30.9° E.

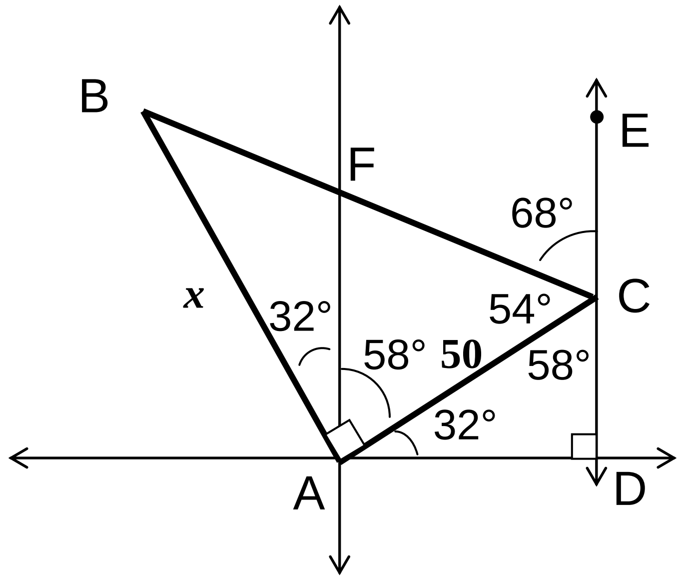
Example: A surveyor wishes to find the distance from A to B. The bearing from A to B is N 32° W. He walks 50 ft. from point A to point C along a line that is at a right angle to AB. At point C, the bearing to point B is N 68° W. Find the distance from A to B.



1. Find: the distance from A to B.
2. Let x = the distance from A to B



- $\angle BAF + \angle FAC$ must = 90°, so $\angle FAC = 58°$.
- $\angle FAC + \angle CAD$ must = 90°, so $\angle CAD = 32°$.
- ΔADC is right, so $\angle ACD = 58°$.
- $\angle ECF + \angle FCA + \angle DCA = 180°$, so $\angle FCA = 54°$



3. $\tan 54° = \dfrac{x}{50}$

$50 \tan 54° = x$

4. $x \approx 68.82$

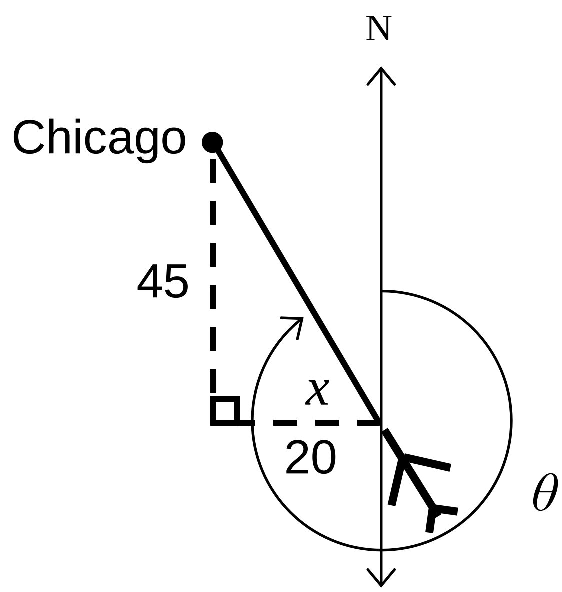
5. Therefore, the distance from A to B is 68.82 meters.

In air navigation, bearings are measured in degrees clockwise from north.

Example: A plane is 45 miles south and 20 miles east of the Chicago airport. What bearing should be taken to fly directly to the airport?

1. Find: the bearing to the airport

2. Let x = the angle of the right triangle
   Let θ = the bearing to the airport



3. $\tan x = 45/20$
   $\arctan x = \arctan(45/20)$
   $x = 66°$

   $\theta = 270° + x$
   $\theta = 270° + 66°$

4. $\theta = 336°$

5. Therefore, the plane must travel on a bearing of 336°.

<u>Note</u>: This would be the same as N 24° W.

Read through the Example of harmonic motion on p. 335-6.

<u>Definition</u>: A point that moves on a coordinate line is said to be in <u>simple harmonic motion</u> if its distance *d* from the origin at time *t* is given by either

$$d = a \sin \omega t \text{ or } d = a \cos \omega t,$$

where a and ω are real numbers such that $\omega > 0$. The motion has amplitude $|a|$, period $\frac{2\pi}{\omega}$, and frequency $\frac{\omega}{2\pi}$.

- If the motion starts from the point of equilibrium (the origin), then the equation is sine.
  - If the point moves up first, then a is positive.
  - If the point moves down first, then a is negative.
- If the motion does not start at the point of equilibrium (the origin), then the equation is cosine.
  - If the point starts above the point of equilibrium, then a is positive.
  - If the point starts below the point of equilibrium, then a is negative.

*If a is negative, think of the sine or cosine function as being reflected over the x-axis.

Example: Write the equation describing the motion of a ball that is attached to a spring suspended from the ceiling, with period 3 seconds and maximum displacement from equilibrium of 7 inches. Assume the ball starts its motion from the origin and travels upwards first.

Because the period is $\frac{2\pi}{\omega}$, = 3, we solve to get ω = $\frac{2\pi}{3}$,.

Because the max displacement is 7, the amplitude is 7.

Since the ball starts its motion from the origin and travels upward first, we use the sine version of the equation and a positive 7.

The equation is then $d = 7 \sin \frac{2\pi}{3} t$.

Example: Find (a) the maximum displacement, (b) the frequency, and (c) the least positive value of t for which d=0 for the equation $d = \frac{1}{2} \cos 20\pi t$.

(a) The amplitude is the max displacement, so the max displacement is $| \frac{1}{2} | = \frac{1}{2}$.

(b) The frequency is $\omega/2\pi$. Our value of ω is 20π, so $\omega/2\pi = 20\pi/2\pi = 10$. Thus the frequency is 10 cycles per unit of time.

(c) Let d = 0. We get 0= ½ cos $20\pi t$

$$0 = \frac{1}{2}\cos 20\pi t$$
$$0 = \cos 20\pi t$$
$$\arccos 0 = \arccos(\cos 20\pi t)$$
$$\frac{\pi}{2} = 20\pi t$$
$$t = \frac{\pi}{2}\cdot\frac{1}{20\pi} = \frac{1}{40}$$

Because this is the cosine function, it starts up at ½, travels downward to – ½ and then back up, etc. It will go through the origin at t = 1/40. If our time is in minute, then that would be 1/40 of a minute after it starts. (i.e. 1.5 seconds)

*Remember that the object travels straight up and down, but we can get an *idea* of its path by looking at this graph: