A.4 Rational Expressions

▶ What you should learn

- How to find domains of algebraic expressions
- How to simplify rational expressions
- How to add, subtract, multiply, and divide rational expressions
- How to simplify complex fractions

▶ Why you should learn it

Rational expressions can be used to solve real-life problems. For instance, in Exercise 78 on page A45, a rational expression is used to model the cost per ounce of precious metals from 1994 through 1999.

Domain of an Algebraic Expression

The set of real numbers for which an algebraic expression is defined is the **domain** of the expression. Two algebraic expressions are **equivalent** if they have the same domain and yield the same values for all numbers in their domain. For instance, $(x + 1) + (x + 2)$ and $2x + 3$ are equivalent because

$$\begin{aligned}(x + 1) + (x + 2) &= x + 1 + x + 2\\ &= x + x + 1 + 2\\ &= 2x + 3.\end{aligned}$$

Example 1 ▶ Finding the Domain of an Algebraic Expression

a. The domain of the polynomial

$$2x^3 + 3x + 4$$

is the set of all real numbers. In fact, the domain of any polynomial is the set of all real numbers, unless the domain is specifically restricted.

b. The domain of the radical expression

$$\sqrt{x - 2}$$

is the set of real numbers greater than or equal to 2, because the square root of a negative number is not a real number.

c. The domain of the expression

$$\frac{x + 2}{x - 3}$$

is the set of all real numbers except $x = 3$, which would produce an undefined division by zero.

The quotient of two algebraic expressions is a *fractional expression.* Moreover, the quotient of two *polynomials* such as

$$\frac{1}{x}, \qquad \frac{2x - 1}{x + 1}, \qquad \text{or} \qquad \frac{x^2 - 1}{x^2 + 1}$$

is a **rational expression.** Recall that a fraction is in simplest form if its numerator and denominator have no factors in common aside from ± 1. To write a fraction in simplest form, divide out common factors.

$$\frac{a \cdot \cancel{c}}{b \cdot \cancel{c}} = \frac{a}{b}, \quad c \neq 0$$

The key to success in simplifying rational expressions lies in your ability to *factor* polynomials.

Simplifying Rational Expressions

When simplifying rational expressions, be sure to factor each polynomial completely before concluding that the numerator and denominator have no factors in common.

STUDY TIP

In Example 2, do not make the mistake of trying to simplify further by dividing out terms.

$$\frac{x+6}{3} = \frac{x+\cancel{6}}{\cancel{3}} = x+2$$

Remember that to simplify fractions, divide out common *factors*, not terms.

Example 2 ▶ Simplifying a Rational Expression

Write $\dfrac{x^2+4x-12}{3x-6}$ in simplest form.

Solution

$$\frac{x^2+4x-12}{3x-6} = \frac{(x+6)\cancel{(x-2)}}{3\cancel{(x-2)}} \qquad \text{Factor completely.}$$

$$= \frac{x+6}{3}, \quad x \neq 2 \qquad \text{Divide out common factors.}$$

Note that the original expression is undefined when $x = 2$ (because division by zero is undefined). To make sure that the simplified expression is *equivalent* to the original expression, you must restrict the domain of the simplified expression by excluding the value $x = 2$.

Sometimes it may be necessary to change the sign of a factor to simplify a rational expression, as shown in Example 3(b).

Example 3 ▶ Simplifying Rational Expressions

Write each expression in simplest form.

a. $\dfrac{x^3-4x}{x^2+x-2}$ **b.** $\dfrac{12+x-x^2}{2x^2-9x+4}$

Solution

a. $$\frac{x^3-4x}{x^2+x-2} = \frac{x(x^2-4)}{(x+2)(x-1)}$$

$$= \frac{x\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-1)} \qquad \text{Factor completely.}$$

$$= \frac{x(x-2)}{(x-1)}, \quad x \neq -2 \qquad \text{Divide out common factors.}$$

b. $$\frac{12+x-x^2}{2x^2-9x+4} = \frac{(4-x)(3+x)}{(2x-1)(x-4)} \qquad \text{Factor completely.}$$

$$= \frac{-\cancel{(x-4)}(3+x)}{(2x-1)\cancel{(x-4)}} \qquad (4-x) = -(x-4)$$

$$= -\frac{3+x}{2x-1}, \quad x \neq 4 \qquad \text{Divide out common factors.}$$

Operations with Rational Expressions

To multiply or divide rational expressions, use the properties of fractions discussed in Section A.1. Recall that to divide fractions, you invert the divisor and multiply.

Example 4 ▶ Multiplying Rational Expressions

$$\frac{2x^2 + x - 6}{x^2 + 4x - 5} \cdot \frac{x^3 - 3x^2 + 2x}{4x^2 - 6x} = \frac{\cancel{(2x - 3)}(x + 2)}{(x + 5)\cancel{(x - 1)}} \cdot \frac{\cancel{x}(x - 2)\cancel{(x - 1)}}{2\cancel{x}\cancel{(2x - 3)}}$$

$$= \frac{(x + 2)(x - 2)}{2(x + 5)}, \qquad x \neq 0, x \neq 1, x \neq \tfrac{3}{2}$$

In this text, when performing operations with rational expressions, the convention of listing *by the simplified expression* all values of x that must be specifically excluded from the domain in order to make the domains of the simplified and original expressions agree is followed. In Example 4, for instance, the restrictions $x \neq 0$, $x \neq 1$, and $x \neq \frac{3}{2}$ are listed with the simplified expression in order to make the two domains agree. Note that the value $x = -5$ is excluded from both domains, so it is not necessary to list this value.

Example 5 ▶ Dividing Rational Expressions

$$\frac{x^3 - 8}{x^2 - 4} \div \frac{x^2 + 2x + 4}{x^3 + 8} = \frac{x^3 - 8}{x^2 - 4} \cdot \frac{x^3 + 8}{x^2 + 2x + 4} \qquad \text{Invert and multiply.}$$

$$= \frac{\cancel{(x - 2)}\cancel{(x^2 + 2x + 4)}}{\cancel{(x + 2)}\cancel{(x - 2)}} \cdot \frac{\cancel{(x + 2)}(x^2 - 2x + 4)}{\cancel{x^2 + 2x + 4}}$$

$$= x^2 - 2x + 4, \qquad x \neq \pm 2 \qquad \text{Divide out common factors.}$$

To add or subtract rational expressions, you can use the LCD (least common denominator) method or the basic definition

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}, \qquad b \neq 0, d \neq 0. \qquad \text{Basic definition}$$

This definition provides an efficient way of adding or subtracting *two* fractions that have no common factors in their denominators.

Example 6 ▶ Subtracting Rational Expressions

$$\frac{x}{x - 3} - \frac{2}{3x + 4} = \frac{x(3x + 4) - 2(x - 3)}{(x - 3)(3x + 4)} \qquad \text{Basic definition}$$

$$= \frac{3x^2 + 4x - 2x + 6}{(x - 3)(3x + 4)} \qquad \text{Distributive Property}$$

$$= \frac{3x^2 + 2x + 6}{(x - 3)(3x + 4)} \qquad \text{Combine like terms.}$$

For three or more fractions, or for fractions with a repeated factor in the denominators, the LCD method works well. Recall that the least common denominator of several fractions consists of the product of all prime factors in the denominators, with each factor given the highest power of its occurrence in any denominator. Here is a numerical example.

$$\frac{1}{6}+\frac{3}{4}-\frac{2}{3}=\frac{1\cdot 2}{6\cdot 2}+\frac{3\cdot 3}{4\cdot 3}-\frac{2\cdot 4}{3\cdot 4} \qquad \text{The LCD is 12.}$$

$$=\frac{2}{12}+\frac{9}{12}-\frac{8}{12}$$

$$=\frac{3}{12}$$

$$=\frac{1}{4}$$

Sometimes the numerator of the answer has a factor in common with the denominator. In such cases the answer should be simplified. For instance, in the example above, $\frac{3}{12}$ was simplified to $\frac{1}{4}$.

Example 7 ▶ Combining Rational Expressions: The LCD Method

Perform the operations and simplify.

$$\frac{3}{x-1}-\frac{2}{x}+\frac{x+3}{x^2-1}$$

Solution

Using the factored denominators $(x-1)$, x, and $(x+1)(x-1)$, you can see that the LCD is $x(x+1)(x-1)$.

$$\frac{3}{x-1}-\frac{2}{x}+\frac{x+3}{(x+1)(x-1)}$$

$$=\frac{3(x)(x+1)}{x(x+1)(x-1)}-\frac{2(x+1)(x-1)}{x(x+1)(x-1)}+\frac{(x+3)(x)}{x(x+1)(x-1)}$$

$$=\frac{3(x)(x+1)-2(x+1)(x-1)+(x+3)(x)}{x(x+1)(x-1)}$$

$$=\frac{3x^2+3x-2x^2+2+x^2+3x}{x(x+1)(x-1)} \qquad \text{Distributive Property}$$

$$=\frac{3x^2-2x^2+x^2+3x+3x+2}{x(x+1)(x-1)} \qquad \text{Group like terms.}$$

$$=\frac{2x^2+6x+2}{x(x+1)(x-1)} \qquad \text{Combine like terms.}$$

$$=\frac{2(x^2+3x+1)}{x(x+1)(x-1)} \qquad \text{Factor.}$$

Complex Fractions

Fractional expressions with separate fractions in the numerator, denominator, or both are called **complex fractions.** Here are two examples.

$$\frac{\left(\frac{1}{x}\right)}{x^2 + 1} \quad \text{and} \quad \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x^2 + 1}\right)}$$

A complex fraction can be simplified by combining the fractions in its numerator into a single fraction and then combining the fractions in its denominator into a single fraction. Then invert the denominator and multiply.

Example 8 ▶ Simplifying a Complex Fraction

$$\frac{\left(\frac{2}{x} - 3\right)}{\left(1 - \frac{1}{x - 1}\right)} = \frac{\left[\frac{2 - 3(x)}{x}\right]}{\left[\frac{1(x - 1) - 1}{x - 1}\right]} \qquad \text{Combine fractions.}$$

$$= \frac{\left(\frac{2 - 3x}{x}\right)}{\left(\frac{x - 2}{x - 1}\right)} \qquad \text{Simplify.}$$

$$= \frac{2 - 3x}{x} \cdot \frac{x - 1}{x - 2} \qquad \text{Invert and multiply.}$$

$$= \frac{(2 - 3x)(x - 1)}{x(x - 2)}, \quad x \neq 1$$

Another way to simplify a complex fraction is to multiply its numerator and denominator by the LCD of all fractions in its numerator and denominator. This method is applied to the fraction in Example 8 as follows.

$$\frac{\left(\frac{2}{x} - 3\right)}{\left(1 - \frac{1}{x - 1}\right)} = \frac{\left(\frac{2}{x} - 3\right)}{\left(1 - \frac{1}{x - 1}\right)} \cdot \frac{x(x - 1)}{x(x - 1)} \qquad \text{LCD is } x(x - 1).$$

$$= \frac{\left(\frac{2 - 3x}{\cancel{x}}\right) \cdot \cancel{x}(x - 1)}{\left(\frac{x - 2}{\cancel{x - 1}}\right) \cdot x(\cancel{x - 1})}$$

$$= \frac{(2 - 3x)(x - 1)}{x(x - 2)}, \quad x \neq 1$$

The next three examples illustrate some methods for simplifying rational expressions involving negative exponents and radicals. These types of expressions occur frequently in calculus.

To simplify an expression with negative exponents, one method is to begin by factoring out the common factor with the smaller exponent. Remember that when factoring, you subtract exponents. For instance, in $3x^{-5/2} + 2x^{-3/2}$ the smaller exponent is $-\frac{5}{2}$ and the common factor is $x^{-5/2}$.

$$\begin{aligned} 3x^{-5/2} + 2x^{-3/2} &= x^{-5/2}[3(1) + 2x^{-3/2-(-5/2)}] \\ &= x^{-5/2}(3 + 2x^{1}) \\ &= \frac{3 + 2x}{x^{5/2}} \end{aligned}$$

Example 9 ▶ Simplifying an Expression

Simplify the following expression containing negative exponents.

$$x(1 - 2x)^{-3/2} + (1 - 2x)^{-1/2}$$

Solution

Begin by factoring out the common factor with the *smaller exponent*.

$$\begin{aligned} x(1 - 2x)^{-3/2} + (1 - 2x)^{-1/2} &= (1 - 2x)^{-3/2}[x + (1 - 2x)^{(-1/2)-(-3/2)}] \\ &= (1 - 2x)^{-3/2}[x + (1 - 2x)^{1}] \\ &= \frac{1 - x}{(1 - 2x)^{3/2}} \end{aligned}$$

A second method for simplifying an expression with negative exponents is shown in the next example.

Example 10 ▶ Simplifying a Complex Fraction

$$\begin{aligned} &\frac{(4 - x^2)^{1/2} + x^2(4 - x^2)^{-1/2}}{4 - x^2} \\ &\qquad = \frac{(4 - x^2)^{1/2} + x^2(4 - x^2)^{-1/2}}{4 - x^2} \cdot \frac{(4 - x^2)^{1/2}}{(4 - x^2)^{1/2}} \\ &\qquad = \frac{(4 - x^2)^{1} + x^2(4 - x^2)^{0}}{(4 - x^2)^{3/2}} \\ &\qquad = \frac{4 - x^2 + x^2}{(4 - x^2)^{3/2}} \\ &\qquad = \frac{4}{(4 - x^2)^{3/2}} \end{aligned}$$

Example 11 ▶ Rewriting a Difference Quotient

The expression from calculus

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

is an example of a *difference quotient*. Rewrite this expression by rationalizing its numerator.

Solution

$$\begin{aligned}\frac{\sqrt{x+h}-\sqrt{x}}{h} &= \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\\ &= \frac{\left(\sqrt{x+h}\right)^2-\left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\ &= \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\ &= \frac{1}{\sqrt{x+h}+\sqrt{x}}, \qquad h \neq 0\end{aligned}$$

Notice that the original expression is undefined when $h = 0$. So, you must exclude $h = 0$ from the domain of the simplified expression so that the expressions are equivalent.

Difference quotients, such as that in Example 11, occur frequently in calculus. Often, they need to be rewritten in an equivalent form that can be evaluated when $h = 0$. Note that the equivalent form is not simpler than the original form, but it has the advantage that it is defined when $h = 0$.

A.4 Exercises

In Exercises 1–8, find the domain of the expression.

1. $3x^2 - 4x + 7$

2. $2x^2 + 5x - 2$

3. $4x^3 + 3, \quad x \geq 0$

4. $6x^2 - 9, \quad x > 0$

5. $\dfrac{1}{x - 2}$

6. $\dfrac{x + 1}{2x + 1}$

7. $\sqrt{x + 1}$

8. $\sqrt{6 - x}$

In Exercises 9 and 10, find the missing factor in the numerator such that the two fractions are equivalent.

9. $\dfrac{5}{2x} = \dfrac{5(\quad)}{6x^2}$

10. $\dfrac{3}{4} = \dfrac{3(\quad)}{4(x + 1)}$

In Exercises 11–28, write the rational expression in simplest form.

11. $\dfrac{15x^2}{10x}$

12. $\dfrac{18y^2}{60y^5}$

13. $\dfrac{3xy}{xy + x}$

14. $\dfrac{2x^2y}{xy - y}$

15. $\dfrac{4y - 8y^2}{10y - 5}$

16. $\dfrac{9x^2 + 9x}{2x + 2}$

17. $\dfrac{x - 5}{10 - 2x}$

18. $\dfrac{12 - 4x}{x - 3}$

19. $\dfrac{y^2 - 16}{y + 4}$

20. $\dfrac{x^2 - 25}{5 - x}$

21. $\dfrac{x^3 + 5x^2 + 6x}{x^2 - 4}$

22. $\dfrac{x^2 + 8x - 20}{x^2 + 11x + 10}$

23. $\dfrac{y^2 - 7y + 12}{y^2 + 3y - 18}$

24. $\dfrac{x^2 - 7x + 6}{x^2 + 11x + 10}$

25. $\dfrac{2 - x + 2x^2 - x^3}{x^2 - 4}$

26. $\dfrac{x^2 - 9}{x^3 + x^2 - 9x - 9}$

27. $\dfrac{z^3 - 8}{z^2 + 2z + 4}$

28. $\dfrac{y^3 - 2y^2 - 3y}{y^3 + 1}$

In Exercises 29 and 30, complete the table. What can you conclude?

29.

x	0	1	2	3	4	5	6
$\dfrac{x^2 - 2x - 3}{x - 3}$							
$x + 1$							

30.

x	0	1	2	3	4	5	6
$\dfrac{x - 3}{x^2 - x - 6}$							
$\dfrac{1}{x + 2}$							

31. ***Error Analysis*** Describe the error.

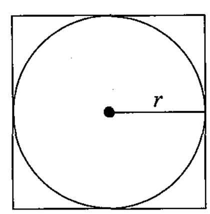
$$\frac{5x^3}{2x^3 + 4} = \frac{5x^3}{2x^3 + 4} = \frac{5}{2 + 4} = \frac{5}{6}$$

32. ***Error Analysis*** Describe the error.

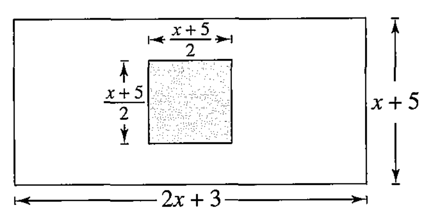
$$\frac{x^3 + 25x}{x^2 - 2x - 15} = \frac{x(x^2 + 25)}{(x - 5)(x + 3)} = \frac{x(x - 5)(x + 5)}{(x - 5)(x + 3)} = \frac{x(x + 5)}{x + 3}$$

Geometry **In Exercises 33 and 34, find the ratio of the area of the shaded portion of the figure to the total area of the figure.**

33.



34.



In Exercises 35–42, perform the multiplication or division and simplify.

35. $\dfrac{5}{x - 1} \cdot \dfrac{x - 1}{25(x - 2)}$

36. $\dfrac{x + 13}{x^3(3 - x)} \cdot \dfrac{x(x - 3)}{5}$

37. $\dfrac{r}{r - 1} \cdot \dfrac{r^2 - 1}{r^2}$

38. $\dfrac{4y - 16}{5y + 15} \cdot \dfrac{2y + 6}{4 - y}$

39. $\dfrac{t^2 - t - 6}{t^2 + 6t + 9} \cdot \dfrac{t + 3}{t^2 - 4}$

40. $\dfrac{x^2 + xy - 2y^2}{x^3 + x^2y} \cdot \dfrac{x}{x^2 + 3xy + 2y^2}$

41. $\dfrac{x^2 - 36}{x} \div \dfrac{x^3 - 6x^2}{x^2 + x}$

42. $\dfrac{x^2 - 14x + 49}{x^2 - 49} \div \dfrac{3x - 21}{x + 7}$

In Exercises 43–52, perform the addition or subtraction and simplify.

43. $\dfrac{5}{x - 1} + \dfrac{x}{x - 1}$

44. $\dfrac{2x - 1}{x + 3} + \dfrac{1 - x}{x + 3}$

45. $6 - \dfrac{5}{x + 3}$

46. $\dfrac{3}{x - 1} - 5$

47. $\dfrac{3}{x - 2} + \dfrac{5}{2 - x}$

48. $\dfrac{2x}{x - 5} - \dfrac{5}{5 - x}$

49. $\dfrac{1}{x^2 - x - 2} - \dfrac{x}{x^2 - 5x + 6}$

50. $\dfrac{2}{x^2 - x - 2} + \dfrac{10}{x^2 + 2x - 8}$

51. $-\dfrac{1}{x} + \dfrac{2}{x^2 + 1} + \dfrac{1}{x^3 + x}$

52. $\dfrac{2}{x + 1} + \dfrac{2}{x - 1} + \dfrac{1}{x^2 - 1}$

In Exercises 53–58, factor the expression by removing the common factor with the smaller exponent.

53. $x^5 - 2x^{-2}$

54. $x^5 - 5x^{-3}$

55. $x^2(x^2 + 1)^{-5} - (x^2 + 1)^{-4}$

56. $2x(x - 5)^{-3} - 4x^2(x - 5)^{-4}$

57. $2x^2(x - 1)^{1/2} - 5(x - 1)^{-1/2}$

58. $4x^3(2x - 1)^{3/2} - 2x(2x - 1)^{-1/2}$

Error Analysis **In Exercises 59 and 60, describe the error.**

59. $$\frac{x + 4}{x + 2} - \frac{3x - 8}{x + 2} = \frac{x + 4 - 3x - 8}{x + 2} = \frac{-2x - 4}{x + 2} = \frac{-2(x + 2)}{x + 2} = -2$$

60. $$\frac{6 - x}{x(x + 2)} + \frac{x + 2}{x^2} + \frac{8}{x^2(x + 2)} = \frac{x(6 - x) + (x + 2)^2 + 8}{x^2(x + 2)} = \frac{6x - x^2 + x^2 + 4 + 8}{x^2(x + 2)} = \frac{6(x + 2)}{x^2(x + 2)} = \frac{6}{x^2}$$

In Exercises 61–70, simplify the complex fraction.

61. $\dfrac{\left(\dfrac{x}{2} - 1\right)}{(x - 2)}$

62. $\dfrac{(x - 4)}{\left(\dfrac{x}{4} - \dfrac{4}{x}\right)}$

63. $\dfrac{\left[\dfrac{x^2}{(x + 1)^2}\right]}{\left[\dfrac{x}{(x + 1)^3}\right]}$

64. $\dfrac{\left(\dfrac{x^2 - 1}{x}\right)}{\left[\dfrac{(x - 1)^2}{x}\right]}$

65. $\dfrac{\left[\dfrac{1}{(x + h)^2} - \dfrac{1}{x^2}\right]}{h}$

66. $\dfrac{\left(\dfrac{x + h}{x + h + 1} - \dfrac{x}{x + 1}\right)}{h}$

67. $\dfrac{\left(\sqrt{x} - \dfrac{1}{2\sqrt{x}}\right)}{\sqrt{x}}$

68. $\dfrac{\left(\dfrac{t^2}{\sqrt{t^2 + 1}} - \sqrt{t^2 + 1}\right)}{t^2}$

69. $\dfrac{3x^{1/3} - x^{-2/3}}{3x^{-2/3}}$

70. $\dfrac{-x^3(1 - x^2)^{-1/2} - 2x(1 - x^2)^{1/2}}{x^4}$

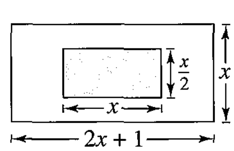
In Exercises 71 and 72, rationalize the numerator of the expression.

71. $\dfrac{\sqrt{x + 2} - \sqrt{x}}{2}$

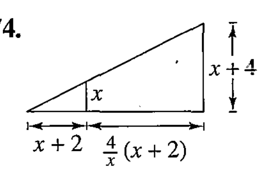
72. $\dfrac{\sqrt{z - 3} - \sqrt{z}}{3}$

Probability **In Exercises 73 and 74, consider an experiment in which a marble is tossed into a box whose base is shown in the figure. The probability that the marble will come to rest in the shaded portion of the box is equal to the ratio of the shaded area to the total area of the figure. Find the probability.**

73.



74.



75. ***Rate*** A photocopier copies at a rate of 16 pages per minute.

(a) Find the time required to copy one page.

(b) Find the time required to copy x pages.

(c) Find the time required to copy 60 pages.

76. ***Finance*** The formula that approximates the annual interest rate r of a monthly installment loan is given by

$$r = \frac{\left[\frac{24(NM - P)}{N}\right]}{\left(P + \frac{NM}{12}\right)}$$

where N is the total number of payments, M is the monthly payment, and P is the amount financed.

(a) Approximate the annual interest rate for a four-year car loan of \$16,000 that has monthly payments of \$400.

(b) Simplify the expression for the annual interest rate r, and then rework part (a).

77. ***Refrigeration*** When food (at room temperature) is placed in a refrigerator, the time required for the food to cool depends on the amount of food, the air circulation in the refrigerator, the original temperature of the food, and the temperature of the refrigerator. The model that gives the temperature of food that has an original temperature of 75°F and is placed in a 40°F refrigerator is

$$T = 10\left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$$

where T is the temperature (in degrees Fahrenheit) and t is the time (in hours).

(a) Complete the table.

t	0	2	4	6	8	10
T						

t	12	14	16	18	20	22
T						

(b) What value of T does the mathematical model appear to be approaching?

78. ***Precious Metals*** The costs per fine ounce of gold and per troy ounce of platinum for the years 1994 through 1999 are shown in the table. (Source: U.S. Bureau of Mines, U.S. Geological Survey)

Year, t	Gold	Platinum
1994	\$385	\$411
1995	\$386	\$425
1996	\$389	\$398
1997	\$332	\$397
1998	\$295	\$373
1999	\$285	\$365

Mathematical models for this data are

$$\text{Cost of gold} = \frac{6.79t^2 - 95.6t + 356}{0.0205t^2 - 0.278t + 1}$$

and

$$\text{Cost of platinum} = \frac{-148.2t + 192}{-0.46t + 1}$$

where $t = 4$ corresponds to the year 1994.

(a) Create a table using the models to estimate the costs of the two metals for the given years.

(b) Compare the estimates given by the models with the actual costs.

(c) Determine a model for the ratio of the cost of gold to the cost of platinum.

(d) Use the model from part (c) to find the ratio over the given years. Over this period of time, did the cost of gold increase or decrease relative to the cost of platinum?

Synthesis

True or False? **In Exercises 79 and 80, determine whether the statement is true or false. Justify your answer.**

79. $\dfrac{x^{2n} - 1^{2n}}{x^n - 1^n} = x^n + 1^n$

80. $\dfrac{x^2 - 3x + 2}{x - 1} = x - 2$ for all values of x.

81. ***Think About It*** How do you determine whether a rational expression is in simplest form?