A.5 Solving Equations

▶ What you should learn

- How to identify different types of equations
- How to solve linear equations in one variable
- How to solve quadratic equations by factoring, extracting square roots, completing the square, and using the Quadratic Formula
- How to solve polynomial equations of degree three or greater
- How to solve equations involving radicals
- How to solve equations involving absolute values

▶ Why you should learn it

Linear equations are used in many real-life applications. For example, in Exercises 185 and 186 on page A57, linear equations can be used to model the relationship between the length of a thigh bone and the height of a person, helping researchers learn about ancient cultures.

Equations and Solutions of Equations

An **equation** in x is a statement that two algebraic expressions are equal. For example

$$3x - 5 = 7,\ x^2 - x - 6 = 0, \text{ and } \sqrt{2x} = 4$$

are equations. To **solve** an equation in x means to find all values of x for which the equation is true. Such values are **solutions.** For instance, $x = 4$ is a solution of the equation

$$3x - 5 = 7$$

because $3(4) - 5 = 7$ is a true statement.

The solutions of an equation depend on the kinds of numbers being considered. For instance, in the set of rational numbers, $x^2 = 10$ has no solution because there is no rational number whose square is 10. However, in the set of real numbers, the equation has the two solutions $\sqrt{10}$ and $-\sqrt{10}$.

An equation that is true for *every* real number in the domain of the variable is called an **identity.** For example

$$x^2 - 9 = (x + 3)(x - 3) \qquad \text{Identity}$$

is an identity because it is a true statement for any real value of x, and

$$\frac{x}{3x^2} = \frac{1}{3x} \qquad \text{Identity}$$

where $x \neq 0$, is an identity because it is true for any nonzero real value of x.

An equation that is true for just *some* (or even none) of the real numbers in the domain of the variable is called a **conditional equation.** For example, the equation

$$x^2 - 9 = 0 \qquad \text{Conditional equation}$$

is conditional because $x = 3$ and $x = -3$ are the only values in the domain that satisfy the equation. The equation $2x - 4 = 2x + 1$ is conditional because there are no real values of x for which the equation is true. Learning to solve conditional equations is the primary focus of this section.

Linear Equations in One Variable

Definition of Linear Equation

A **linear equation in one variable** x is an equation that can be written in the standard form

$$ax + b = 0$$

where a and b are real numbers with $a \neq 0$.

A linear equation has exactly one solution. To see this, consider the following steps. (Remember that $a \neq 0$.)

$ax + b = 0$	Write original equation.
$ax = -b$	Subtract b from each side.
$x = -\dfrac{b}{a}$	Divide each side by a.

To solve a conditional equation in x, isolate x on one side of the equation by a sequence of **equivalent** (and usually simpler) **equations,** each having the same solution(s) as the original equation. The operations that yield equivalent equations come from the Substitution Principle and simplification techniques.

Generating Equivalent Equations

An equation can be transformed into an *equivalent equation* by one or more of the following steps.

	Given Equation	*Equivalent Equation*
1. Remove symbols of grouping, combine like terms, or simplify fractions on one or both sides of the equation.	$2x - x = 4$	$x = 4$
2. Add (or subtract) the same quantity to (from) *each* side of the equation.	$x + 1 = 6$	$x = 5$
3. Multiply (or divide) *each* side of the equation by the same *nonzero* quantity.	$2x = 6$	$x = 3$
4. Interchange the two sides of the equation.	$2 = x$	$x = 2$

STUDY TIP

After solving an equation, you should check each solution in the original equation. For instance, you can check the solution to Example 1(a) as follows.

$3x - 6 = 0$	Write original equation.
$3(2) - 6 \stackrel{?}{=} 0$	Substitute 2 for x.
$0 = 0$	Solution checks. ✓

Try checking the solution to Example 1(b).

Example 1 ▶ Solving a Linear Equation

a. $3x - 6 = 0$	Original equation
$3x = 6$	Add 6 to each side.
$x = 2$	Divide each side by 3.
b. $5x + 4 = 3x - 8$	Original equation
$2x + 4 = -8$	Subtract $3x$ from each side.
$2x = -12$	Subtract 4 from each side.
$x = -6$	Divide each side by 2.

To solve an equation involving fractional expressions, find the least common denominator (LCD) of all terms and multiply every term by the LCD.

Example 2 ▶ An Equation Involving Fractional Expressions

Solve $\frac{x}{3} + \frac{3x}{4} = 2$.

Solution

$$\frac{x}{3} + \frac{3x}{4} = 2 \qquad \text{Write original equation.}$$

$$(12)\frac{x}{3} + (12)\frac{3x}{4} = (12)2 \qquad \text{Multiply each term by the LCD of 12.}$$

$$4x + 9x = 24 \qquad \text{Divide out and multiply.}$$

$$13x = 24 \qquad \text{Combine like terms.}$$

$$x = \frac{24}{13} \qquad \text{Divide each side by 13.}$$

The solution is $x = \frac{24}{13}$. Check this in the original equation.

When multiplying or dividing an equation by a *variable* quantity, it is possible to introduce an extraneous solution. An **extraneous solution** is one that does not satisfy the original equation.

Example 3 ▶ An Equation with an Extraneous Solution

Solve $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$.

Solution

The LCD is $x^2 - 4$, or $(x + 2)(x - 2)$. Multiply each term by this LCD.

$$\frac{1}{x-2}(x+2)(x-2) = \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2)$$

$$x + 2 = 3(x - 2) - 6x, \qquad x \neq \pm 2$$

$$x + 2 = 3x - 6 - 6x$$

$$x + 2 = -3x - 6$$

$$4x = -8$$

$$x = -2$$

In the original equation, $x = -2$ yields a denominator of zero. So, $x = -2$ is an extraneous solution, and the original equation has *no solution*.

Quadratic Equations

A **quadratic equation** in x is an equation that can be written in the general form

$$ax^2 + bx + c = 0$$

where a, b, and c are real numbers, with $a \neq 0$. A quadratic equation in x is also known as a **second-degree polynomial equation** in x.

You should be familiar with the following four methods of solving quadratic equations.

Solving a Quadratic Equation

Factoring: If $ab = 0$, then $a = 0$ or $b = 0$.

Example:

$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0 \quad \Rightarrow \quad x = 3$$
$$x + 2 = 0 \quad \Rightarrow \quad x = -2$$

Square Root Principle: If $u^2 = c$, where $c > 0$, then $u = \pm\sqrt{c}$.

Example:

$$(x + 3)^2 = 16$$
$$x + 3 = \pm 4$$
$$x = -3 \pm 4$$
$$x = 1 \quad \text{or} \quad x = -7$$

Completing the Square: If $x^2 + bx = c$, then

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$
$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}.$$

Example:

$$x^2 + 6x = 5$$
$$x^2 + 6x + 3^2 = 5 + 3^2$$
$$(x + 3)^2 = 14$$
$$x + 3 = \pm\sqrt{14}$$
$$x = -3 \pm \sqrt{14}$$

Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example:

$$2x^2 + 3x - 1 = 0$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{17}}{4}$$

STUDY TIP

The Square Root Principle is also referred to as *extracting square roots*.

STUDY TIP

You can solve every quadratic equation by completing the square or using the Quadratic Formula.

Example 4 ▶ Solving a Quadratic Equation by Factoring

a. $2x^2 + 9x + 7 = 3$ — Original equation

$2x^2 + 9x + 4 = 0$ — Write in general form.

$(2x + 1)(x + 4) = 0$ — Factor.

$2x + 1 = 0 \implies x = -\frac{1}{2}$ — Set 1st factor equal to 0.

$x + 4 = 0 \implies x = -4$ — Set 2nd factor equal to 0.

The solutions are $x = -\frac{1}{2}$ and $x = -4$. Check these in the original equation.

b. $6x^2 - 3x = 0$ — Original equation

$3x(2x - 1) = 0$ — Factor.

$3x = 0 \implies x = 0$ — Set 1st factor equal to 0.

$2x - 1 = 0 \implies x = \frac{1}{2}$ — Set 2nd factor equal to 0.

The solutions are $x = 0$ and $x = \frac{1}{2}$. Check these in the original equation.

Note that the method of solution in Example 4 is based on the Zero-Factor Property from Section A.1. Be sure you see that this property works *only* for equations written in general form (in which the right side of the equation is zero). So, all terms must be collected on one side *before* factoring. For instance, in the equation

$$(x - 5)(x + 2) = 8$$

it is *incorrect* to set each factor equal to 8. Try to solve this equation correctly.

Example 5 ▶ Extracting Square Roots

Solve each equation by extracting square roots.

a. $4x^2 = 12$ **b.** $(x - 3)^2 = 7$

Solution

a. $4x^2 = 12$ — Write original equation.

$x^2 = 3$ — Divide each side by 4.

$x = \pm\sqrt{3}$ — Extract square roots.

The solutions are $x = \sqrt{3}$ and $x = -\sqrt{3}$. Check these in the original equation.

b. $(x - 3)^2 = 7$ — Write original equation.

$x - 3 = \pm\sqrt{7}$ — Extract square roots.

$x = 3 \pm \sqrt{7}$ — Add 3 to each side.

The solutions are $x = 3 \pm \sqrt{7}$. Check these in the original equation.

Example 6 ▶ The Quadratic Formula: Two Distinct Solutions

Use the Quadratic Formula to solve

$$x^2 + 3x = 9.$$

Solution

$x^2 + 3x = 9$	Write original equation.
$x^2 + 3x - 9 = 0$	Write in general form.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$x = \dfrac{-3 \pm \sqrt{(3)^2 - 4(1)(-9)}}{2(1)}$	Substitute $a = 1$, $b = 3$, and $c = -9$.
$x = \dfrac{-3 \pm \sqrt{45}}{2}$	Simplify.
$x = \dfrac{-3 \pm 3\sqrt{5}}{2}$	Simplify.

The equation has two solutions:

$$x = \frac{-3 + 3\sqrt{5}}{2} \quad \text{and} \quad x = \frac{-3 - 3\sqrt{5}}{2}.$$

Check these in the original equation.

Example 7 ▶ The Quadratic Formula: One Solution

Use the Quadratic Formula to solve

$$8x^2 - 24x + 18 = 0.$$

Solution

$8x^2 - 24x + 18 = 0$	Write original equation.
$4x^2 - 12x + 9 = 0$	Divide out common factor of 2.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic Formula
$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$	Substitute.
$x = \dfrac{12 \pm \sqrt{0}}{8}$	Simplify.
$x = \dfrac{3}{2}$	Simplify.

This quadratic equation has only one solution: $x = \frac{3}{2}$. Check this in the original equation.

STUDY TIP

A common mistake that is made in solving an equation such as that in Example 8 is to divide each side of the equation by the variable factor x^2. This loses the solution $x = 0$. When solving an equation, always write the equation in general form, then factor the equation and set each factor equal to zero. Do not divide each side of an equation by a variable factor in an attempt to simplify the equation.

Polynomial Equations of Higher Degree

The methods used to solve quadratic equations can sometimes be extended to polynomials of higher degree.

Example 8 ▶ Solving a Polynomial Equation by Factoring

Solve $3x^4 = 48x^2$.

Solution

First write the polynomial equation in general form with zero on one side, factor the other side, and then set each factor equal to zero.

$3x^4 = 48x^2$		Write original equation.
$3x^4 - 48x^2 = 0$		Write in general form.
$3x^2(x^2 - 16) = 0$		Factor out common factor.
$3x^2(x + 4)(x - 4) = 0$		Write in factored form.
$3x^2 = 0$	➡ $x = 0$	Set 1st factor equal to 0.
$x + 4 = 0$	➡ $x = -4$	Set 2nd factor equal to 0.
$x - 4 = 0$	➡ $x = 4$	Set 3rd factor equal to 0.

You can check these solutions by substituting in the original equation, as follows.

Check

$3x^4 = 48x^2$	Write original equation.
$3(0)^4 = 48(0)^2$	0 checks. ✓
$3(-4)^4 = 48(-4)^2$	-4 checks. ✓
$3(4)^4 = 48(4)^2$	4 checks. ✓

So, you can conclude that the solutions are $x = 0$, $x = -4$, and $x = 4$.

Example 9 ▶ Solving a Polynomial Equation by Factoring

Solve $x^3 - 3x^2 - 3x + 9 = 0$.

Solution

$x^3 - 3x^2 - 3x + 9 = 0$		Write original equation.
$x^2(x - 3) - 3(x - 3) = 0$		Factor by grouping.
$(x - 3)(x^2 - 3) = 0$		Distributive Property
$x - 3 = 0$	➡ $x = 3$	Set 1st factor equal to 0.
$x^2 - 3 = 0$	➡ $x = \pm\sqrt{3}$	Set 2nd factor equal to 0.

The solutions are $x = 3$, $x = \sqrt{3}$, and $x = -\sqrt{3}$.

Equations Involving Radicals

The steps involved in solving the remaining equations in this section will often introduce *extraneous solutions.* Extraneous solutions occur during operations such as squaring each side of an equation, raising each side of an equation to a rational power, and multiplying each side by a variable quantity. So, when you use any of these operations, checking is crucial.

STUDY TIP

The essential operations in Example 10 are isolating the square root and squaring each side. In Example 11, this is equivalent to isolating the factor with the rational exponent and raising each side to the *reciprocal power.*

Example 10 ▶ Solving Equations Involving Radicals

a.

$\sqrt{2x+7} - x = 2$	Original equation
$\sqrt{2x+7} = x + 2$	Isolate radical.
$2x + 7 = x^2 + 4x + 4$	Square each side.
$0 = x^2 + 2x - 3$	Write in general form.
$0 = (x+3)(x-1)$	Factor.
$x + 3 = 0 \Rightarrow x = -3$	Set 1st factor equal to 0.
$x - 1 = 0 \Rightarrow x = 1$	Set 2nd factor equal to 0.

By checking these values, you can determine that the only solution is $x = 1$.

b.

$\sqrt{2x-5} - \sqrt{x-3} = 1$	Original equation
$\sqrt{2x-5} = \sqrt{x-3} + 1$	Isolate $\sqrt{2x-5}$.
$2x - 5 = x - 3 + 2\sqrt{x-3} + 1$	Square each side.
$2x - 5 = x - 2 + 2\sqrt{x-3}$	Combine like terms.
$x - 3 = 2\sqrt{x-3}$	Isolate $2\sqrt{x-3}$.
$x^2 - 6x + 9 = 4(x-3)$	Square each side.
$x^2 - 10x + 21 = 0$	Write in general form.
$(x-3)(x-7) = 0$	Factor.
$x - 3 = 0 \Rightarrow x = 3$	Set 1st factor equal to 0.
$x - 7 = 0 \Rightarrow x = 7$	Set 2nd factor equal to 0.

The solutions are $x = 3$ and $x = 7$. Check these in the original equation.

Example 11 ▶ Solving an Equation Involving a Rational Exponent

$(x-4)^{2/3} = 25$	Original equation
$x - 4 = 25^{3/2}$	Raise each side to the $\frac{3}{2}$ power.
$x - 4 = 125$	Simplify.
$x = 129$	Add 4 to each side.

The solution is $x = 129$. Check this in the original equation.

Equations with Absolute Values

To solve an equation involving an absolute value, remember that the expression inside the absolute value signs can be positive or negative. This results in *two* separate equations, each of which must be solved. For instance, the equation

$$|x - 2| = 3$$

results in the two equations $x - 2 = 3$ and $-(x - 2) = 3$, which implies that the equation has two solutions: $x = 5$ and $x = -1$.

Example 12 ▶ Solving an Equation Involving Absolute Value

Solve $|x^2 - 3x| = -4x + 6$.

Solution

Because the variable expression inside the absolute value signs can be positive or negative, you must solve the following two equations.

First Equation

$x^2 - 3x = -4x + 6$		Use positive expression.
$x^2 + x - 6 = 0$		Write in general form.
$(x + 3)(x - 2) = 0$		Factor.
$x + 3 = 0$	$\Rightarrow$ $x = -3$	Set 1st factor equal to 0.
$x - 2 = 0$	$\Rightarrow$ $x = 2$	Set 2nd factor equal to 0.

Second Equation

$-(x^2 - 3x) = -4x + 6$		Use negative expression.
$x^2 - 7x + 6 = 0$		Write in general form.
$(x - 1)(x - 6) = 0$		Factor.
$x - 1 = 0$	$\Rightarrow$ $x = 1$	Set 1st factor equal to 0.
$x - 6 = 0$	$\Rightarrow$ $x = 6$	Set 2nd factor equal to 0.

Check

$\lvert(-3)^2 - 3(-3)\rvert \stackrel{?}{=} -4(-3) + 6$	Substitute -3 for x.
$18 = 18$	-3 checks. ✓
$\lvert(2)^2 - 3(2)\rvert \stackrel{?}{=} -4(2) + 6$	Substitute 2 for x.
$2 \neq -2$	2 does not check.
$\lvert(1)^2 - 3(1)\rvert \stackrel{?}{=} -4(1) + 6$	Substitute 1 for x.
$2 = 2$	1 checks. ✓
$\lvert(6)^2 - 3(6)\rvert \stackrel{?}{=} -4(6) + 6$	Substitute 6 for x.
$18 \neq -18$	6 does not check.

The solutions are $x = -3$ and $x = 1$.

A.5

In Exercis identity c

1. $2(x$
2. $3(x$
3. $-6($
4. $3(x$
5. $4(x$
6. $-7($
7. $x^2 -$
8. $x^2 +$
9. $3 +$

In Exercis solution.

11. $x +$
13. $7 -$
15. $8x -$
17. $2(x$
18. $3(x$
19. $x -$
20. $9x -$
21. $\frac{5x}{4} +$
23. $\frac{3}{2}(z +$
24. $\frac{3x}{2} +$
25. 0.25
26. 0.60

In Exercis solution.

27. $x +$
28. $8(x$
29. $\frac{100}{}$
30. $\frac{17 +}{y}$
31. $\frac{5x -}{5x +}$

A.5 Exercises

In Exercises 1–10, determine whether the equation is an identity or a conditional equation.

1. $2(x - 1) = 2x - 2$
2. $3(x + 2) = 5x + 4$
3. $-6(x - 3) + 5 = -2x + 10$
4. $3(x + 2) - 5 = 3x + 1$
5. $4(x + 1) - 2x = 2(x + 2)$
6. $-7(x - 3) + 4x = 3(7 - x)$
7. $x^2 - 8x + 5 = (x - 4)^2 - 11$
8. $x^2 + 2(3x - 2) = x^2 + 6x - 4$
9. $3 + \frac{1}{x + 1} = \frac{4x}{x + 1}$
10. $\frac{5}{x} + \frac{3}{x} = 24$

In Exercises 11–26, solve the equation and check your solution.

11. $x + 11 = 15$
12. $7 - x = 19$
13. $7 - 2x = 25$
14. $7x + 2 = 23$
15. $8x - 5 = 3x + 20$
16. $7x + 3 = 3x - 17$
17. $2(x + 5) - 7 = 3(x - 2)$
18. $3(x + 3) = 5(1 - x) - 1$
19. $x - 3(2x + 3) = 8 - 5x$
20. $9x - 10 = 5x + 2(2x - 5)$
21. $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$
22. $\frac{x}{5} - \frac{x}{2} = 3 + \frac{3x}{10}$
23. $\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24) = 0$
24. $\frac{3x}{2} + \frac{1}{4}(x - 2) = 10$
25. $0.25x + 0.75(10 - x) = 3$
26. $0.60x + 0.40(100 - x) = 50$

In Exercises 27–48, solve the equation and check your solution. (If not possible, explain why.)

27. $x + 8 = 2(x - 2) - x$
28. $8(x + 2) - 3(2x + 1) = 2(x + 5)$
29. $\frac{100 - 4x}{3} = \frac{5x + 6}{4} + 6$
30. $\frac{17 + y}{y} + \frac{32 + y}{y} = 100$
31. $\frac{5x - 4}{5x + 4} = \frac{2}{3}$
32. $\frac{10x + 3}{5x + 6} = \frac{1}{2}$
33. $10 - \frac{13}{x} = 4 + \frac{5}{x}$
34. $\frac{15}{x} - 4 = \frac{6}{x} + 3$
35. $\frac{x}{x + 4} + \frac{4}{x + 4} + 2 = 0$
36. $3 = 2 + \frac{2}{z + 2}$
37. $\frac{1}{x} + \frac{2}{x - 5} = 0$
38. $\frac{7}{2x + 1} - \frac{8x}{2x - 1} = -4$
39. $\frac{2}{(x - 4)(x - 2)} = \frac{1}{x - 4} + \frac{2}{x - 2}$
40. $\frac{4}{x - 1} + \frac{6}{3x + 1} = \frac{15}{3x + 1}$
41. $\frac{1}{x - 3} + \frac{1}{x + 3} = \frac{10}{x^2 - 9}$
42. $\frac{1}{x - 2} + \frac{3}{x + 3} = \frac{4}{x^2 + x - 6}$
43. $\frac{3}{x^2 - 3x} + \frac{4}{x} = \frac{1}{x - 3}$
44. $\frac{6}{x} - \frac{2}{x + 3} = \frac{3(x + 5)}{x^2 + 3x}$
45. $(x + 2)^2 + 5 = (x + 3)^2$
46. $(x + 1)^2 + 2(x - 2) = (x + 1)(x - 2)$
47. $(x + 2)^2 - x^2 = 4(x + 1)$
48. $(2x + 1)^2 = 4(x^2 + x + 1)$

In Exercises 49–54, write the quadratic equation in general form.

49. $2x^2 = 3 - 8x$
50. $x^2 = 16x$
51. $(x - 3)^2 = 3$
52. $13 - 3(x + 7)^2 = 0$
53. $\frac{1}{5}(3x^2 - 10) = 18x$
54. $x(x + 2) = 5x^2 + 1$

In Exercises 55–68, solve the quadratic equation by factoring.

55. $6x^2 + 3x = 0$
56. $9x^2 - 1 = 0$
57. $x^2 - 2x - 8 = 0$
58. $x^2 - 10x + 9 = 0$
59. $x^2 + 10x + 25 = 0$
60. $4x^2 + 12x + 9 = 0$
61. $3 + 5x - 2x^2 = 0$
62. $2x^2 = 19x + 33$
63. $x^2 + 4x = 12$
64. $-x^2 + 8x = 12$

65. $\frac{3}{4}x^2 + 8x + 20 = 0$ **66.** $\frac{1}{8}x^2 - x - 16 = 0$

67. $x^2 + 2ax + a^2 = 0$ **68.** $(x + a)^2 - b^2 = 0$

In Exercises 69–82, solve the equation by extracting square roots. List both the exact solution and the decimal solution rounded to two decimal places.

69. $x^2 = 49$ **70.** $x^2 = 169$

71. $x^2 = 11$ **72.** $x^2 = 32$

73. $3x^2 = 81$ **74.** $9x^2 = 36$

75. $(x - 12)^2 = 16$ **76.** $(x + 13)^2 = 25$

77. $(x + 2)^2 = 14$ **78.** $(x - 5)^2 = 30$

79. $(2x - 1)^2 = 18$ **80.** $(4x + 7)^2 = 44$

81. $(x - 7)^2 = (x + 3)^2$ **82.** $(x + 5)^2 = (x + 4)^2$

In Exercises 83–92, solve the quadratic equation by completing the square.

83. $x^2 - 2x = 0$ **84.** $x^2 + 4x = 0$

85. $x^2 + 4x - 32 = 0$ **86.** $x^2 - 2x - 3 = 0$

87. $x^2 + 6x + 2 = 0$ **88.** $x^2 + 8x + 14 = 0$

89. $9x^2 - 18x = -3$ **90.** $9x^2 - 12x = 14$

91. $8 + 4x - x^2 = 0$ **92.** $4x^2 - 4x - 99 = 0$

In Exercises 93–116, use the Quadratic Formula to solve the equation.

93. $2x^2 + x - 1 = 0$ **94.** $2x^2 - x - 1 = 0$

95. $16x^2 + 8x - 3 = 0$ **96.** $25x^2 - 20x + 3 = 0$

97. $2 + 2x - x^2 = 0$ **98.** $x^2 - 10x + 22 = 0$

99. $x^2 + 14x + 44 = 0$ **100.** $6x = 4 - x^2$

101. $x^2 + 8x - 4 = 0$ **102.** $4x^2 - 4x - 4 = 0$

103. $12x - 9x^2 = -3$ **104.** $16x^2 + 22 = 40x$

105. $9x^2 + 24x + 16 = 0$

106. $36x^2 + 24x - 7 = 0$

107. $4x^2 + 4x = 7$ **108.** $16x^2 - 40x + 5 = 0$

109. $28x - 49x^2 = 4$ **110.** $3x + x^2 - 1 = 0$

111. $8t = 5 + 2t^2$ **112.** $25h^2 + 80h + 61 = 0$

113. $(y - 5)^2 = 2y$ **114.** $(z + 6)^2 = -2z$

115. $\frac{1}{2}x^2 + \frac{3}{8}x = 2$ **116.** $(\frac{5}{7}x - 14)^2 = 8x$

In Exercises 117–124, use the Quadratic Formula to solve the equation. (Round your answers to three decimal places.)

117. $5.1x^2 - 1.7x - 3.2 = 0$

118. $2x^2 - 2.50x - 0.42 = 0$

119. $-0.067x^2 - 0.852x + 1.277 = 0$

120. $-0.005x^2 + 0.101x - 0.193 = 0$

121. $422x^2 - 506x - 347 = 0$

122. $1100x^2 + 326x - 715 = 0$

123. $12.67x^2 + 31.55x + 8.09 = 0$

124. $-3.22x^2 - 0.08x + 28.651 = 0$

In Exercises 125–134, solve the equation using a convenient method.

125. $x^2 - 2x - 1 = 0$ **126.** $11x^2 + 33x = 0$

127. $(x + 3)^2 = 81$ **128.** $x^2 - 14x + 49 = 0$

129. $x^2 - x - \frac{11}{4} = 0$ **130.** $x^2 + 3x - \frac{3}{4} = 0$

131. $(x + 1)^2 = x^2$ **132.** $a^2x^2 - b^2 = 0$

133. $3x + 4 = 2x^2 - 7$

134. $4x^2 + 2x + 4 = 2x + 8$

In Exercises 135–152, find all solutions of the equation. Check your solutions in the original equation.

135. $4x^4 - 18x^2 = 0$ **136.** $20x^3 - 125x = 0$

137. $x^4 - 81 = 0$ **138.** $x^6 - 64 = 0$

139. $x^3 + 216 = 0$ **140.** $27x^3 - 512 = 0$

141. $5x^3 + 30x^2 + 45x = 0$

142. $9x^4 - 24x^3 + 16x^2 = 0$

143. $x^3 - 3x^2 - x + 3 = 0$

144. $x^3 + 2x^2 + 3x + 6 = 0$

145. $x^4 - x^3 + x - 1 = 0$

146. $x^4 + 2x^3 - 8x - 16 = 0$

147. $x^4 - 4x^2 + 3 = 0$

148. $x^4 + 5x^2 - 36 = 0$

149. $4x^4 - 65x^2 + 16 = 0$

150. $36t^4 + 29t^2 - 7 = 0$

151. $x^6 + 7x^3 - 8 = 0$

152. $x^6 + 3x^3 + 2 = 0$

In Exercises 153–170, find all solutions of the equation. Check your solutions in the original equation.

153. $\sqrt{2x} - 10 = 0$ **154.** $4\sqrt{x} - 3 = 0$

155. $\sqrt{x - 10} - 4 = 0$ **156.** $\sqrt{5 - x} - 3 = 0$

157. $\sqrt[3]{2x + 5} + 3 = 0$ **158.** $\sqrt[3]{3x + 1} - 5 = 0$

159. $-\sqrt{26 - 11x} + 4 = x$

160. $x + \sqrt{31 - 9x} = 5$

161. $\sqrt{x + 1} = \sqrt{3x + 1}$

162. $\sqrt{x+5} = \sqrt{x-5}$

163. $(x-5)^{3/2} = 8$ **164.** $(x+3)^{3/2} = 8$

165. $(x+3)^{2/3} = 8$ **166.** $(x+2)^{2/3} = 9$

167. $(x^2-5)^{3/2} = 27$

168. $(x^2-x-22)^{3/2} = 27$

169. $3x(x-1)^{1/2} + 2(x-1)^{3/2} = 0$

170. $4x^2(x-1)^{1/3} + 6x(x-1)^{4/3} = 0$

In Exercises 171–184, find all solutions of the equation. Check your solutions in the original equation.

171. $\dfrac{20-x}{x} = x$ **172.** $\dfrac{4}{x} - \dfrac{5}{3} = \dfrac{x}{6}$

173. $\dfrac{1}{x} - \dfrac{1}{x+1} = 3$ **174.** $\dfrac{x}{x^2-4} + \dfrac{1}{x+2} = 3$

175. $x = \dfrac{3}{x} + \dfrac{1}{2}$ **176.** $4x + 1 = \dfrac{3}{x}$

177. $\dfrac{4}{x+1} - \dfrac{3}{x+2} = 1$ **178.** $\dfrac{x+1}{3} - \dfrac{x+1}{x+2} = 0$

179. $|2x-1| = 5$ **180.** $|3x+2| = 7$

181. $|x| = x^2 + x - 3$ **182.** $|x^2 + 6x| = 3x + 18$

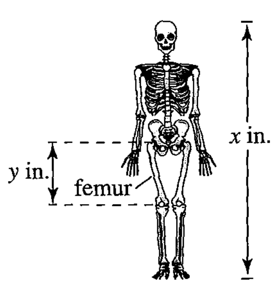
183. $|x+1| = x^2 - 5$ **184.** $|x-10| = x^2 - 10x$

Anthropology **In Exercises 185 and 186, use the following information. The relationship between the length of an adult's femur (thigh bone) and the height of the adult can be approximated by the linear equations**

$y = 0.432x - 10.44$ **Female**

$y = 0.449x - 12.15$ **Male**

where y is the length of the femur in inches and x is the height of the adult in inches (see figure).



185. An anthropologist discovers a femur belonging to an adult human female. The bone is 16 inches long. Estimate the height of the female.

186. From the foot bones of an adult human male, an anthropologist estimates that the person's height was 69 inches. A few feet away from the site where the foot bones were discovered, the anthropologist discovers a male adult femur that is 19 inches long. Is it likely that the foot bones and the femur came from the same person?

187. ***Operating Cost*** A delivery company has a fleet of vans. The annual operating cost C per van is

$$C = 0.32m + 2500$$

where m is the number of miles traveled by a van in a year. What number of miles will yield an annual operating cost of \$10,000?

188. ***Flood Control*** A river has risen 8 feet above its flood stage. The water begins to recede at a rate of 3 inches per hour. Write a mathematical model that shows the number of feet above flood stage after t hours. If the water continually recedes at this rate, when will the river be 1 foot above its flood stage?

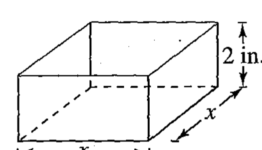
189. ***Floor Space*** The floor of a one-story building is 14 feet longer than it is wide. The building has 1632 square feet of floor space.

(a) Draw a diagram that gives a visual representation of the floor space. Represent the width as w and show the length in terms of w.

(b) Write a quadratic equation in terms of w.

(c) Find the length and width of the floor of the building.

190. ***Packaging*** An open box with a square base (see figure) is to be constructed from 84 square inches of material. The height of the box is 2 inches. What are the dimensions of the box? (*Hint:* The surface area is $S = x^2 + 4xh$.)



191. ***Geometry*** The hypotenuse of an isosceles right triangle is 5 centimeters long. How long are its sides?

192. ***Geometry*** An equilateral triangle has a height of 10 inches. How long is one of its sides? (*Hint:* Use the height of the triangle to partition the triangle into two congruent right triangles.)