A.6 Solving Inequalities

▶ What you should learn

- How to represent solutions of linear inequalities in one variable
- How to solve linear inequalities in one variable
- How to solve inequalities involving absolute values
- How to solve polynomial and rational inequalities

▶ Why you should learn it

Inequalities can be used to model and solve real-life problems. For instance, in Exercise 142 on page A69, you will use a linear inequality to analyze data about the maximum weight a weightlifter can bench press.

Introduction

Simple inequalities were reviewed in Section A.1. There, you used the inequality symbols $<$, $\leq$, $>$, and $\geq$ to compare two numbers and to denote subsets of real numbers. For instance, the simple inequality

$$x \geq 3$$

denotes all real numbers x that are greater than or equal to 3.

In this section you will expand your work with inequalities to include more involved statements such as

$$5x - 7 < 3x + 9$$

and

$$-3 \leq 6x - 1 < 3.$$

As with an equation, you **solve an inequality** in the variable x by finding all values of x for which the inequality is true. Such values are **solutions** and are said to **satisfy** the inequality. The set of all real numbers that are solutions of an inequality is the **solution set** of the inequality. For instance, the solution set of

$$x + 1 < 4$$

is all real numbers that are less than 3.

The set of all points on the real number line that represent the solution set is the **graph of the inequality.** Graphs of many types of inequalities consist of intervals on the real number line. See Section A.1 to review the nine basic types of intervals on the real number line. Note that each type of interval can be classified as *bounded* or *unbounded.*

Example 1 ▶ Intervals and Inequalities

Write an inequality to represent each interval, and state whether the interval is bounded or unbounded.

a. $(-3, 5]$
b. $(-3, \infty)$
c. $[0, 2]$
d. $(-\infty, \infty)$

Solution

a. $(-3, 5]$ corresponds to $-3 < x \leq 5$. Bounded
b. $(-3, \infty)$ corresponds to $-3 < x$. Unbounded
c. $[0, 2]$ corresponds to $0 \leq x \leq 2$. Bounded
d. $(-\infty, \infty)$ corresponds to $-\infty < x < \infty$. Unbounded

Properties of Inequalities

The procedures for solving linear inequalities in one variable are much like those for solving linear equations. To isolate the variable, you can make use of the **Properties of Inequalities.** These properties are similar to the properties of equality, but there are two important exceptions. When each side of an inequality is multiplied or divided by a negative number, the direction of the inequality symbol must be reversed. Here is an example.

$-2 < 5$	Original inequality
$(-3)(-2) > (-3)(5)$	Multiply each side by -3 and reverse inequality.
$6 > -15$	Simplify.

Two inequalities that have the same solution set are **equivalent.** For instance, the inequalities

$$x + 2 < 5$$

and

$$x < 3$$

are equivalent. To obtain the second inequality from the first, you can subtract 2 from each side of the inequality. The following list describes the operations that can be used to create equivalent inequalities.

Properties of Inequalities

Let a, b, c, and d be real numbers.

1. Transitive Property

$$a < b \text{ and } b < c \Longrightarrow a < c$$

2. Addition of Inequalities

$$a < b \text{ and } c < d \Longrightarrow a + c < b + d$$

3. Addition of a Constant

$$a < b \Longrightarrow a + c < b + c$$

4. Multiplication by a Constant

$$\text{For } c > 0,\ a < b \Longrightarrow ac < bc$$

$$\text{For } c < 0,\ a < b \Longrightarrow ac > bc$$

Each of the properties above is true if the symbol $<$ is replaced by $\le$ and the symbol $>$ is replaced by $\ge$. For instance, another form of the multiplication property would be as follows.

$$\text{For } c > 0,\ a \le b \Longrightarrow ac \le bc$$

$$\text{For } c < 0,\ a \le b \Longrightarrow ac \ge bc$$

Linear Inequalities

The simplest type of inequality is a **linear inequality** in one variable. For instance, $2x + 3 > 4$ is a linear inequality in x.

In the following examples, pay special attention to the steps in which the inequality symbol is reversed. Remember that when you multiply or divide by a negative number, you must reverse the inequality symbol.

STUDY TIP

Checking the solution set of an inequality is not as simple as checking the solutions of an equation. You can, however, get an indication of the validity of a solution set by substituting a few convenient values of x.

Example 2 ▶ Solving Linear Inequalities

Solve each inequality.

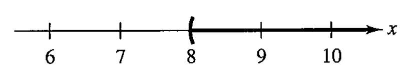
a. $5x - 7 > 3x + 9$

b. $1 - \dfrac{3x}{2} \geq x - 4$

Solution

a. $5x - 7 > 3x + 9$	Write original inequality.
$2x - 7 > 9$	Subtract $3x$ from each side.
$2x > 16$	Add 7 to each side.
$x > 8$	Divide each side by 2.

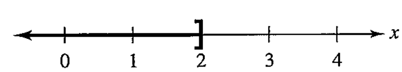
The solution set is all real numbers that are greater than 8, which is denoted by $(8, \infty)$. The graph of this solution set is shown in Figure A.7.



Solution interval: $(8, \infty)$
FIGURE A.7

b. $1 - \dfrac{3x}{2} \geq x - 4$	Write original inequality.
$2 - 3x \geq 2x - 8$	Multiply each side by 2.
$2 - 5x \geq -8$	Subtract $2x$ from each side.
$-5x \geq -10$	Subtract 2 from each side.
$x \leq 2$	Divide each side by -5 and reverse the inequality.

The solution set is all real numbers that are less than or equal to 2, which is denoted by $(-\infty, 2]$. The graph of this solution set is shown in Figure A.8.



Solution interval: $(-\infty, 2]$
FIGURE A.8

Sometimes it is possible to write two inequalities as a **double inequality.** For instance, you can write the two inequalities $-4 \le 5x - 2$ and $5x - 2 < 7$ more simply as

$$-4 \le 5x - 2 < 7.$$

This form allows you to solve the two inequalities together, as demonstrated in Example 3.

Example 3 ▶ Solving a Double Inequality

To solve a double inequality, you can isolate x as the middle term.

$$-3 \le 6x - 1 < 3 \qquad \text{Write original inequality.}$$

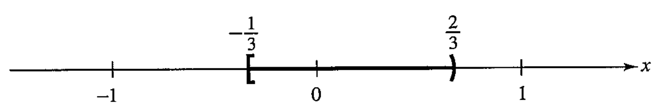
$$-3 + 1 \le 6x - 1 + 1 < 3 + 1 \qquad \text{Add 1 to each part.}$$

$$-2 \le 6x < 4 \qquad \text{Simplify.}$$

$$\frac{-2}{6} \le \frac{6x}{6} < \frac{4}{6} \qquad \text{Divide each part by 6.}$$

$$-\frac{1}{3} \le x < \frac{2}{3} \qquad \text{Simplify.}$$

The solution set is all real numbers that are greater than or equal to $-\frac{1}{3}$ and less than $\frac{2}{3}$, which is denoted by $\left[-\frac{1}{3}, \frac{2}{3}\right)$. The graph of this solution set is shown in Figure A.9.



Solution interval: $\left[-\frac{1}{3}, \frac{2}{3}\right)$

FIGURE A.9

The double inequality in Example 3 could have been solved in two parts as follows.

$$-3 \le 6x - 1 \quad \text{and} \quad 6x - 1 < 3$$

$$-2 \le 6x \qquad\qquad 6x < 4$$

$$-\frac{1}{3} \le x \qquad\qquad x < \frac{2}{3}$$

The solution set consists of all real numbers that satisfy *both* inequalities. In other words, the solution set is the set of all values of x for which

$$-\frac{1}{3} \le x < \frac{2}{3}.$$

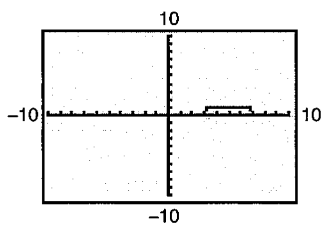
When combining two inequalities to form a double inequality, be sure that the inequalities satisfy the Transitive Property. For instance, it is *incorrect* to combine the inequalities $3 < x$ and $x \le -1$ as $3 < x \le -1$. This "inequality" is wrong because 3 is not less than -1.

Technology

A graphing utility can be used to identify the solution set of an inequality. For instance, to find the solution set of $|x - 5| < 2$ (see Example 4a), enter

$$\text{Y1} = \text{abs}(\text{X} - 5) < 2$$

and press the graph key. The graph should look like the one shown below.



The solution set is indicated by the line segment above the x-axis.

Inequalities Involving Absolute Values

Solving an Absolute Value Inequality

Let x be a variable or an algebraic expression and let a be a real number such that $a \geq 0$.

1. The solutions of $|x| < a$ are all values of x that lie between $-a$ and a.

   $|x| < a$ if and only if $-a < x < a$.

2. The solutions of $|x| > a$ are all values of x that are less than $-a$ or greater than a.

   $|x| > a$ if and only if $x < -a$ or $x > a$.

These rules are also valid if $<$ is replaced by $\leq$ and $>$ is replaced by $\geq$.

Example 4 ▶ Solving an Absolute Value Inequality

Solve each inequality.

a. $|x - 5| < 2$ **b.** $|x + 3| \geq 7$

Solution

a.

$\lvert x - 5\rvert < 2$	Write original inequality.
$-2 < x - 5 < 2$	Write equivalent inequalities.
$-2 + 5 < x - 5 + 5 < 2 + 5$	Add 5 to each part.
$3 < x < 7$	Simplify.

The solution set is all real numbers that are greater than 3 and less than 7, which is denoted by $(3, 7)$. The graph of this solution set is shown in Figure A.10.

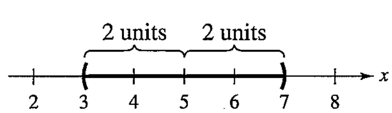
b.

$\lvert x + 3\rvert \geq 7$			Write original inequality.
$x + 3 \leq -7$	or	$x + 3 \geq 7$	Write equivalent inequalities.
$x + 3 - 3 \leq -7 - 3$		$x + 3 - 3 \geq 7 - 3$	Subtract 3 from each side.
$x \leq -10$		$x \geq 4$	Simplify.

The solution set is all real numbers that are less than or equal to -10 *or* greater than or equal to 4. The interval notation for this solution set is $(-\infty, -10] \cup [4, \infty)$. The symbol $\cup$ is called a *union* symbol and is used to denote the combining of two sets. The graph of this solution set is shown in Figure A.11.

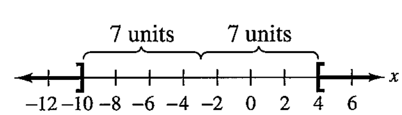
STUDY TIP

Note that the graph of the inequality $|x - 5| < 2$ can be described as all real numbers within two units of 5, as shown in Figure A.10.



$|x - 5| < 2$: *Solutions lie inside* $(3, 7)$

FIGURE A.10



$|x + 3| \geq 7$: *Solutions lie outside* $(-10, 4)$

FIGURE A.11

Other Types of Inequalities

To solve a polynomial inequality, you can use the fact that a polynomial can change signs only at its zeros (the x-values that make the polynomial equal to zero). Between two consecutive zeros, a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes. These zeros are the **critical numbers** of the inequality, and the resulting intervals are the **test intervals** for the inequality.

Example 5 ▶ Solving a Polynomial Inequality

Solve

$$x^2 - x - 6 < 0.$$

Solution

By factoring the polynomial as

$$x^2 - x - 6 = (x + 2)(x - 3)$$

you can see that the critical numbers are

$$x = -2 \quad \text{and} \quad x = 3.$$

So, the polynomial's test intervals are

$$(-\infty, -2), \quad (-2, 3), \quad \text{and} \quad (3, \infty). \qquad \text{Test intervals}$$

In each test interval, choose a representative x-value and evaluate the polynomial.

Interval	*x-Value*	*Polynomial Value*	*Conclusion*
$(-\infty, -2)$	$x = -3$	$(-3)^2 - (-3) - 6 = 6$	Positive
$(-2, 3)$	$x = 0$	$(0)^2 - (0) - 6 = -6$	Negative
$(3, \infty)$	$x = 4$	$(4)^2 - (4) - 6 = 6$	Positive

From this you can conclude that the inequality is satisfied for all x-values in $(-2, 3)$. This implies that the solution of the inequality $x^2 - x - 6 < 0$ is the interval $(-2, 3)$, as shown in Figure A.12.

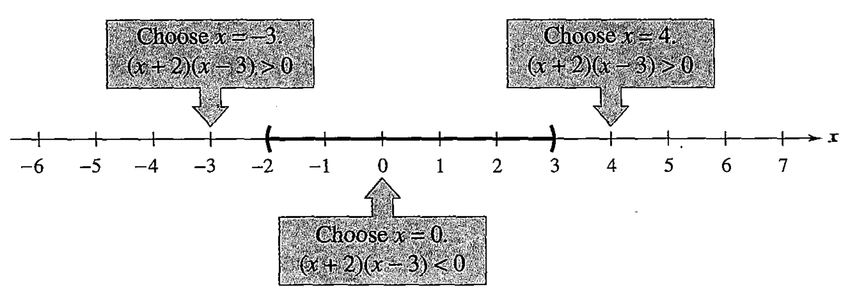


FIGURE A.12

STUDY TIP

As with linear inequalities, you can check the reasonableness of a solution by substituting x-values into the original inequality. For instance, to check the solution found in Example 5, try substituting several x-values from the interval $(-2, 3)$ into the inequality

$$x^2 - x - 6 < 0.$$

Regardless of which x-values you choose, the inequality should be satisfied.

The concepts of critical numbers and test intervals can be extended to rational inequalities. To do this, use the fact that the value of a rational expression can change sign only at its zeros (the x-values for which its numerator is zero) and its *undefined values* (the x-values for which its denominator is zero). These two types of numbers make up the *critical numbers* of a rational inequality.

Example 6 ▶ Solving a Rational Inequality

Solve $\dfrac{2x-7}{x-5} \le 3$.

Solution

Begin by writing the rational inequality in general form.

$$\frac{2x-7}{x-5} \le 3$$ Write original inequality.

$$\frac{2x-7}{x-5} - 3 \le 0$$ Write in general form.

$$\frac{2x-7-3x+15}{x-5} \le 0$$ Add fractions.

$$\frac{-x+8}{x-5} \le 0$$ Simplify.

Critical numbers: $x = 5, x = 8$ Zeros and undefined values of rational expression

Test intervals: $(-\infty, 5), (5, 8), (8, \infty)$

Test: Is $\dfrac{-x+8}{x-5} \le 0$?

After testing these intervals, as shown in Figure A.13, you can see that the inequality is satisfied on the open intervals $(-\infty, 5)$ and $(8, \infty)$. Moreover, because $(-x+8)/(x-5) = 0$ when $x = 8$, you can conclude that the solution set consists of all real numbers in the intervals $(-\infty, 5) \cup [8, \infty)$. (Be sure to use a closed interval to indicate that x can equal 8.)

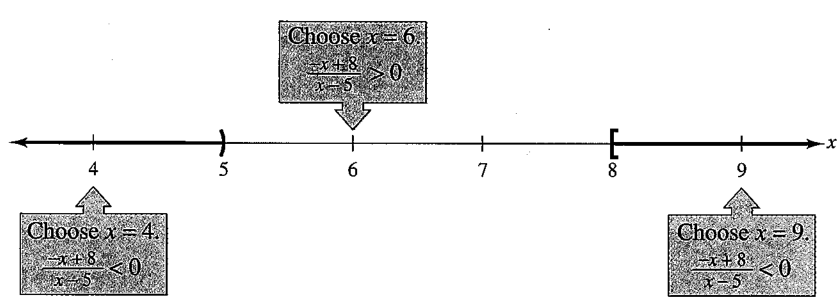


FIGURE A.13

A.6 Exercises

In Exercises 1–6, write an inequality that represents the interval, and state whether the interval is bounded or unbounded.

1. $[-1, 5]$ **2.** $(2, 10]$
3. $(11, \infty)$ **4.** $[-5, \infty)$
5. $(-\infty, -2)$ **6.** $(-\infty, 7]$

In Exercises 7–12, match the inequality with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

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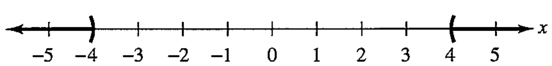
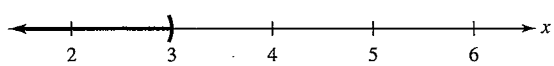


(b)


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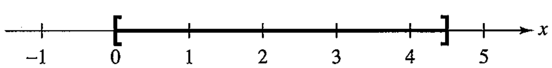

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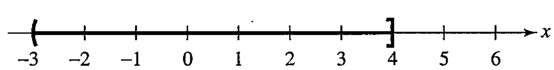

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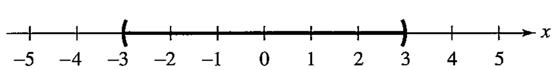

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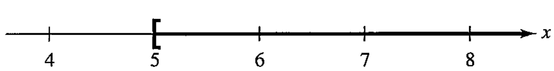


7. $x < 3$ **8.** $x \geq 5$
9. $-3 < x \leq 4$ **10.** $0 \leq x \leq \frac{9}{2}$
11. $|x| < 3$ **12.** $|x| > 4$

In Exercises 13–18, determine whether each value of *x* is a solution of the inequality.

Inequality	*Values*	
13. $5x - 12 > 0$	(a) $x = 3$	(b) $x = -3$
	(c) $x = \frac{5}{2}$	(d) $x = \frac{3}{2}$
14. $2x + 1 < -3$	(a) $x = 0$	(b) $x = -\frac{1}{4}$
	(c) $x = -4$	(d) $x = -\frac{3}{2}$
15. $0 < \dfrac{x-2}{4} < 2$	(a) $x = 4$	(b) $x = 10$
	(c) $x = 0$	(d) $x = \frac{7}{2}$
16. $-1 < \dfrac{3-x}{2} \leq 1$	(a) $x = 0$	(b) $x = -5$
	(c) $x = 1$	(d) $x = 5$
17. $|x - 10| \geq 3$	(a) $x = 13$	(b) $x = -1$
	(c) $x = 14$	(d) $x = 9$
18. $|2x - 3| < 15$	(a) $x = -6$	(b) $x = 0$
	(c) $x = 12$	(d) $x = 7$

In Exercises 19–44, solve the inequality and sketch the solution on the real number line. (Some inequalities have no solution.)

19. $4x < 12$ **20.** $10x < -40$
21. $-2x > -3$ **22.** $-6x > 15$
23. $x - 5 \geq 7$ **24.** $x + 7 \leq 12$
25. $2x + 7 < 3 + 4x$ **26.** $3x + 1 \geq 2 + x$
27. $2x - 1 \geq 1 - 5x$ **28.** $6x - 4 \leq 2 + 8x$
29. $4 - 2x < 3(3 - x)$ **30.** $4(x + 1) < 2x + 3$
31. $\frac{3}{4}x - 6 \leq x - 7$ **32.** $3 + \frac{2}{7}x > x - 2$
33. $\frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2}$ **34.** $9x - 1 < \frac{3}{4}(16x - 2)$
35. $3.6x + 11 \geq -3.4$
36. $15.6 - 1.3x < -5.2$
37. $1 < 2x + 3 < 9$
38. $-8 \leq -(3x + 5) < 13$
39. $-4 < \dfrac{2x-3}{3} < 4$ **40.** $0 \leq \dfrac{x+3}{2} < 5$
41. $\frac{3}{4} > x + 1 > \frac{1}{4}$ **42.** $-1 < 2 - \dfrac{x}{3} < 1$
43. $3.2 \leq 0.4x - 1 \leq 4.4$
44. $4.5 > \dfrac{1.5x + 6}{2} > 10.5$

In Exercises 45–60, solve the inequality and sketch the solution on the real number line. (Some inequalities have no solution.)

45. $|x| < 6$ **46.** $|x| > 4$
47. $\left|\dfrac{x}{2}\right| > 1$ **48.** $\left|\dfrac{x}{5}\right| > 3$
49. $|x - 5| < -1$ **50.** $|x - 5| \geq 0$
51. $|x - 20| \leq 6$ **52.** $|x - 7| < -5$
53. $|3 - 4x| \geq 9$ **54.** $|1 - 2x| < 5$
55. $\left|\dfrac{x-3}{2}\right| \geq 4$ **56.** $\left|1 - \dfrac{2x}{3}\right| < 1$
57. $|9 - 2x| - 2 < -1$ **58.** $|x + 14| + 3 > 17$
59. $2|x + 10| \geq 9$ **60.** $3|4 - 5x| \leq 9$

Graphical Analysis **In Exercises 61–68, use a graphing utility to graph the inequality and identify the solution set.**

61. $6x > 12$

62. $3x - 1 \leq 5$

63. $5 - 2x \geq 1$

64. $3(x + 1) < x + 7$

65. $|x - 8| \leq 14$

66. $|2x + 9| > 13$

67. $2|x + 7| \geq 13$

68. $\frac{1}{2}|x + 1| \leq 3$

In Exercises 69–74, find the interval(s) on the real number line for which the radicand is nonnegative (greater than or equal to zero).

69. $\sqrt{x - 5}$

70. $\sqrt{x - 10}$

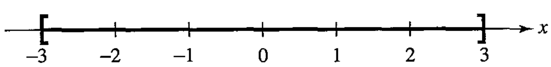
71. $\sqrt{x + 3}$

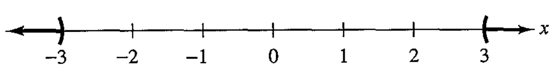
72. $\sqrt{3 - x}$

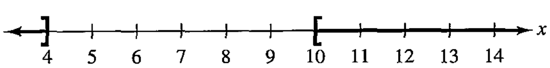
73. $\sqrt[4]{7 - 2x}$

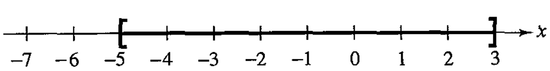
74. $\sqrt[4]{6x + 15}$

In Exercises 75–82, use absolute value notation to define the interval (or pair of intervals) on the real number line.

75. 

76. 

77. 

78. 

79. All real numbers within 10 units of 12

80. All real numbers at least 5 units from 8

81. All real numbers more than 5 units from -3

82. All real numbers no more than 7 units from -6

In Exercises 83–86, determine whether each value of x is a solution of the inequality.

Inequality	*Values*	
83. $x^2 - 3 < 0$	(a) $x = 3$	(b) $x = 0$
	(c) $x = \frac{3}{2}$	(d) $x = -5$
84. $x^2 - x - 12 \geq 0$	(a) $x = 5$	(b) $x = 0$
	(c) $x = -4$	(d) $x = -3$
85. $\dfrac{x + 2}{x - 4} \geq 3$	(a) $x = 5$	(b) $x = 4$
	(c) $x = -\frac{9}{2}$	(d) $x = \frac{9}{2}$
86. $\dfrac{3x^2}{x^2 + 4} < 1$	(a) $x = -2$	(b) $x = -1$
	(c) $x = 0$	(d) $x = 3$

In Exercises 87–90, find the critical numbers.

87. $2x^2 - x - 6$

88. $9x^3 - 25x^2$

89. $2 + \dfrac{3}{x - 5}$

90. $\dfrac{x}{x + 2} - \dfrac{2}{x - 1}$

In Exercises 91–106, solve the inequality and graph the solution on the real number line.

91. $x^2 \leq 9$

92. $x^2 < 36$

93. $(x + 2)^2 < 25$

94. $(x - 3)^2 \geq 1$

95. $x^2 + 4x + 4 \geq 9$

96. $x^2 - 6x + 9 < 16$

97. $x^2 + x < 6$

98. $x^2 + 2x > 3$

99. $x^2 + 2x - 3 < 0$

100. $x^2 - 4x - 1 > 0$

101. $x^2 + 8x - 5 \geq 0$

102. $-2x^2 + 6x + 15 \leq 0$

103. $x^3 - 3x^2 - x + 3 > 0$

104. $x^3 + 2x^2 - 4x - 8 \leq 0$

105. $x^3 - 2x^2 - 9x - 2 \geq -20$

106. $2x^3 + 13x^2 - 8x - 46 \geq 6$

In Exercises 107–112, solve the inequality and write the solution set in interval notation.

107. $4x^3 - 6x^2 < 0$

108. $4x^3 - 12x^2 > 0$

109. $x^3 - 4x \geq 0$

110. $2x^3 - x^4 \leq 0$

111. $(x - 1)^2(x + 2)^3 \geq 0$

112. $x^4(x - 3) \leq 0$

In Exercises 113–126, solve the inequality and graph the solution on the real number line.

113. $\dfrac{1}{x} - x > 0$

114. $\dfrac{1}{x} - 4 < 0$

115. $\dfrac{x + 6}{x + 1} - 2 < 0$

116. $\dfrac{x + 12}{x + 2} - 3 \geq 0$

117. $\dfrac{3x - 5}{x - 5} > 4$

118. $\dfrac{5 + 7x}{1 + 2x} < 4$

119. $\dfrac{4}{x+5} > \dfrac{1}{2x+3}$

120. $\dfrac{5}{x-6} > \dfrac{3}{x+2}$

121. $\dfrac{1}{x-3} \le \dfrac{9}{4x+3}$

122. $\dfrac{1}{x} \ge \dfrac{1}{x+3}$

123. $\dfrac{x^2+2x}{x^2-9} \le 0$

124. $\dfrac{x^2+x-6}{x} \ge 0$

125. $\dfrac{5}{x-1} - \dfrac{2x}{x+1} < 1$

126. $\dfrac{3x}{x-1} \le \dfrac{x}{x+4} + 3$

In Exercises 127–132, find the domain of x in the expression.

127. $\sqrt{4-x^2}$

128. $\sqrt{x^2-4}$

129. $\sqrt{x^2-7x+12}$

130. $\sqrt{144-9x^2}$

131. $\sqrt{\dfrac{x}{x^2-2x-35}}$

132. $\sqrt{\dfrac{x}{x^2-9}}$

In Exercises 133–138, solve the inequality. (Round your answers to two decimal places.)

133. $0.4x^2 + 5.26 < 10.2$

134. $-1.3x^2 + 3.78 > 2.12$

135. $-0.5x^2 + 12.5x + 1.6 > 0$

136. $1.2x^2 + 4.8x + 3.1 < 5.3$

137. $\dfrac{1}{2.3x-5.2} > 3.4$ **138.** $\dfrac{2}{3.1x-3.7} > 5.8$

139. ***Car Rental*** You can rent a midsize car from Company A for $250 per week with unlimited mileage. A similar car can be rented from Company B for $150 per week plus 25 cents for each mile driven. How many miles must you drive in a week in order for the rental fee for Company B to be greater than that for Company A?

140. ***Copying Costs*** Your department sends its copying to the photocopy center of your company. The center bills your department $0.10 per page. You have investigated the possibility of buying a departmental copier for $3000. With your own copier, the cost per page would be $0.03. The expected life of the copier is 4 years. How many copies must you make in the four-year period to justify buying the copier?

141. ***Investment*** In order for an investment of $1000 to grow to more than $1062.50 in 2 years, what must the annual interest rate be? $[A = P(1 + rt)]$

142. ***Athletics*** For 60 men enrolled in a weightlifting class, the relationship between body weight x (in pounds) and maximum bench-press weight y (in pounds) can be modeled by the equation $y = 1.266x - 35.766$. Use this model to estimate the range of body weights of the men in this group that can bench press more than 200 pounds.

143. ***Height*** The heights h of two-thirds of the members of a population satisfy the inequality

$$\left|\frac{h-68.5}{2.7}\right| \le 1$$

where h is measured in inches. Determine the interval on the real number line in which these heights lie.

144. ***Meteorology*** An electronic device is to be operated in an environment with relative humidity h in the interval defined by

$$|h - 50| \le 30.$$

What are the minimum and maximum relative humidities for the operation of this device?

145. ***Geometry*** A rectangular playing field with a perimeter of 100 meters is to have an area of at least 500 square meters. Within what bounds must the length of the rectangle lie?

146. ***Geometry*** A rectangular parking lot with a perimeter of 440 feet is to have an area of at least 8000 square feet. Within what bounds must the length of the rectangle lie?

147. ***Investment*** P dollars, invested at interest rate r compounded annually, increases to an amount

$$A = P(1 + r)^2$$

in 2 years. An investment of $1000 is to increase to an amount greater than $1100 in 2 years. The interest rate must be greater than what percent?

148. ***Cost, Revenue, and Profit*** The revenue and cost equations for a product are

$$R = x(50 - 0.0002x)$$

and

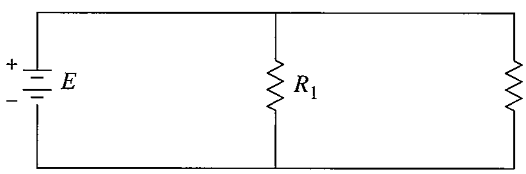
$$C = 12x + 150{,}000$$

where R and C are measured in dollars and x represents the number of units sold. How many units must be sold to obtain a profit of at least \$1,650,000?

149. ***Resistors*** When two resistors of resistance R_1 and R_2 are connected in parallel (see figure), the total resistance R satisfies the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Find R_1 for a parallel circuit in which $R_2 = 2$ ohms and R must be at least 1 ohm.



150. ***Safe Load*** The maximum safe load uniformly distributed over a one-foot section of a two-inch-wide wooden beam is approximated by the model

$$\text{Load} = 168.5d^2 - 472.1$$

where d is the depth of the beam.

(a) Evaluate the model for $d = 4$, $d = 6$, $d = 8$, $d = 10$, and $d = 12$. Use the results to create a bar graph.

(b) Determine the minimum depth of the beam that will safely support a load of 2000 pounds.

Synthesis

True or False? **In Exercises 151–154, determine whether the statement is true or false. Justify your answer.**

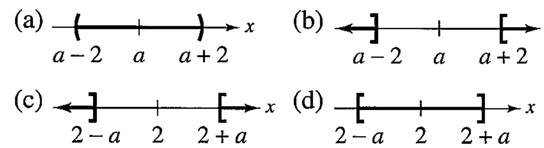
151. If a, b, and c are real numbers, and $a \le b$, then $ac \le bc$.

152. If $-10 \le x \le 8$, then $-10 \ge -x$ and $-x \ge -8$.

153. The zeros of the polynomial $x^3 - 2x^2 - 11x + 12 \ge 0$ divide the real number line into four test intervals.

154. The solution set of the inequality $\frac{3}{2}x^2 + 3x + 6 \ge 0$ is the set of real numbers.

155. Identify the graph of the inequality $|x - a| \ge 2$.



156. Find sets of values for a, b, and c such that $0 \le x \le 10$ is a solution of the inequality $|ax - b| \le c$.

157. ***Think About It*** The graph of $|x - 5| < 3$ can be described as all real numbers within 3 units of 5. Give a similar description of $|x - 10| < 8$.

158. ***Think About It*** The graph of $|x - 2| > 5$ can be described as all real numbers more than 5 units from 2. Give a similar description of $|x - 8| > 4$.

Exploration **In Exercises 159–162, find the interval for b such that the equation has at least one real solution.**

159. $x^2 + bx + 4 = 0$

160. $x^2 + bx - 4 = 0$

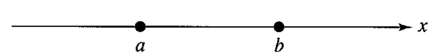
161. $3x^2 + bx + 10 = 0$

162. $2x^2 + bx + 5 = 0$

163. ***Conjecture*** Write a conjecture about the interval for b in Exercises 159–162. Explain your reasoning.

164. ***Think About It*** What is the center of the interval for b in Exercises 159–162?

165. Consider the polynomial $(x - a)(x - b)$ and the real number line shown below.



(a) Identify the points on the line at which the polynomial is zero.

(b) In each of the three subintervals of the line, write the sign of each factor and the sign of the product.

(c) For what x-values does the polynomial change signs?