A.3 Polynomials and Factoring

What you should learn

- How to write polynomials in standard form
- How to add, subtract, and multiply polynomials
- How to use special products to multiply polynomials
- How to remove common factors from polynomials
- How to factor special polynomial forms
- How to factor trinomials as the product of two binomials
- How to factor polynomials by grouping

Why you should learn it

Polynomials can be used to model and solve real-life problems. For instance, in Exercise 178 on page A34, a polynomial is used to model the stopping distance of an automobile.

Polynomials

The most common type of algebraic expression is the **polynomial.** Some examples are

$$2x + 5, \quad 3x^4 - 7x^2 + 2x + 4, \quad \text{and} \quad 5x^2y^2 - xy + 3.$$

The first two are *polynomials in x* and the third is a *polynomial in x and y*. The terms of a polynomial in x have the form ax^k, where a is the **coefficient** and k is the **degree** of the term. For instance, the polynomial

$$2x^3 - 5x^2 + 1 = 2x^3 + (-5)x^2 + (0)x + 1$$

has coefficients 2, -5, 0, and 1.

Definition of Polynomial in x

Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and let n be a nonnegative integer. A polynomial in x is an expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdot \cdot \cdot + a_1x + a_0$$

where $a_n \neq 0$. The polynomial is of **degree** n, a_n is the **leading coefficient,** and a_0 is the **constant term.**

Polynomials with one, two, and three terms are called **monomials, binomials,** and **trinomials,** respectively. In **standard form,** a polynomial is written with descending powers of x.

Example 1 ▶ **Writing Polynomials in Standard Form**

	Polynomial	*Standard Form*	*Degree*
a.	$4x^2 - 5x^7 - 2 + 3x$	$-5x^7 + 4x^2 + 3x - 2$	7
b.	$4 - 9x^2$	$-9x^2 + 4$	2
c.	8	$8 \ (8 = 8x^0)$	0

A polynomial that has all zero coefficients is called the zero polynomial, denoted by 0. No degree is assigned to this particular polynomial. For polynomials in more than one variable, the degree of a *term* is the sum of the exponents of the variables in the term. The degree of the *polynomial* is the degree of the highest-degree term. The leading coefficient of the polynomial is the coefficient of the highest-degree term. Expressions such as the following are not polynomials.

$x^3 - \sqrt{3x} = x^3 - (3x)^{1/2}$ The exponent "1/2" is not an integer.

$x^2 + 5x^{-1}$ The exponent "-1" is not a nonnegative integer.

Operations with Polynomials

You can add and subtract polynomials in much the same way you add and subtract real numbers. Simply add or subtract the *like terms* (terms having the same variables to the same powers) by adding their coefficients. For instance, $-3xy^2$ and $5xy^2$ are like terms and their sum is

$$-3xy^2 + 5xy^2 = (-3 + 5)xy^2 = 2xy^2.$$

STUDY TIP

A common mistake is to fail to change the sign of each term inside parentheses preceded by a negative sign. For instance, note that

$$-(x^2 - x + 3) = -x^2 + x - 3$$

and

$$-(x^2 - x + 3) \neq -x^2 - x + 3.$$

Example 2 ▶ Sums and Differences of Polynomials

Perform the operation on the polynomials.

a. $(5x^3 - 7x^2 - 3) + (x^3 + 2x^2 - x + 8)$

b. $(7x^4 - x^2 - 4x + 2) - (3x^4 - 4x^2 + 3x)$

Solution

a. $(5x^3 - 7x^2 - 3) + (x^3 + 2x^2 - x + 8)$

$= (5x^3 + x^3) + (2x^2 - 7x^2) - x + (8 - 3)$ Group like terms.

$= 6x^3 - 5x^2 - x + 5$ Combine like terms.

b. $(7x^4 - x^2 - 4x + 2) - (3x^4 - 4x^2 + 3x)$

$= 7x^4 - x^2 - 4x + 2 - 3x^4 + 4x^2 - 3x$ Distributive Property

$= (7x^4 - 3x^4) + (4x^2 - x^2) + (-3x - 4x) + 2$ Group like terms.

$= 4x^4 + 3x^2 - 7x + 2$ Combine like terms.

To find the **product** of two polynomials, use the left and right Distributive Properties.

Example 3 ▶ Multiplying Polynomials: The FOIL Method

Multiply $(3x - 2)$ by $(5x + 7)$.

Solution

$$\begin{aligned}(3x - 2)(5x + 7) &= 3x(5x + 7) - 2(5x + 7)\\ &= (3x)(5x) + (3x)(7) - (2)(5x) - (2)(7)\\ &= \underbrace{15x^2}_{\text{Product of First terms}} + \underbrace{21x}_{\text{Product of Outer terms}} - \underbrace{10x}_{\text{Product of Inner terms}} - \underbrace{14}_{\text{Product of Last terms}}\\ &= 15x^2 + 11x - 14\end{aligned}$$

Note that in this **FOIL Method** for binomials, the outer (O) and inner (I) terms are like terms and can be combined.

Special Products

Some binomial products have special forms that occur frequently in algebra.

Special Products

Let u and v be real numbers, variables, or algebraic expressions.

Special Product	*Example*
Sum and Difference of Same Terms	
$(u + v)(u - v) = u^2 - v^2$	$(x + 4)(x - 4) = x^2 - 4^2$ $= x^2 - 16$
Square of a Binomial	
$(u + v)^2 = u^2 + 2uv + v^2$	$(x + 3)^2 = x^2 + 2(x)(3) + 3^2$ $= x^2 + 6x + 9$
$(u - v)^2 = u^2 - 2uv + v^2$	$(3x - 2)^2 = (3x)^2 - 2(3x)(2) + 2^2$ $= 9x^2 - 12x + 4$
Cube of a Binomial	
$(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$	$(x + 2)^3 = x^3 + 3x^2(2) + 3x(2^2) + 2^3$ $= x^3 + 6x^2 + 12x + 8$
$(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3$	$(x - 1)^3 = x^3 - 3x^2(1) + 3x(1^2) - 1^3$ $= x^3 - 3x^2 + 3x - 1$

Example 4 ▶ Multiplying Polynomials: Special Products

Find each product.

a. $(5x + 9)(5x - 9)$ **b.** $(3x + 2)^3$ **c.** $(x + y - 2)(x + y + 2)$

Solution

a. The product of a sum and a difference of the same two terms has no middle term and takes the form $(u + v)(u - v) = u^2 - v^2$.

$$\begin{aligned}(5x + 9)(5x - 9) &= (5x)^2 - 9^2 \\ &= 25x^2 - 81\end{aligned}$$

b.
$$\begin{aligned}(3x + 2)^3 &= (3x)^3 + 3(3x)^2(2) + 3(3x)(2)^2 + 2^3 \\ &= 27x^3 + 54x^2 + 36x + 8\end{aligned}$$

c. By grouping $x + y$ in parentheses, you can write the product of the trinomials as a special product.

$$\begin{aligned}(x + y - 2)(x + y + 2) &= [(x + y) - 2][(x + y) + 2] \\ &= (x + y)^2 - 2^2 \\ &= x^2 + 2xy + y^2 - 4\end{aligned}$$

Factoring

The process of writing a polynomial as a product is called **factoring.** It is an important tool for solving equations and for simplifying rational expressions.

Unless noted otherwise, when you are asked to factor a polynomial, you can assume that you are looking for factors with integer coefficients. If a polynomial cannot be factored using integer coefficients, then it is **prime** or **irreducible over the integers.** For instance, the polynomial $x^2 - 3$ is irreducible over the integers. Over the *real numbers*, this polynomial can be factored as

$$x^2 - 3 = \left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right).$$

A polynomial is **completely factored** when each of its factors is prime. For instance,

$$x^3 - x^2 + 4x - 4 = (x - 1)(x^2 + 4) \qquad \text{Completely factored}$$

is completely factored, but

$$x^3 - x^2 - 4x + 4 = (x - 1)(x^2 - 4) \qquad \text{Not completely factored}$$

is not completely factored. Its complete factorization is

$$x^3 - x^2 - 4x + 4 = (x - 1)(x + 2)(x - 2).$$

The simplest type of factoring involves a polynomial that can be written as the product of a monomial and another polynomial. The technique used here is the Distributive Property, $a(b + c) = ab + ac$, in the *reverse* direction.

$$ab + ac = a(b + c) \qquad a \text{ is a common factor.}$$

Removing (factoring out) a common factor is the first step in completely factoring a polynomial.

Example 5 ▶ Removing Common Factors

Factor each expression.

a. $6x^3 - 4x$

b. $-4x^2 + 12x - 16$

c. $(x - 2)(2x) + (x - 2)(3)$

Solution

a. $6x^3 - 4x = 2x(3x^2) - 2x(2)$ — $2x$ is a common factor.

$\qquad = 2x(3x^2 - 2)$

b. $-4x^2 + 12x - 16 = -4(x^2) + (-4)(-3x) + (-4)4$ — -4 is a common factor.

$\qquad = -4(x^2 - 3x + 4)$

c. $(x - 2)(2x) + (x - 2)(3) = (x - 2)(2x + 3)$ — $x - 2$ is a common factor.

Factoring Special Polynomial Forms

Some polynomials have special forms that you should learn to recognize so that you can factor such polynomials easily.

Factoring Special Polynomial Forms

Factored Form	*Example*
Difference of Two Squares	
$u^2 - v^2 = (u + v)(u - v)$	$9x^2 - 4 = (3x)^2 - 2^2 = (3x + 2)(3x - 2)$
Perfect Square Trinomial	
$u^2 + 2uv + v^2 = (u + v)^2$	$x^2 + 6x + 9 = x^2 + 2(x)(3) + 3^2 = (x + 3)^2$
$u^2 - 2uv + v^2 = (u - v)^2$	$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2 = (x - 3)^2$
Sum or Difference of Two Cubes	
$u^3 + v^3 = (u + v)(u^2 - uv + v^2)$	$x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$
$u^3 - v^3 = (u - v)(u^2 + uv + v^2)$	$27x^3 - 1 = (3x)^3 - 1^3 = (3x - 1)(9x^2 + 3x + 1)$

One of the easiest special polynomial forms to factor is the difference of two squares. Think of this form as follows.

$$u^2 - v^2 = (u + v)(u - v)$$

Difference   Opposite signs

To recognize perfect square terms, look for coefficients that are squares of integers and variables raised to *even powers*.

STUDY TIP

In Example 6, note that the first step in factoring a polynomial is to check for a common factor. Once the common factor has been removed, it is often possible to recognize patterns that were not immediately obvious.

Example 6 ▶ **Removing a Common Factor First**

Factor $3 - 12x^2$.

Solution

$$\begin{aligned} 3 - 12x^2 &= 3(1 - 4x^2) && \text{3 is a common factor.} \\ &= 3[1^2 - (2x)^2] \\ &= 3(1 + 2x)(1 - 2x) && \text{Difference of two squares} \end{aligned}$$

Example 7 ▶ **Factoring the Difference of Two Squares**

a. $(x + 2)^2 - y^2 = [(x + 2) + y][(x + 2) - y] = (x + 2 + y)(x + 2 - y)$

b. $16x^4 - 81 = (4x^2)^2 - 9^2$

$$\begin{aligned} &= (4x^2 + 9)(4x^2 - 9) && \text{Difference of two squares} \\ &= (4x^2 + 9)[(2x)^2 - 3^2] \\ &= (4x^2 + 9)(2x + 3)(2x - 3) && \text{Difference of two squares} \end{aligned}$$

A perfect square trinomial is the square of a binomial, and it has the following form.

$$u^2 + 2uv + v^2 = (u + v)^2 \qquad \text{or} \qquad u^2 - 2uv + v^2 = (u - v)^2$$

Like signs Like signs

Note that the first and last terms are squares and the middle term is twice the product of u and v.

Example 8 ▶ Factoring Perfect Square Trinomials

Factor each trinomial.

a. $x^2 - 10x + 25$ **b.** $16x^2 + 8x + 1$

Solution

a. $x^2 - 10x + 25 = x^2 - 2(x)(5) + 5^2$

$= (x - 5)^2$

b. $16x^2 + 8x + 1 = (4x)^2 + 2(4x)(1) + 1^2$

$= (4x + 1)^2$

The next two formulas show the sums and differences of cubes. Pay special attention to the signs of the terms.

Like signs Like signs

$$u^3 + v^3 = (u + v)(u^2 - uv + v^2) \qquad u^3 - v^3 = (u - v)(u^2 + uv + v^2)$$

Unlike signs Unlike signs

Example 9 ▶ Factoring the Difference of Cubes

Factor $x^3 - 27$.

Solution

$x^3 - 27 = x^3 - 3^3$ Rewrite 27 as 3^3.

$= (x - 3)(x^2 + 3x + 9)$ Factor.

Example 10 ▶ Factoring the Sum of Cubes

a. $y^3 + 8 = y^3 + 2^3$ Rewrite 8 as 2^3.

$= (y + 2)(y^2 - 2y + 4)$ Factor.

b. $3(x^3 + 64) = 3(x^3 + 4^3)$ Rewrite 64 as 4^3.

$= 3(x + 4)(x^2 - 4x + 16)$ Factor.

Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c$, use the following pattern.

Factors of a

$$ax^2 + bx + c = (\quad x + \quad)(\quad x + \quad)$$

Factors of c

The goal is to find a combination of factors of a and c such that the outer and inner products add up to the middle term bx. For instance, in the trinomial $6x^2 + 17x + 5$, you can write

$$\begin{aligned}(2x + 5)(3x + 1) &= \overset{\text{F}}{6x^2} + \overset{\text{O}}{2x} + \overset{\text{I}}{15x} + \overset{\text{L}}{5} \\ &\qquad\quad \text{O} + \text{I} \\ &= 6x^2 + 17x + 5.\end{aligned}$$

Note that the outer (O) and inner (I) products add up to $17x$.

Example 11 ▶ Factoring a Trinomial: Leading Coefficient Is 1

Factor $x^2 - 7x + 12$.

Solution

The possible factorizations are

$(x - 2)(x - 6)$, $(x - 1)(x - 12)$, and $(x - 3)(x - 4)$.

Testing the middle term, you will find the correct factorization to be

$$x^2 - 7x + 12 = (x - 3)(x - 4).$$

Example 12 ▶ Factoring a Trinomial: Leading Coefficient Is Not 1

Factor $2x^2 + x - 15$.

Solution

The eight possible factorizations are as follows.

$(2x - 1)(x + 15)$	$(2x + 1)(x - 15)$
$(2x - 3)(x + 5)$	$(2x + 3)(x - 5)$
$(2x - 5)(x + 3)$	$(2x + 5)(x - 3)$
$(2x - 15)(x + 1)$	$(2x + 15)(x - 1)$

Testing the middle term, you will find the correct factorization to be

$$2x^2 + x - 15 = (2x - 5)(x + 3). \qquad \text{O} + \text{I} = 6x - 5x = x$$

STUDY TIP

If the original trinomial has no common monomial factor, its binomial factors cannot have common monomial factors. For instance, when factoring $4x^2 - 3x - 10$, you do not have to test factors, such as $(2x - 2)$, that have a common factor of 2.

Factoring by Grouping

Sometimes polynomials with more than three terms can be factored by a method called **factoring by grouping.** It is not always obvious which terms to group, and sometimes several different groupings will work.

Example 13 ▶ Factoring By Grouping

Use factoring by grouping to factor $x^3 - 2x^2 - 3x + 6$.

Solution

$$x^3 - 2x^2 - 3x + 6 = (x^3 - 2x^2) - (3x - 6) \quad \text{Group terms.}$$
$$= x^2(x - 2) - 3(x - 2) \quad \text{Factor groups.}$$
$$= (x - 2)(x^2 - 3) \quad \text{Distributive Property}$$

STUDY TIP

Another way to factor the polynomial in Example 13 is to group the terms as follows.

$$x^3 - 2x^2 - 3x + 6$$
$$= (x^3 - 3x) - (2x^2 - 6)$$
$$= x(x^2 - 3) - 2(x^2 - 3)$$
$$= (x^2 - 3)(x - 2)$$

As you can see, you obtain the same result as in Example 13.

Factoring a trinomial can involve quite a bit of trial and error. Some of this trial and error can be lessened by using factoring by grouping. The key to this method of factoring is knowing how to rewrite the middle term. In general, to factor a trinomial $ax^2 + bx + c$ by grouping, choose factors of the product ac that add up to b and use these factors to rewrite the middle term. This technique is illustrated in Example 14.

Example 14 ▶ Factoring a Trinomial By Grouping

Use factoring by grouping to factor $2x^2 + 5x - 3$.

Solution

In the trinomial $2x^2 + 5x - 3$, $a = 2$ and $c = -3$, which implies that the product ac is -6. Now, -6 factors as $(6)(-1)$ and $6 - 1 = 5 = b$. So, you can rewrite the middle term as $5x = 6x - x$. This produces the following.

$$2x^2 + 5x - 3 = 2x^2 + 6x - x - 3 \quad \text{Rewrite middle term.}$$
$$= (2x^2 + 6x) - (x + 3) \quad \text{Group terms.}$$
$$= 2x(x + 3) - (x + 3) \quad \text{Factor groups.}$$
$$= (x + 3)(2x - 1) \quad \text{Distributive Property}$$

So, the trinomial factors as $2x^2 + 5x - 3 = (x + 3)(2x - 1)$.

Guidelines for Factoring Polynomials

1. Factor out any common factors using the Distributive Property.
2. Factor according to one of the special polynomial forms.
3. Factor as $ax^2 + bx + c = (mx + r)(nx + s)$.
4. Factor by grouping.

A.3 Exercises

In Exercises 1–6, match the polynomial with its description. [The polynomials are labeled (a), (b), (c), (d), (e), and (f).]

(a) $3x^2$ (b) $1 - 2x^3$

(c) $x^3 + 3x^2 + 3x + 1$ (d) 12

(e) $-3x^5 + 2x^3 + x$ (f) $\frac{2}{3}x^4 + x^2 + 10$

1. A polynomial of degree 0
2. A trinomial of degree 5
3. A binomial with leading coefficient -2
4. A monomial of positive degree
5. A trinomial with leading coefficient $\frac{2}{3}$
6. A third-degree polynomial with leading coefficient 1

In Exercises 7–10, write a polynomial that fits the description. (There are many correct answers.)

7. A third-degree polynomial with leading coefficient -2
8. A fifth-degree polynomial with leading coefficient 6
9. A fourth-degree binomial with a negative leading coefficient
10. A third-degree binomial with an even leading coefficient

In Exercises 11–16, find the degree and leading coefficient of the polynomial.

11. $3 + 2x$
12. $-3x^4 + 2x^2 - 5$
13. $1 - x + 6x^4 - 4x^5$
14. 3
15. $4x^3y - 3xy^2 + x^2y^3$
16. $-x^5y + 2x^2y^2 + xy^4$

In Exercises 17–22, is the expression a polynomial? If so, write the polynomial in standard form.

17. $2x - 3x^3 + 8$
18. $2x^3 + x - 3x^{-1}$
19. $\dfrac{3x + 4}{x}$
20. $\dfrac{x^2 + 2x - 3}{2}$
21. $y^2 - y^4 + y^3$
22. $\sqrt{y^2 - y^4}$

In Exercises 23–38, perform the operation and write the result in standard form.

23. $(2x^2 + 1) - (x^2 - 2x + 1)$
24. $-(5x^2 - 1) - (-3x^2 + 5)$
25. $(15x^2 - 6) - (-8.3x^3 - 14.7x^2 - 17)$
26. $(15.2x^4 - 18x - 19.1) - (13.9x^4 - 9.6x + 15)$
27. $5z - [3z - (10z + 8)]$
28. $(y^3 + 1) - [(y^2 + 1) + (3y - 7)]$
29. $3x(x^2 - 2x + 1)$
30. $y^2(4y^2 + 2y - 3)$
31. $-5z(3z - 1)$
32. $(-3x)(5x + 2)$
33. $(1 - x^3)(4x)$
34. $-4x(3 - x^3)$
35. $(2.5x^2 + 3)(3x)$
36. $(2 - 3.5y)(2y^3)$
37. $-4x\left(\frac{1}{8}x + 3\right)$
38. $2y\left(4 - \frac{7}{8}y\right)$

In Exercises 39–72, multiply or find the special product.

39. $(x + 3)(x + 4)$
40. $(x - 5)(x + 10)$
41. $(3x - 5)(2x + 1)$
42. $(7x - 2)(4x - 3)$
43. $(2x + 3)^2$
44. $(4x + 5)^2$
45. $(2x - 5y)^2$
46. $(5 - 8x)^2$
47. $(x + 10)(x - 10)$
48. $(2x + 3)(2x - 3)$
49. $(x + 2y)(x - 2y)$
50. $(2x + 3y)(2x - 3y)$
51. $[(m - 3) + n][(m - 3) - n]$
52. $[(x + y) + 1][(x + y) - 1]$
53. $[(x - 3) + y]^2$
54. $[(x + 1) - y]^2$
55. $(2r^2 - 5)(2r^2 + 5)$
56. $(3a^3 - 4b^2)(3a^3 + 4b^2)$
57. $(x + 1)^3$
58. $(x - 2)^3$
59. $(2x - y)^3$
60. $(4x^3 - 3)^2$
61. $\left(\frac{1}{2}x - 3\right)^2$
62. $\left(\frac{2}{3}t + 5\right)^2$
63. $\left(\frac{1}{3}x - 2\right)\left(\frac{1}{3}x + 2\right)$
64. $\left(2x + \frac{1}{5}\right)\left(2x - \frac{1}{5}\right)$
65. $(1.2x + 3)^2$
66. $(1.5y - 3)^2$
67. $(1.5x - 4)(1.5x + 4)$
68. $(2.5y + 3)(2.5y - 3)$
69. $5x(x + 1) - 3x(x + 1)$
70. $(2x - 1)(x + 3) + 3(x + 3)$

71. $(u + 2)(u - 2)(u^2 + 4)$

72. $(x + y)(x - y)(x^2 + y^2)$

In Exercises 73–76, find the product. The expressions are not polynomials, but the formulas can still be used.

73. $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

74. $(5 + \sqrt{x})(5 - \sqrt{x})$

75. $(x - \sqrt{5})^2$

76. $(x + \sqrt{3})^2$

In Exercises 77–80, determine whether the polynomial is completely factored. If not, give the complete factorization.

77. $x^3 + 2x^2 + x + 2 = (x + 2)(x^2 + 1)$

78. $x^3 + 3x^2 - 9x - 27 = (x + 3)(x^2 - 9)$

79. $x^3 + x^2 - 7x - 7 = (x^2 - 7)(x + 1)$

80. $4x^4 + 12x^3 - x^2 - 3x = (x^2 + 3x)(4x^2 - 1)$

In Exercises 81–88, factor out the common factor.

81. $3x + 6$

82. $5y - 30$

83. $2x^3 - 6x$

84. $4x^3 - 6x^2 + 12x$

85. $x(x - 1) + 6(x - 1)$

86. $3x(x + 2) - 4(x + 2)$

87. $(x + 3)^2 - 4(x + 3)$

88. $(3x - 1)^2 + (3x - 1)$

In Exercises 89–92, find the greatest common factor such that the remaining factors have only integer coefficients.

89. $\frac{1}{2}x^3 + 2x^2 - 5x$

90. $\frac{1}{3}y^4 - 5y^2 + 2y$

91. $\frac{2}{3}x(x - 3) - 4(x - 3)$

92. $\frac{4}{5}y(y + 1) - 2(y + 1)$

In Exercises 93–100, factor the difference of two squares.

93. $16y^2 - 9$

94. $49 - 9y^2$

95. $16x^2 - \frac{1}{9}$

96. $\frac{4}{25}y^2 - 64$

97. $(x - 1)^2 - 4$

98. $25 - (z + 5)^2$

99. $9u^2 - 4v^2$

100. $25x^2 - 16y^2$

In Exercises 101–108, factor the perfect square trinomial.

101. $x^2 - 4x + 4$

102. $x^2 + 10x + 25$

103. $36y^2 - 108y + 81$

104. $9x^2 - 12x + 4$

105. $9u^2 + 24uv + 16v^2$

106. $4x^2 - 4xy + y^2$

107. $x^2 - \frac{4}{3}x + \frac{4}{9}$

108. $z^2 + z + \frac{1}{4}$

In Exercises 109–116, factor the sum or difference of cubes.

109. $x^3 - 8$

110. $x^3 - 27$

111. $y^3 + 64$

112. $z^3 + 125$

113. $8t^3 - 1$

114. $27x^3 + 8$

115. $u^3 + 27v^3$

116. $64x^3 - y^3$

In Exercises 117–128, factor the trinomial.

117. $x^2 + x - 2$

118. $x^2 + 5x + 6$

119. $s^2 - 5s + 6$

120. $t^2 - t - 6$

121. $20 - y - y^2$

122. $24 + 5z - z^2$

123. $3x^2 - 5x + 2$

124. $2x^2 - x - 1$

125. $5x^2 + 26x + 5$

126. $12x^2 + 7x + 1$

127. $-9z^2 + 3z + 2$

128. $-5u^2 - 13u + 6$

In Exercises 129–134, factor by grouping.

129. $x^3 - x^2 + 2x - 2$

130. $x^3 + 5x^2 - 5x - 25$

131. $2x^3 - x^2 - 6x + 3$

132. $6 + 2x - 3x^3 - x^4$

133. $6x^3 - 2x + 3x^2 - 1$

134. $8x^5 - 6x^2 + 12x^3 - 9$

In Exercises 135–138, factor the trinomial by grouping.

135. $3x^2 + 10x + 8$

136. $2x^2 + 9x + 9$

137. $15x^2 - 11x + 2$

138. $12x^2 - 13x + 1$

In Exercises 139–160, completely factor the expression.

139. $6x^2 - 54$

140. $12x^2 - 48$

141. $x^3 - 4x^2$

142. $x^3 - 9x$

143. $2x^2 + 4x - 2x^3$

144. $2y^3 - 7y^2 - 15y$

145. $3x^3 + x^2 + 15x + 5$

146. $13x + 6 + 5x^2$

147. $\frac{1}{81}x^2 + \frac{2}{9}x - 8$

148. $\frac{1}{8}x^2 - \frac{1}{96}x - \frac{1}{16}$

149. $x^4 - 4x^3 + x^2 - 4x$

150. $3u - 2u^2 + 6 - u^3$

151. $(x^2 + 1)^2 - 4x^2$

152. $(x^2 + 8)^2 - 36x^2$

153. $2t^3 - 16$

154. $5x^3 + 40$

155. $4x(2x - 1) + (2x - 1)^2$

156. $5(3 - 4x)^2 - 8(3 - 4x)(5x - 1)$

157. $7(3x + 2)^2(1 - x)^2 + (3x + 2)(1 - x)^3$

158. $7x(2)(x^2 + 1)(2x) - (x^2 + 1)^2(7)$

159. $3(x - 2)^2(x + 1)^4 + (x - 2)^3(4)(x + 1)^3$

160. $5(x^6 + 1)^4(6x^5)(3x + 2)^3 + 3(3x + 2)^2(3)(x^6 + 1)^5$

In Exercises 161–164, find all values of b for which the trinomial can be factored.

161. $x^2 + bx - 15$

162. $x^2 + bx + 50$

163. $x^2 + bx - 12$

164. $x^2 + bx + 24$

In Exercises 165–168, find two integer values of c such that the trinomial can be factored. (There are many correct answers.)

165. $2x^2 + 5x + c$

166. $3x^2 - 10x + c$

167. $3x^2 - x + c$

168. $2x^2 + 9x + c$

169. ***Cost, Revenue, and Profit*** An electronics manufacturer can produce and sell x radios per week. The total cost C (in dollars) for producing x radios is

$$C = 73x + 25{,}000$$

and the total revenue R (in dollars) is

$$R = 95x.$$

Find the profit P obtained by selling 5000 radios per week.

170. ***Cost, Revenue, and Profit*** An artist can produce and sell x craft items per month. The total cost C (in dollars) for producing x craft items is

$$C = 460 + 12x$$

and the total revenue R (in dollars) is

$$R = 36x.$$

Find the profit P obtained by selling 42 craft items per month.

171. ***Compound Interest*** After 2 years, an investment of \$500 compounded annually at an interest rate r will yield an amount of

$$500(1 + r)^2.$$

(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r in the table.

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1 + r)^2$					

(c) What conclusion can you make from the table?

172. ***Compound Interest*** After 3 years, an investment of \$1200 compounded annually at an interest rate r will yield an amount of

$$1200(1 + r)^3.$$

(a) Write this polynomial in standard form.

(b) Use a calculator to evaluate the polynomial for the values of r in the table.

r	2%	3%	$3\frac{1}{2}\%$	4%	$5\frac{1}{2}\%$
$1200(1 + r)^3$					

(c) What conclusion can you make from the table?